

A Skeptic's Menagerie: Conflictors, Preemptors, Reinstaters, and Zombies in Nonmonotonic Inheritance

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Abstract

Subtle differences in the method of constructing arguments in inheritance systems can result in profound differences in both the conclusions reached and the efficiency of inference. This paper focuses on issues surrounding the defeat of arguments in nonmonotonic inheritance. Looking primarily at skeptical reasoners, we analyze several types of defeat that may be encountered, especially the defeat of defeaters. Finally, we raise some questions specific to networks that mix strict and defeasible links.

1 Introduction

In earlier work we presented a *skeptical* approach to nonmonotonic inheritance reasoning [4] that differed in several respects from Touretzky's original *credulous* approach [15]. The main difference is that conflicting paths such as the well-known Nixon diamond generate multiple extensions in a credulous reasoner, while a skeptical reasoner produces a single extension in which all conflicted paths are excluded. As discussed in our "Clash of Intuitions" paper [16], inheritance systems of either type may differ in several other technical respects, such as the direction in which arguments are extended in computing inheritance (upward vs. downward reasoning), the precise definition of the preemption relation, the treatment of negative information, and the admission of strict (as opposed to defeasible) links.

In this paper we analyze another major point of difference among nonmonotonic reasoners: the treatment of defeated paths, primarily in skeptical systems. We define several types of possible interactions among paths according to the types of defeat involved. This "skeptics's menagerie" provides new insights into inheritance reasoning, and helps us to evaluate the computational consequences of alternative axiomatizations.

2 Defeat in Inheritance Systems

Let Γ be an inheritance network containing defeasible positive or negative links of form $x \rightarrow y$ or $x \not\rightarrow y$. A path σ through a network is a sequence of links, e.g., the path $x \rightarrow y \not\rightarrow z$ is composed of the links $x \rightarrow y$ and $y \not\rightarrow z$. Let Φ be the extension of Γ containing all paths through Γ permitted by some upward skeptical inheritance rule, such as the one in [4]. By definition, $\Gamma \subseteq \Phi$. We say that Γ "permits" σ (written $\Gamma \vdash \sigma$) iff $\sigma \in \Phi$.

Definition 1 A path ρ is a *situator* of $x \rightarrow \tau \rightarrow w \not\rightarrow y$ with respect to $x \rightarrow \sigma \rightarrow z \rightarrow y$ iff ρ has the form $x \rightarrow \tau_1 \rightarrow w \rightarrow \tau_2 \rightarrow z$. (Similarly for paths of opposite polarity.)

A situator establishes the "betweenness" required by the inferential distance metric for preemption to hold. The above definition produces "off-path preemption" [16], originally proposed by Sandewall [11]. For "on-path preemption" we require a closer relationship between the subject path and the situator: $x \rightarrow \sigma \rightarrow z$ must be a subpath of ρ .

Definition 2 A path $x \rightarrow \sigma \rightarrow z \rightarrow y$ is *preempted* in Γ iff $\Gamma \vdash x \rightarrow \sigma \rightarrow z$ and there is a preemptor path $x \rightarrow \tau \rightarrow w \not\rightarrow y$ with a permitted situator. (Similarly for paths of opposite polarity.)

In the famous "Clyde the elephant" example, the path Clyde \rightarrow Elephant \rightarrow Gray is defeated by the preemptor Clyde \rightarrow Royal-Elephant $\not\rightarrow$ Gray, since Γ permits the situator Clyde \rightarrow Royal-Elephant \rightarrow Elephant. In our definition the situator must be a permitted path, but some inheritance schemes only require the two subpaths $x \rightarrow \tau_1 \rightarrow w$ and $w \rightarrow \tau_2 \rightarrow z$ to be permitted [2; 6]. Inheritance systems also disagree about whether the preemptor must be permitted; some require only the initial segment $x \rightarrow \tau \rightarrow w$ to be permitted.

Definition 3 A *conflictor* of a path $x \rightarrow \sigma \rightarrow y$ is a path $x \rightarrow \tau \not\rightarrow y$ having no permitted situator with respect to the former path. (Similarly for paths of opposite polarity.)

Definition 4 A path $x \rightarrow \sigma \rightarrow y$ is *conflicted* in Γ iff $\Gamma \vdash x \rightarrow \sigma$ and there is a conflictor $x \rightarrow \tau \not\rightarrow y$ such that $\Gamma \vdash x \rightarrow \tau$. (Similarly for paths of opposite polarity.)

In the classic Nixon diamond example, the conflicted paths are Nixon \rightarrow Quaker \rightarrow Pacifist and Nixon \rightarrow Republican $\not\rightarrow$ Pacifist. Each is a conflictor of the other because neither is situated with respect to the other.

Conflictors, being unsituated, always generate multiple extensions in a credulous theory, or no conclusion in a skeptical theory, unless they are preempted by a direct link. We view direct links as preemptors and not conflictors because a direct link $x \not\rightarrow y$ is always situated with respect to any path from x to y ; therefore it can never generate multiple extensions or no conclusion the way a true conflictor can. Returning to the elephant example, the direct link Royal-Elephant $\not\rightarrow$ Gray preempts the path Royal-Elephant \rightarrow Elephant \rightarrow Gray, but does not conflict with it.

Definition 5 An inheritance *defeater* is either a conflictor or a situated preemptor.

The concept of a defeater as a general category of paths that interfere with a particular argument is due originally to Pollock [19, 103]. Since his overall theoretical framework is so different from ours, however, it is hard to find any exact correspondence between his notion of a defeater and the one presented here.

The definition of inheritability varies depending on whether one is using upward or downward reasoning. For upward reasoning we have:

Definition 6 A path $x \rightarrow \sigma \rightarrow y \rightarrow z$ is *inheritable* iff $x \rightarrow \sigma \rightarrow y$ is permitted and $y \rightarrow z \in \Gamma$. (Similarly for negative paths.)

All links in Γ are permitted. Permission of longer paths is defined in terms of inheritability and defeat:

Definition 7 A path is *permitted* if it is inheritable and has no defeaters.

Finally, we need to define the conclusions an inheritance theory allows one to draw from a network:

Definition 8 Γ supports the conclusion $x \rightarrow y$ iff its extension contains some path $x \rightarrow \sigma \rightarrow y$. (Similarly for negative paths.)

3 Defeater Defeaters

Some inheritance systems prohibit defeated paths from themselves acting as defeaters. In such a system, the ability of a path to act as a defeater may itself be defeated by some other path. Thus, these systems are said to admit "defeater-defeaters," another term due to Pollock, and later used by Loui [7]. Since in these systems defeating a path prevents it from defeating other paths, adding a defeater-defeater to a nonmonotonic network can reinstate a previously defeated path. Given the two types of inheritance defeat defined in the preceding section, there are five types¹ of defeater-defeater in an inheritance system:

- preemptor-preemptor
- preemptor-conflictor
- conflictor-preemptor
- conflictor-conflictor
- situator-preemptor

These different types of defeater-defeater can have different effects in inheritance networks, and in some cases there is disagreement about the best way to handle them. We shall consider several of these issues in the following sections.

4 Reinstatement

Consider the network in Figure 1. The path $X \rightarrow \text{Chicken} \rightarrow \text{Bird} \rightarrow \text{Flies}$ is a preemptor of $X \rightarrow \text{Chicken} \rightarrow \text{Bird} \rightarrow \text{Flies}$; it is situated by $X \rightarrow \text{Chicken} \rightarrow \text{Bird}$. It is not necessary that $X \rightarrow \text{Chicken} \rightarrow \text{Flies}$ actually be a permitted path for it to be a preemptor. (In fact, $X \rightarrow \text{Chicken} \rightarrow \text{Flies}$ is defeated by the direct link $X \rightarrow \text{Flies}$.) But suppose we decided that preemptors must be permitted. This would cause the longer path to go through, since it has no *permitted* preemptor. In this case the path $X \rightarrow \text{Flies}$ would be acting as a "reinstater" of $X \rightarrow \text{Chicken} \rightarrow \text{Bird} \rightarrow \text{Flies}$.

¹This missing type, situator-conflictor, will be discussed in section 5.

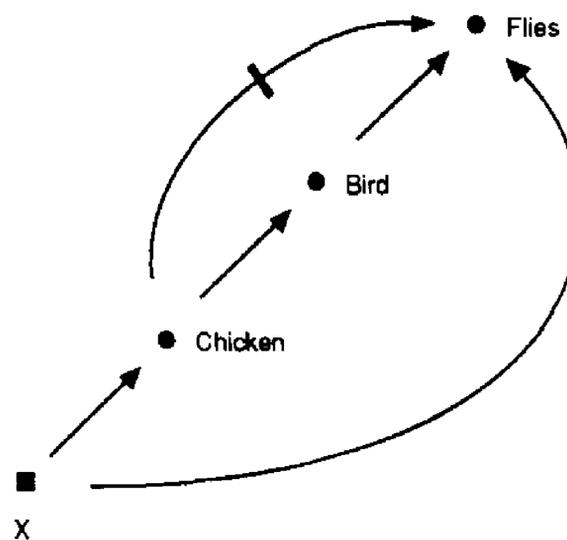


Figure 1; The intuitiveness of reinstatement seems to depend on the identity of the node X.

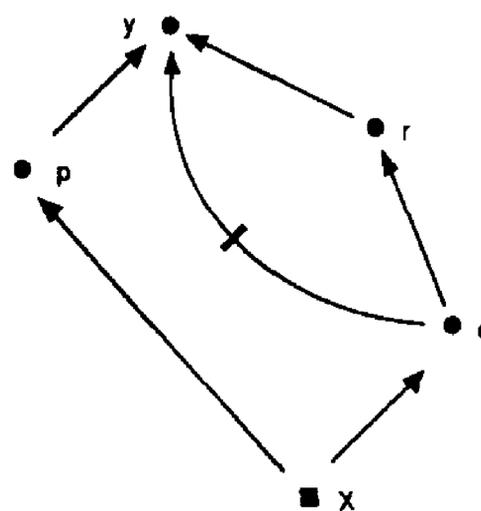


Figure 2: Conflictors can be reinstates only in credulous theories.

Definition 9 A *reinstater* is a path whose permission defeats preemptors of other paths to the same conclusion, thereby allowing them to also go through.

Reinstates are a type of defeater-defeater; specifically, they are preemptor defeaters. In skeptical systems, reinstates must be preemptor-preemptors, not preemptor-conflictors, since a reinstating path by definition must be permitted; in a skeptical system conflictors are never permitted. But in Figure 2, for example, note that $x \rightarrow p \rightarrow y$ is a conflictor of $x \rightarrow q \rightarrow y$, which is in turn a preemptor of $x \rightarrow q \rightarrow r \rightarrow y$. The path $x \rightarrow p \rightarrow y$ will be permitted in one of the two credulous extensions, and in that extension it can reinstate $x \rightarrow q \rightarrow r \rightarrow y$ if the inheritance axioms permit reinstatement.

Reinstatement may not be desirable in inheritance reasoning. Suppose node X in Figure 1 stands for the concept "chicken with a jet pack." We have directly asserted that chickens with jet packs can fly, but the reason for their flying has nothing to do with their being birds. Therefore the

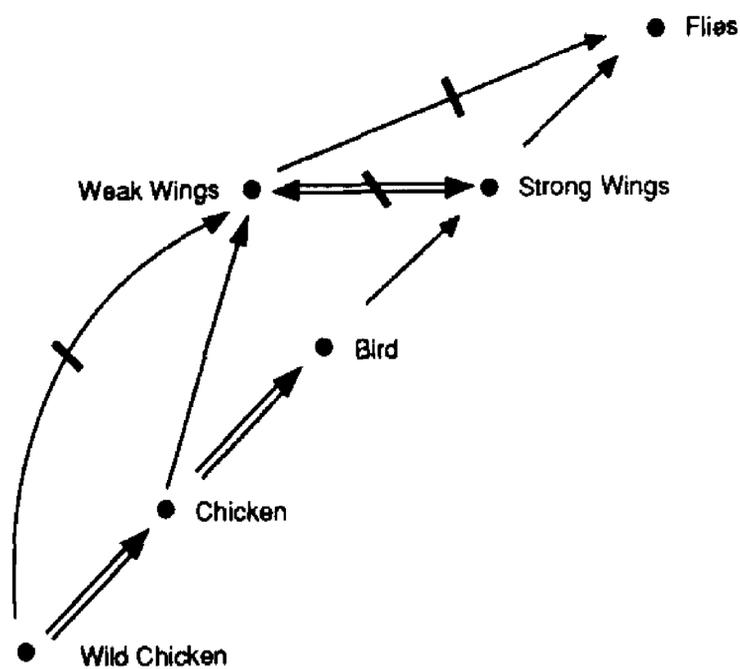


Figure 3: Wild chickens fly because they cancel an exceptional property of chickens.

direct link does not logically support the argument the reinstated path represents, which is that X can fly because birds fly. In general, the problem with reinstates is that they allow *any* defeated argument to go through if its conclusion holds, even if the argument's reasons are undercut by more specific information. Therefore we conclude that reinstates are undesirable.

To test our example further, suppose that X stands for the concept "wild chicken," and that wild chickens, having stronger wings than their domestic cousins, can fly. In this case the wild chicken's flying ability really is a consequence of its being a bird. This may seem to be evidence in favor of reinstatement, but what it really shows, we believe, is that the network of Figure 1 doesn't capture all of our knowledge about the relationship of wild chickens to ordinary chickens. In particular, it doesn't express the fact that the reason why wild chickens fly is that they cancel precisely those exceptional properties of chickens that prevent them from flying. Figure 3 shows one way to express this knowledge. The network mixes strict and defeasible links, and under the definition of inheritance given in [31], the path Wild-Chicken \Rightarrow Chicken \Rightarrow Bird \Rightarrow Strong-Wings \Rightarrow Flies is permitted. It is not, however, "reinstated" (as we have defined the term), as there is no preemptor that could have defeated it. This follows from the fact that no positive path from Wild-Chicken can reach Weak-Wings.

Reinstatement can affect not just the arguments a network permits, but also the conclusions it reaches. In Figure 4, if we do not allow reinstatement, the link $x \rightarrow o$ does not cause Γ to permit the situator path $x \rightarrow m \rightarrow n \rightarrow o$. Therefore the two paths $x \rightarrow m \not\rightarrow y$ and $x \rightarrow o \rightarrow y$ conflict, and so a skeptical reasoner will draw no conclusion about whether x is a y . But if reinstatement is allowed, the permitted situator $x \rightarrow m \rightarrow n \rightarrow o$ causes $x \rightarrow m \not\rightarrow y$ to preempt $x \rightarrow o \rightarrow y$, causing us to conclude $x \not\rightarrow y$.

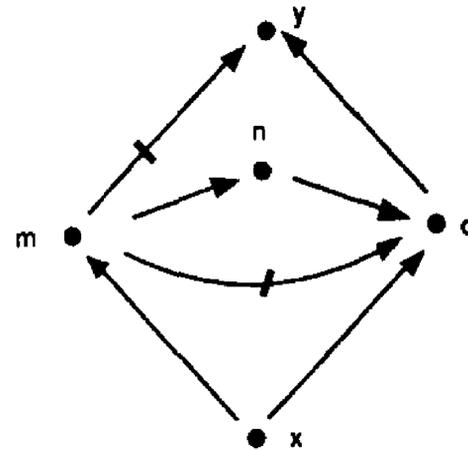


Figure 4: A network where reinstatement affects the conclusions that can be drawn.

Although reinstatement may be semantically undesirable, the parallel marker propagation algorithms described in [4] for defeasible nets and [17] for mixed nets depend on this property in order to compute preemption efficiently. One of the lemmas in the correctness proof of the algorithm in [4] states (roughly) that if $x \rightarrow z_1 \in \Gamma$, $z_i \rightarrow z_{i+1} \in \Gamma$ for i from 1 to $n - 1$, and Γ supports $x \rightarrow z_i$ for i from 1 to n , then $\Gamma \vdash x \rightarrow z_1 \rightarrow \dots \rightarrow z_n$.

Consider what happens when the inheritance query algorithm is asked to determine whether x is a y in Figure 4. Processing nodes in degree order, the algorithm puts the "true" mark M_T on x , m , n , and o , indicating that Γ supports $x \rightarrow m$, $x \rightarrow n$, and $x \rightarrow o$. Then it is ready to determine whether Γ supports the conclusion $x \rightarrow y$ or $x \not\rightarrow y$. It marks all nodes with M_T having a direct positive or negative link to y with marker M_{dir} ; both m and o are so marked. Next, any nodes on preempted paths must be eliminated. A preemption mark, M_{pre} , is used to kill off nodes whose link to y is preempted by some lower node. This is done in a very simple way: Every node with M_{dir} sends M_{pre} up " \rightarrow " links, and M_{pre} is propagated upward across " \rightarrow " links into nodes with M_T as far as it will go. So node m first sends M_{pre} to node n , and node o sends it nowhere because no node above o has M_T . Propagation then causes M_{pre} to spread upward from n to o , killing off o . Finally, only those nodes that have M_{dir} and not M_{pre} are allowed to send M_T up " \rightarrow " links or M_F up " $\not\rightarrow$ " links to y . Since m is the only node meeting these conditions, y is marked with M_F , meaning Γ supports $x \not\rightarrow y$.

The important thing to note here is that the preemption marker M_{pre} that originates at m flows from n to o precisely because m , n , and o are all marked with M_T , meaning Γ supports $x \rightarrow m$, $x \rightarrow n$, and $x \rightarrow o$. But the preemption is only valid because, as the lemma tells us, this guarantees that $\Gamma \vdash x \rightarrow m \rightarrow n \rightarrow o$. The lemma holds for the system in [4] because its definition of inheritance requires that preemptors not have preemptors of their own; this is what produces reinstatement.

In more recent work, Horty has developed an alternative axiomatization of skeptical preemption that does not admit reinstates [5]. In this scheme preemptors need not be permitted; they can defeat other paths even if they themselves are preempted by more specific paths.

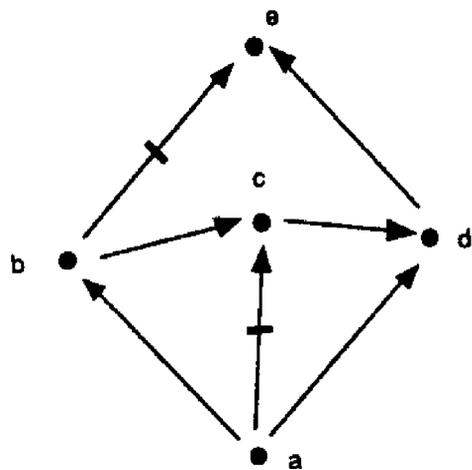


Figure 5: An example of a situator-preemptor.

It is an open question whether preemption without reinstatement can be computed efficiently without enumerating all possible situator paths. It cannot be done using just parallel marker propagation with a bounded number of markers. The reason is that, referring to Figure 4 again, there can be an arbitrary number of paths between m and o . Only one of these need go through in order to situate the preemptor $x \rightarrow m \not\rightarrow y$ with respect to $x \rightarrow o \rightarrow y$, and each situator must be examined independently.

Thus we see the possibility for an interesting tradeoff between inheritance definitions: some give the most "correct" results, while others have efficient algorithms that are correct in most cases, but will produce different results in certain situations.²

5 Defeat of Situators

The previous section looked at defeat of preemption by defeating the preemptor. It's also possible to defeat preemption by defeating the situator. This form of defeat can result in a conflict (skepticism or multiple extensions) rather than the replacement of a conclusion with its opposite.

Figure 5 shows an example of a situator-preemptor. The path $a \rightarrow d \rightarrow e$ has $p r e e m p t a \rightarrow b \not\rightarrow e$, with situator $a \rightarrow b \rightarrow c \rightarrow d$. But the preemptor isn't situated because the situator isn't permitted; the link $a \not\rightarrow c$ preempts its initial segment $a \rightarrow b \rightarrow c$. Unsitated preemptors are conflictors, so we must be skeptical about whether a is an e .

Situator-conflictors are excluded from our list of defeater-defeaters because they have no independent effect. Let p be a path of form $x \rightarrow \tau_1 \rightarrow w \rightarrow \tau_2 \rightarrow z$. Suppose p is a situator of $x \rightarrow \tau \rightarrow w \not\rightarrow y$, a potential preemptor of the subject path, $x \rightarrow \sigma \rightarrow z \rightarrow y$. Then a conflictor $\xi \not\rightarrow p$ must

The tradeoff is further complicated by the fact that *theoretical* well-behavedness may not be the same as intuitive correctness. In [5], it is shown that skeptical extensions allowing reinstatement can be defined directly through a fixedpoint equation, while the only known definitions of skeptical extensions without reinstatement rely on an iterative process using degree. To the extent that fixedpoint approaches in nonmonotonic reasoning seem more declarative than iterative definitions, this may provide a theoretical reason for preferring reinstatement.

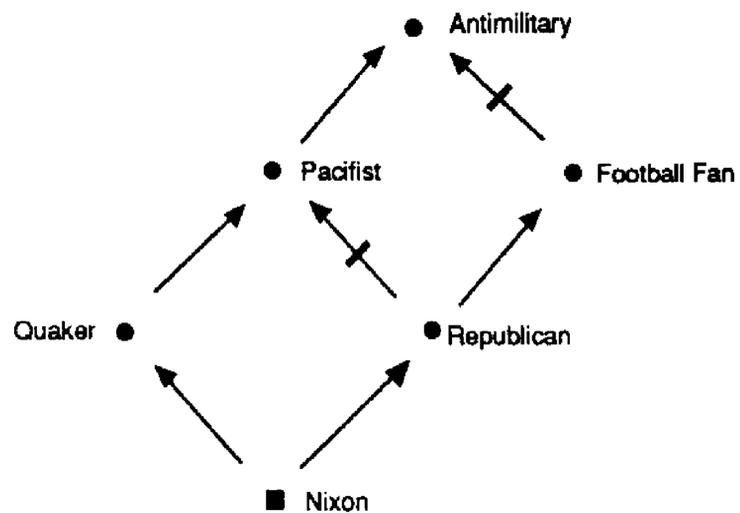


Figure 6: A conflicted conflictor.

be of form $x \rightarrow \mu \not\rightarrow z$, which also makes it a conflictor of $x \rightarrow \sigma \rightarrow z$, an initial segment of the subject path. In all skeptical definitions, if an initial segment of a subject path is conflicted, the subject path is not even potentially inheritable; it will not be permitted. It therefore does not matter whether the subject path's preemptors have permitted situators.

In credulous theories there are two cases to consider. Let ξ be a conflictor of the situator p . In extensions that support $x \not\rightarrow z$ (because ξ is permitted), the subject path has a conflicted initial segment and will not be inheritable. In extensions that support $x \rightarrow z$, p will be a permitted path, and hence the situator will not be conflicted. So in credulous as well as skeptical theories, situator-conflictors do not function as defeater-defeaters,

6 Conflicted Paths

An important difference between defeat by preemptors vs. defeat by conflictors is that preempted paths cannot be extended further; they are dead. Conflicted paths will hold in some credulous extension, and thus can be extended there, perhaps giving rise to other instances of defeat.

Since skeptical reasoning as defined in [4] does not allow conflicted paths to be extended, one way of making a path go through is to conflict some initial segment of its defeater. In the double-diamond example reproduced in Figure 6, because Nixon is conflicted about Pacifist, the negative path to Anti-Military goes through unopposed. Thus, the path $Nixon \rightarrow Republican \not\rightarrow Pacifist$ is acting as a conflictor-conflictor, another type of defeater-defeater.

An alternative notion of skepticism to the one presented here is one where the extension is the intersection of all credulous extensions. In [16] we called this "ambiguity propagation," and Stein and others view it as a more rational or pure form of skepticism [14; 8]. Conflicted paths in such a system cannot be salvaged by conflictor-conflictors. This observation has led Makinson and Senlechts to propose the notion of "zombies:" conflicted paths that are dead but can still kill other paths [8]. In their proposal, conflictor-conflictors would not be defeater-defeaters. Thus, even though we are conflicted about Nixon's pacifism, we would still go on to form the zombie path $Nixon \rightarrow Quaker \rightarrow Pacifist \rightarrow Anti-Military$

10 conflict with $\text{Nixon} \rightarrow \text{Football-Fan} \not\rightarrow \text{Ami-Military}$, causing us to be skeptical about the upper diamond in Figure 6 as well as the lower.

Stein [13; 14] and Makinson and Schlechta [8] have shown that the intersection of all credulous extensions may not support some of the conclusions that all extensions support. This occurs when a conclusion is reached via different paths in two extensions. (Makinson and Schlechta call this a "floating conclusion.") Thus, marker propagation algorithms (which by our definition are limited to a constant number of markers) cannot support this ideal form of skepticism, since there is no way to keep track of which conclusions were reached in which extensions.

Recently, Schlechta has shown that no path-based approach to skeptical reasoning (for any "reasonable" form of skepticism) can produce the intersection of credulous extensions [12]. Hence, we cannot even axiomatize ideal skepticism in our purely path-based formalism. However, the analysis of various defeater-defeater situations reported here applies to other formulations of inheritance as well.

7 Defeat in Mixed Nets

Networks that mix strict and defeasible links, as in [3] and [5], use similar definitions of defeat to the ones presented here, except that strict extensions of paths must be taken into account. (The strict extension of a path is the set of nodes reachable by purely strict links from the path's conclusion.) For example, a reinstater need not have the same conclusion as the reinstated path; the reinstated path's conclusion simply needs to be in the reinstater's strict extension.

One very interesting idea for mixed nets is to require that situators be strict paths. This is in accordance with Braehman's observation [1] that inclusion in natural hierarchies is strict; only properties are defeasible. Note that our proposal does not reduce the network to a simple class/property system as defined in [15], since it is still possible to chain off of defeasible links, defeasible inferences can have strict extensions, and they can generate conflicts. However, the preemption relation is simplified by requiring situators to be strict, since only defeasible paths have the potential to be reinstated, and these can never be situators.

Unfortunately, we still cannot allow reinstatement without affecting the conclusions the system will reach. Figure 7 shows an example of why this is true. The path $x \Rightarrow v \not\rightarrow m \Leftarrow y$ preempts $x \Rightarrow v \Rightarrow u \rightarrow y$, with a strict situator $x \Rightarrow v \Rightarrow u$. The preemptor is itself preempted by the direct link $x \rightarrow m$. But with no reinstatement, this does not restore any path from x to y . Therefore we should reach no conclusion about y . An axiomatization in which preemptors must be unpreempted (leading to reinstatement) would conclude that x is a y .

8 Conclusions

Inheritance theory has a richer structure than we previously imagined. Complex patterns of defeat and reinstatement were known to exist, but had not been systematically analyzed. Even within a narrow family of reasoners, such as skeptical, upward, purely defeasible systems, we find that differences in axiomatization can affect both the results produced and the efficiency of inference.

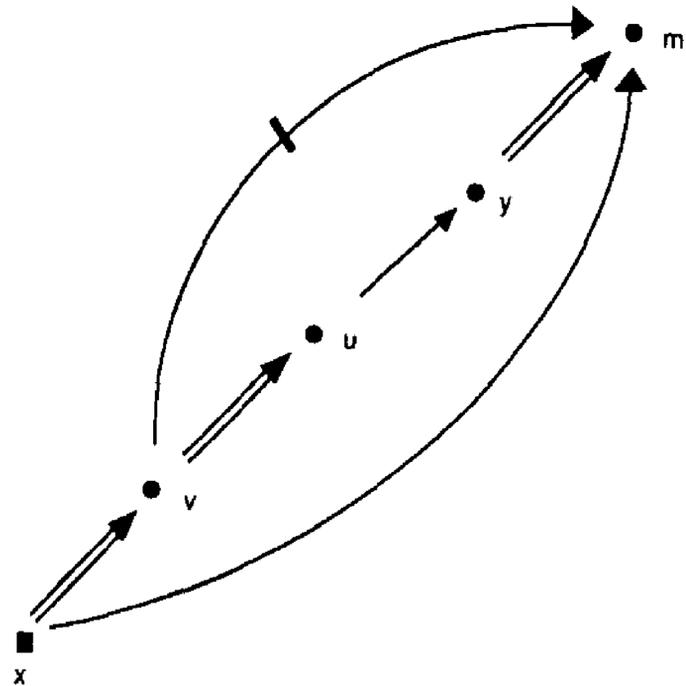


Figure 7: A mixed network whose conclusions are affected by reinstatement.

There are two immediate observations to be drawn from our investigations. First, axiomatizations of inheritance should avoid reinstatement, because it is semantically undesirable. But, second, marker propagation systems³ have implicitly relied on reinstatement, by assuming that preemptors will be permitted paths, in order to compute permission efficiently. Since they also cannot implement ideal skepticism, we conclude that simple marker propagation architectures may not be as well-suited to inheritance reasoning as previously thought. They may still be useful as fast query/retrieval devices, provided that the correctness of the query algorithm is enforced by other means, such as the conditioning algorithms of [15], or is shown experimentally to provide correct results for most naturally occurring networks.

Acknowledgements

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³ As we have defined them, using a small fixed number of markers with no internal structure.

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