New Canonical Representations by Augmenting OBDDs with Conjunctive Decomposition (Extended Abstract)*

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Abstract

We identify two families of canonical representations called ROBDD[^b_t][i]c and ROBDD[^b_C][i]T by augmenting ROBDD with two types of conjunctive decompositions. These representations cover the three existing languages ROBDD, ROBDD with as many implied literals as possible (ROBDD-L_\infty), and AND/OR BDD. We introduce a new time efficiency criterion called rapidity which reflects the idea that exponential operations may be preferable if the language can be exponentially more succinct. Then we demonstrate that the expressivity, succinctness and operation rapidity do not decrease from ROBDD[^b_t][i]T to ROBDD[^b_t][i]c, and then to ROBDD[^b_t][i+1]c. We also demonstrate that ROBDD[^b_t][i]c (i > 1) and ROBDD[^b_t][j]T are not less tractable than ROBDD-L_\infty and ROBDD, respectively. Finally, we develop a compiler for ROBDD[^b_t][i]c which significantly advances the compiling efficiency of canonical representations.

1 Introduction

Canonicity, an important property of knowledge compilation (KC) languages, provides equivalence tests with constant time complexity and plays a critical role in the performance of compiling methods [Darwiche, 2011; Van den Broeck and Darwiche, 2015]. The reduced ordered binary decision diagram (ROBDD) [Bryant, 1986] is one of the most influential canonical languages in the KC literature.

Despite its current success, ROBDD has a well-known weakness in succinctness, which reflects the explosion in size for many types of knowledge bases in practice. Deterministic, decomposable negation normal form (d-DNNF) [Darwiche, 2001] is a strict non-canonical superset of ROBDD, and most efficient compilers (e.g., OBDD compilers) can be seen as special d-DNNF compilers [Huang and Darwiche, 2007]. A recent trend in the KC field is to identify new canonical representations in d-DNNF to mitigate the size explosion problem of ROBDD without sacrificing its main advantages. Researchers has proposed many canonical languages, including AND/OR BDD (AOBDD) [Mateescu et al., 2008], compressed and trimmed SDD over a fixed vtree V (CSDD_V) [Darwiche, 2011], ROBDD with as many implied literals as possible (ROBDD-L_\infty) [Lai et al., 2013]. However, the corresponding compilers cannot yet compile many problems that the state-of-the-art d-DNNF compiler DSHARP can compile [Muir et al., 2012]. Furthermore, the relationships between these canonical representations, which are indispensable for choosing appropriate representations in practical applications, are not well studied.

Decomposability is an important factor behind the strong succinctness and tractability of d-DNNF. The ideas of ROBDD-L_\infty and AOBDD are to use two special types of conjunctive decomposability to relax the linear variable ordering of ROBDD. We generalize these two types of ∧-decompositions to propose bounded ∧-decomposition parameterized by integer i (∧_i-decomposition), and ∧_1-decomposition respecting tree T (∧_{T,i}-decomposition). Then we identify a family of canonical languages in d-DNNF called ROBDD[^b_t][i]c by imposing reducenedness, the finest ∧_j-decomposability, and ordered decision respecting a chain C; and another family of canonical languages called ROBDD[^b_{T,i}][j]T by imposing reducenedness, the finest ∧_{T,i}-decomposability, and ordered decision respecting T. We demonstrate that these two families of languages cover the three previous languages ROBDD, AOBDD and ROBDD-L_\infty, as depicted in Figure 1.

We evaluate the theoretical properties of the two families of canonical languages from four aspects, and the obtained results significantly extend the current KC map:

(a) We analyze the expressivity and demonstrate that ROBDD[^b_t][j]c is complete while ROBDD[^b_{T,i}][j]T is incomplete. We also demonstrate that if i ≤ j, ROBDD[^b_{T,i}][j]T is not more expressive than ROBDD[^b_{T,i}][j]T (resp. CSDD_V).

(b) We analyze the succinctness and demonstrate that ROBDD[^b_t][i]c (resp. MODS) is strictly less succinct than ROBDD[^b_t][j]c for ⋀ i < j. We also demonstrate that ROBDD[^b_{T,i}][j]T is at most as succinct as ROBDD[^b_t][j]c.

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Figure 1: The relationship between many canonical representations in d-DNNF, where each formula in MODS is a set of models, \( \mathcal{T} \) is not a chain, \( \mathcal{C} \) is a topological order of \( \mathcal{T} \), and \( \mathcal{V} \) is the vtree corresponding to \( \mathcal{T} \).

(c) We analyze the time efficiency in terms of tractability. We demonstrate that ROBDD\( [\land_{\mathcal{T},i}] \) is not less tractable than ROBDD. We also demonstrate that ROBDD\( [\land_{\mathcal{T},i}] \) (\( i \geq 2 \)) maintains the same tractability as ROBDD-L\( _\infty \), which is more tractable than d-DNNF.

(d) We also analyze the time efficiency in terms of a new notion called rapidity. We prove that each operation on ROBDD\( [\land_{\mathcal{T},i}] \) (resp. ROBDD\( [\land_{\mathcal{T},i}] \) and MODS) is at most as rapid as the same operation on ROBDD\( [\land_{\mathcal{T},2}] \). For practical purposes, we developed an ROBDD\( [\land_{\mathcal{T},2}] \) compiler with high efficiency. Preliminary experimental results indicate that our compiler is significantly more efficient than the state-of-the-art compilers of ROBDD-L\( _\infty \) and CSDD\( _\mathcal{V} \); moreover, only our compiler is comparable to DS-HARP in both compiling time and resulting sizes.

2 Basic Concepts

This paper uses \( x \) to denote a propositional or Boolean variable, and \( PV = \{x_1, \ldots, x_n, \ldots\} \) to denote a countably infinite set of variables. A formula \( \varphi \) is constructed from constants \( \text{true}, \text{false} \) and variables in \( PV \) using negation operator \( \neg \), conjunction operator \( \land \), disjunction operator \( \lor \), equality operator \( \equiv \) and decision operator \( \psi \land \psi' = (\neg x \lor \psi) \lor (x \lor \psi') \), and we use \( Vars(\varphi) \) to denote the set of variables appearing in \( \varphi \).

We assume that \( PV \) is associated with some strict (partial) order \( \prec \). We focus on the tree-structured orders which are defined as the ancestor-descendant relationships on trees of variables. Given a tree \( \mathcal{T} \) over variables, we denote its depth by \( \text{dep}(\mathcal{T}) \), and the corresponding order by \( \prec_{\mathcal{T}} \).

Definition 1 (\( \land \)-decomposition, \( \land_{\mathcal{T}} \)-decomposition and \( \land_{\mathcal{T},i} \)-decomposition). Given a formula \( \varphi \), a formula set \( \Psi \) is its \( \land \)-decomposition, iff \( \varphi = \bigwedge_{\psi \in \Psi} \psi \land \{Vars(\psi) : \psi \in \Psi\} \) partitions \( Vars(\varphi) \). \( \Psi \) is bounded by an integer \( 0 \leq i \leq \infty \) (\( \land_{\mathcal{T}} \)-decomposition) iff there exists at most one \( \psi \in \Psi \) with \( |Vars(\psi)| > i \). \( \Psi \) respects a tree \( \mathcal{T} \) over variables (\( \land_{\mathcal{T},i} \)-decomposition) iff any two formulas \( \psi, \psi' \in \Psi \) satisfy that \( Vars(\psi) \) and \( Vars(\psi') \) are from two disjoint subtrees.

[Lai et al., 2013] and [Mateescu et al., 2008] implicitly discussed \( \land_1 \)-decomposition and \( \land_{\mathcal{T}} \)-decomposition, respectively. Given two \( \land \)-decompositions \( \Psi \) and \( \Psi' \) of a formula, \( \Psi \) is strict iff \( |\Psi| > 1 \); and \( \Psi \) is finer than \( \Psi' \) iff \( \{Vars(\psi) : \psi \in \Psi\} \) is a finer partition than \( \{Vars(\psi) : \psi \in \Psi'\} \).

Proposition 1. From the viewpoint of equivalence, each non-trivial formula has exactly one finest \( \land_{\mathcal{T}} \)-decomposition (resp. \( \land_{\mathcal{T},i} \)-decomposition).

The finest \( \land_{\mathcal{T}} \)-decomposition (resp. \( \land_{\mathcal{T},i} \)-decomposition) is hereafter denoted by \( \land_{\mathcal{T},\mathcal{C}} \)-decomposition (resp. \( \land_{\mathcal{T},i,\mathcal{C}} \)-decomposition), and a \( \land_{\mathcal{T}} \)-decomposition bounded by integer \( i \) is denoted by \( \land_{\mathcal{T},i,\mathcal{C}} \)-decomposition.

3 ROBDD\( [\land_{\mathcal{C}}] \) and ROBDD\( [\land_{\mathcal{T},i}] \)

We first define binary decision diagram with conjunctive decomposition (BDD[\( \land \)]):

Definition 2 (BDD[\( \land \)]). A BDD[\( \land \)] is a rooted DAG. Each vertex \( v \) is labeled with a symbol \( \text{sym}(v) \). If \( v \) is a leaf, \( \text{sym}(v) = \perp \lor \top \); otherwise, \( \text{sym}(v) \) is a variable or operator \( \land \). Each internal vertex \( v \) has a set of children \( Ch(v) \). For a vertex labeled with variable, \( Ch(v) = \{lo(v), hi(v)\} \), where \( lo(v) \) and \( hi(v) \) are called low and high children, and are connected by dashed and solid arcs, respectively; for a \( \land \)-vertex, \( \{\vartheta(w) : w \in Ch(v)\} \) is a strict \( \land \)-decomposition of \( \vartheta(v) \). Each vertex represents a formula defined as follows:

\[
\vartheta(v) = \begin{cases} \text{false/true} & \text{sym}(v) = \perp/\top; \\
\land_{w \in Ch(v)} \vartheta(w) & \text{sym}(v) = \land; \\
\vartheta(lo(v)) \circ \text{sym}(v) \vartheta(hi(v)) & \text{otherwise.}
\end{cases}
\]

The formula represented by the BDD[\( \land \)] is defined as the one represented by its root.

Given two vertices, we say that they are identical with each other, if they are leaf vertices with the same symbol, or they are internal vertices with the same symbol and children. Next we define some constraints on BDD[\( \land \)]:

Definition 3 (constraints on BDD[\( \land \)]). Given an integer \( i \), a partial order \( \prec \) over variables, and a tree \( \mathcal{T} \) over variables,

- A BDD[\( \land \)] is ordered over \( \prec \) (OBDD[\( \land_{\mathcal{C}} \)]) iff each \( \prec \)-vertex \( u \) and its \( \prec \)-descendant \( v \) satisfy \( \text{sym}(u) \prec \text{sym}(v) \);
- A BDD[\( \land \)] is reduced (RBDD[\( \land \)]) iff no two vertices are identical and no \( \prec \)-vertex has two identical children;
- A BDD[\( \land \)] is, respectively, \( \land_{1} \)-decomposable and \( \land_{\mathcal{T},i} \)-decomposable (BDD[\( \land_{\mathcal{C}} \)] and BDD[\( \land_{\mathcal{T},i,\mathcal{C}} \)]) iff each \( \prec \)-vertex is a \( \land_{1} \)-decomposition and a \( \land_{\mathcal{T},i} \)-decomposition;
- A BDD[\( \land \)] is, respectively, \( \land_{\mathcal{T}} \)-decomposable and \( \land_{\mathcal{T},i} \)-decomposable (BDD[\( \land_{\mathcal{C}} \)] and BDD[\( \land_{\mathcal{T},i,\mathcal{C}} \)]) iff each \( \prec \)-vertex is a \( \land_{\mathcal{T}} \)-decomposition and a \( \land_{\mathcal{T},i} \)-decomposition, and the \( \land_{\mathcal{C}} \)-decomposition and \( \land_{\mathcal{T},i,\mathcal{C}} \)-decomposition of each \( \prec \)-vertex \( \vartheta \) are \( \{\vartheta(w)\} \).

In the subsequent sections, we focus on OBDD[\( \land_{\mathcal{C}} \)] where \( \prec \) is tree-structured: the ancestor-descendant relationship \( \prec_{\mathcal{T}} \) on a tree \( \mathcal{T} \) and particularly \( \prec_{\mathcal{C}} \) over a chain \( \mathcal{C} \). We assume that \( \mathcal{C} \) and \( \mathcal{T} \) always satisfy \( \prec_{\mathcal{T}} \subset \prec_{\mathcal{C}} \), and we sometimes use \( \mathcal{C} \) and \( \mathcal{T} \) to denote \( \prec_{\mathcal{C}} \) and \( \prec_{\mathcal{T}} \), respectively. We will analyze the canonicity, expressivity, and space-time efficiency of ROBDD[\( \land_{\mathcal{C}} \)] and ROBDD[\( \land_{\mathcal{T},i,\mathcal{C}} \)]. Note that each ROBDD over \( \mathcal{C} \) is an ROBDD[\( \land_{\mathcal{C}} \)] and the mutual transformation
between an ROBDD-$L_{\infty}$ over $C$ (resp. AOBDD over $T$) and the equivalent ROBDD[$\land \neg_i]\ C$ (resp. ROBDD[$\land \neg_{\infty}$$])$ can be performed in linear time.

### 4 Canonicity and Expressivity

We first state that ROBDD[$\land \neg_i]$ is canonical and complete:

**Theorem 1.** Fixing integer $i$ and chain $C$ over $PV$, there is exactly one ROBDD[$\land \neg_i$] representing a given formula.

ROBDD[$\land \neg_{\infty}$]$_{T}$ is also canonical, but incomplete:

**Theorem 2.** Fixing tree $T$ over $PV$, there is at most one ROBDD[$\land \neg_{T}$]$_{T}$ to represent a given formula, and ROBDD[$\land \neg_{T}$]$_{T}$ is incomplete when $T$ is not a chain.

Due to the incompleteness of ROBDD[$\land \neg_{T}$]$_{T}$, we draw the expressivity relation between different ROBDD[$\land \neg_{T}$]$_{T}$:

**Theorem 3.** Given a tree $T$ over variables and two integers $i$ and $j$, ROBDD[$\land \neg_{i}$]$_{T}$ is at most as expressive as ROBDD[$\land \neg_{j}$]$_{T}$ if $i \leq j$.

Given a vtree $V$ corresponding to $T$, ROBDD[$\land \neg_{T}$]$_{T}$ is at most as expressive as CSDD$_V$, because the former is a subset of the latter.

### 5 Succinctness

In this section, we first analyze the succinctness relationship between ROBDD[$\land \neg_i$]$_{C}$ and ROBDD[$\land \neg_j$]$_{C}$, and we then analyze the succinctness relationship between ROBDD[$\land \neg_i$]$_{C}$ and ROBDD[$\land \neg_{T}$]$_{T}$. Note that the standard definition of succinctness in the KC map only applies to complete languages. We can extend it to handle the case of incomplete languages by only comparing the sizes of formulas that can be represented in both languages.

We can present an algorithm (see Algorithm DECOMPOSE in [Lai et al., 2017]) to transform an OBDD[$\land \neg_i$]$_{C}$ into the equivalent ROBDD[$\land \neg_i$]$_{C}$. Along with some counter-examples, we can state the following succinctness results:

**Theorem 4.** ROBDD[$\land \neg_i$]$_{C}$ is at most as succinct as ROBDD[$\land \neg_j$]$_{C}$ if $i \leq j$.

The above theorem indicates that ROBDD[$\land \neg_i$]$_{C}$ can further mitigate the size explosion problem of ROBDD from a theoretical perspective. We then analyze the succinctness relationship between ROBDD[$\land \neg_{T}$]$_{T}$ and ROBDD[$\land \neg_i$]$_{C}$:

**Theorem 5.** ROBDD[$\land \neg_{T}$]$_{T}$ is as succinct as ROBDD[$\land \neg_i$]$_{C}$ if $i = 0$ or $\text{dep}(T) < \infty$, and ROBDD[$\land \neg_{T}$]$_{T}$ is strictly less succinct than ROBDD[$\land \neg_i$]$_{C}$ otherwise.

The succinctness relationship between ROBDD[$\land \neg_i$]$_{C}$ (resp. ROBDD[$\land \neg_{T}$]$_{T}$) and the ones in ROBDD, ROBDD-$L_{\infty}$, AOBDD} is immediate from Theorems 4–5. The succinctness relationship between ROBDD[$\land \neg_i$]$_{C}$ ($i \geq 1$) and CSDD$_V$ is incomparable, because some class of circular bit-shift functions can be represented in ROBDD[$\land \neg_i$]$_{C}$ but not in CSDD$_V$ in polyisize [Pipatsrisawat, 2010]. Finally, ROBDD[$\land \neg_{T}$]$_{T}$ is strictly more succinct than MODS if some variable in $T$ has an infinite number of disjoint subtrees of which has at least two vertices.

### 6 Operating Efficiency

We now analyze the time efficiency of operating ROBDD[$\land \neg_i$]$_{C}$ and ROBDD[$\land \neg_{T}$]$_{T}$ in terms of tractability and a new notion called rapidity. We can write an operation as a set of triples with the form $(\varphi_1, \ldots, \varphi_n, \alpha, \beta)$, where $\varphi_1, \ldots, \varphi_n$ represent the input formulas, $\alpha$ represents the supplementary input, and $\beta$ represents the output.

#### 6.1 Tractability Evaluation

We examine the tractability of ROBDD[$\land \neg_i$]$_{C}$ and ROBDD[$\land \neg_{T}$]$_{T}$ with respect to the criteria proposed in [Darwiche and Marquis, 2002] in Table 1.

According to the results in Table 1, we know that ROBDD[$\land \neg_i$]$_{C}$ ($i \geq 2$) is as tractable as ROBDD-$L_{\infty}$. In other-words, compared with ROBDD-$L_{\infty}$ over $C$, ROBDD[$\land \neg_i$]$_{C}$ indeed improves the succinctness under the premise of maintaining the same tractability. Moreover, we know that ROBDD[$\land \neg_{T}$]$_{T}$ is at least as tractable as ROBDD; in particular, ROBDD[$\land \neg_{T}$]$_{T}$ even has more tractability than ROBDD if $\text{dep}(T) < \infty$. Finally, we know that ROBDD[$\land \neg_i$]$_{C}$ is more tractable than d-DNNF, and ROBDD[$\land \neg_{T}$]$_{T}$ is more tractable than CSDD$_V$ [Van den Broeck and Darwiche, 2015].

#### 6.2 New Perspective on Time Efficiency

Due to distinct succinctness, it can sometimes be insufficient to compare the time efficiency of two languages solely by comparing their tractability. Consider an operation $OP$, and two languages $L$ and $L'$ such that $L$ does not satisfy $OP$ but $L'$ satisfy $OP$. Assume that the number of basic arithmetic operations involved in performing $OP$ on $(\varphi_1, \ldots, \varphi_n, \alpha)$ (resp. $(\varphi'_1, \ldots, \varphi'_n, \alpha)$) be $2^m$ (resp. $n$), where $\varphi_i \in L$ (resp. $\varphi'_i \in L'$), and $m = |\alpha| + \sum_{1 \leq i \leq n} |\varphi_i| \ (\text{resp.} \ n = |\alpha| + \sum_{1 \leq i \leq n} |\varphi'_i|)$. Then, performing $OP$ on $(\varphi_1, \ldots, \varphi_n, \alpha)$ can be exponentially (in $m$) more time-consuming than performing $OP$ on $(\varphi'_1, \ldots, \varphi'_n, \alpha)$ when $n = 2^{m^2}$. To overcome this problem, we define a new notion to compare time efficiency from a different perspective, supplementing the concept of tractability:

**Definition 4 (rapidity).** An operation $OP$ on a canonical language $L_1$ is at most as rapid as $OP$ on another canonical language $L_2$ ($L_1 \leq_r^OP L_2$), if for each algorithm ALG

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performing $OP$ on $L_1$, there exists some polynomial $p$ and some algorithm $ALG'$ performing $OP$ on $L_2$ such that for every valid input $(\varphi_1, \ldots, \varphi_n, \alpha)$ of $OP$ on $L_1$ and every valid input $(\varphi_1', \ldots, \varphi_n', \alpha)$ of $OP$ on $L_2$ satisfying $\varphi_i \equiv \varphi_i'$, $ALG'(\varphi_1', \ldots, \varphi_n', \alpha)$ can be done in time $p(t + |\varphi_1| + \cdots + |\varphi_n| + |\alpha|)$, where $\alpha$ is any element of supplementary information and $t$ is the running time of $ALG(\varphi_1, \ldots, \varphi_n, \alpha)$.

Assume that an operation $OP$ satisfies $L_1 \leq_r L_2$. Let $(\varphi_1, \ldots, \varphi_n, \alpha)$ be a valid input of $OP$ on $L_1$ and $(\varphi_1', \ldots, \varphi_n', \alpha)$ be a valid input of $OP$ on $L_2$, where $\varphi_i \equiv \varphi_i'$ for $1 \leq i \leq n$. We know that if performing $OP$ on $(\varphi_1, \ldots, \varphi_n, \alpha)$ can be done in polytime, then performing $OP$ on $(\varphi_1', \ldots, \varphi_n', \alpha)$ can also be done in polytime in $|\alpha| + \sum_{1 \leq i \leq n} |\varphi_i|$. Therefore, for applications needing canonical languages, we suggest that users choose a language by the following steps rather than by the traditional viewpoint of the KC map: first, identify the set $L$ of canonical languages meeting the expressivity requirement; second, identify the set $OP$ of necessary operations and identify the subset $L'$ of $L$ meeting the tractability requirement; third, add each language $L \in L'$ satisfying $\exists L'|L' \in OP \wedge L' \leq_r L$ to $L'$; and finally, choose one of the most succinct languages in $L'$.

We can propose algorithms (see Algorithms CONVERT-Down and CONVERTTree in [Lai et al., 2017]) which respectively transform $ROBDD[\land]_c$ and $ROBDD[\land]_c$ into $ROBDD[\land\lor\land]_c$ and $ROBDD[\land\lor\land]_T (i \leq j)$ and whose time complexities are polynomial in the sizes of outputs. Then we can obtain the following rapidity results:

**Theorem 6.** Given two integers $i$ and $j$, a chain $C$ and a tree $T$ over variables, and an operation $OP$, $MODS \leq_r \land OP ROBDD[\land]_c \leq_r \land OP ROBDD[\land]_c$ if $i \leq j$; and $ROBDD[\land\lor\land]_T \leq_r \land OP ROBDD[\land]_c$.

It was mentioned that for the operation $OP$ corresponding to $SE$, $SFO$, $\land BC$ or $\lor BC$, $OP$ on $ROBDD[\land]_c$ can be performed in polytime but $OP$ on $ROBDD[\land]_c$ $(i > 0)$ cannot be performed in polytime unless $P = NP$. Therefore, if we only consider the tractability of $OP$, it may create the illusion that the time efficiency of performing $OP$ on $ROBDD[\land]_c$ is pessimistically lower than that of performing $OP$ on $ROBDD[\land]_c$. Actually, $OP$ on $ROBDD[\land]_c$ can also be performed in polytime in the sizes of equivalent $ROBDD[\land]_c$. According to this new perspective, an application requiring $OP$ prefers $ROBDD[\land]_c$ to $ROBDD[\land]_c$.

**7 Preliminary Experimental Results**

We developed an $ROBDD[\land]_c$ compiler, and compare it with three compilers of two canonical languages $ROBDD\_\infty$ and $CSDD\_\infty$ and a non-canonical language d-DNNF. The three canonical languages can be seen as subsets of d-DNNF. The state-of-the-art $ROBDD\_\infty$, $CSDD\_\infty$ and d-DNNF compilers are reported in [Lai et al., 2013; Öztok and Darwiche, 2015; Muise et al., 2012], and called BDDjLu, miniC2D and DSHARP, respectively. Individual runs were limited to a one-hour time-out. Table 2 shows the overall performance of the four compilers over the eight domains. The experimental results show that the $ROBDD[\land]_c$ compiler outperformed both BDDjLu and miniC2D on three domains; and it succeeded in 18 (resp. 30) instances more than BDDjLu (resp. miniC2D). The $ROBDD[\land]_c$ compiler and DSHARP outperformed each other on two domains. However, DSHARP succeeded in six more instances than the $ROBDD[\land]_c$ compiler, since the latter was relatively inefficient in sortnet. The reason behind the inefficiency of $ROBDD[\land]_c$ compiler in sortnet is that the ordering heuristic has a negative effect on this domain. Specifically, the $ROBDD[\land]_c$ compiler succeeded in all instances in sortnet when we used the lexicographical order of variables.

Figure 2 analyzes the detailed compiling time and resulting size performance between our $ROBDD[\land]_c$ compiler and DSHARP across the eight domains in Table 2. The experimental results show that the performance of the $ROBDD[\land]_c$ compiler is comparable with that of DSHARP.

**8 Conclusions**

We proposed two families of canonical languages, and analyzed their theoretical properties in terms of the existing criteria of expressivity, succinctness and tractability, as well as the new criterion rapidity. These results provide an important complement to the existing KC map, and the notion of rapidity sheds new light on identifying more succinct canonical representations without the worry of losing the tractability of KC languages. We also developed an efficient $ROBDD[\land]_c$ compiler, which significantly advances the state-of-the-art of compiling efficiency of canonical representations, and has compiling efficiency even comparable with that of DSHARP.
References


