Layered Mereotopology

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Abstract

In fields such as medicine, geography, and mechanics, spatial reasoning involves reasoning about entities—for example cavities and invading particles—that may coincide without overlapping. The purpose of this paper is to develop a mereotopology for domains that include coincident entities, such as material objects, holes, geopolitical entities and spatial regions. In addition, I construct mathematical models of this mereotopology in which nontrivial coincidence relations are defined.

1 Introduction

Two objects *coincide* when they occupy overlapping regions of space.[1] My hand and my body (partially) coincide. The Mississippi River and Minnesota (partially) coincide. Note that the objects in both pairs actually stand in a stronger relation than coincidence. The objects in each pair overlap (i.e. share parts). My hand is a part of both itself and my body. The first ten kilometers of the Mississippi River are part of both the river and the state. Any two overlapping entities are, trivially, coincident—their locations are identical at their common parts.

It is an assumption of this paper that the relation of coincidence is broader than that of overlap. In other words, I assume that there are pairs of coincident objects which do not share parts. The food that is currently being digested in my stomach cavity coincides with, but does not overlap, my stomach cavity. The U.S. Embassy in Paris coincides with, but does not overlap, France. Any object coincides with, but does not overlap, the spatial region at which it is located at a given point of time.

A mereotopology is a formal theory of parthood and connection relations. It has long been recognized that mereotopology forms an essential part of formal ontology. Several different mereotopologies have been proposed in recent literature, including [Asher and Vieu, 1995; Borgo *et al,* 1996; Gotts *et al.,* 1996; Smith, 1996]. These theories are ultimately intended for reasoning about spatial relations among material objects. However, it is assumed in nearly all of this work that the immediate domains of application are restricted to spatial regions.[2] When material objects are introduced, as in [Cohn, 2001], mereotopological relations are still restricted to regions. Each object's spatial properties are determined by those of the region at which it is located. Thus, distinctive coincidence relations are not usually introduced into standard mereotopologies: on domains of regions, coincidence is just overlap.

Likewise, mathematical models for these theories typically use domains consisting of subsets of a topological space. See, for example, [Biacino and Gerla, 1991; Asher and Vieu, 1995; Cohn and Varzi, 1998]. On these domains, there is no natural way of defining a coincidence relation that is distinct from overlap.

The goal of this paper is to construct a mereotopology for domains that include coincident but non-overlapping entities. In particular, domains for the theory may include both material objects and the regions at which they are located, in addition to other types of entities, such as holes or geopolitical entities, which may coincide with material objects. My theory allows mereotopological relations to apply directly to all entities within the domain, be they regions, material objects, holes, or what have you. It also extends mereotopology by adding coincidence relations and by making explicit the relation between an object and the region at which it is located. To guide the development of the formal theory, I construct mathematical models in which a coincidence relation distinct from overlap is defined.

The formal theory presented in this paper borrows much from the theory of location of [Casati and Varzi, 1999]. It differs in that it divides the domain into differ-

[1] Note that with this usage, coincident objects need only occupy *overlapping* spatial regions. I will use the term "complete coincidence" for the stronger relation that holds between objects that occupy *identical* spatial regions.

[2] Here and throughout this paper the term "region" designates any part of an immaterial space in which objects are located. 1 leave open the question of whether these parts may be of mixed dimensionalities. This contrasts with the usage in, for example [Gotts *et al.,* 1996], where the term "region" does not apply to lower-dimensional entities, such as boundaries or points.

ent layers, each of which is mereotopologically independent of the others, and also in providing mathematical models that allow distinct members of the domain to be located at the same region. Additional work on combining either coincidence or location relations with other spatial relations can be found in [Bennett, 2001; Borgo *et al.,* 1997; Cohn, 2001; Schulz and Hahn, 2001].

2 Layered Models

I introduce a class of mathematical structures in which members of the domain can coincide without overlapping. I call these structures *Layered Models* because their domains are partitioned into equivalence classes, called *layers.* Members of the same layer coincide only when they overlap. Members of different layers never overlap and cannot be parts of the same object, though they may coincide. Examples of types of entities assigned to distinct layers are: 1. spatial regions, 2. material objects, 3. holes within these material objects. All regions are parts of a special layer that covers the entire space.

Layered Models are the target models of Layered Mereotopology, the formal theory developed in §3 – §5.

Layered Models are defined as follows. Let $T = <X, cl>$ be a topological space, where X is the set of points and cl is the closure operator. Let I be either, *N,* the set of all natural numbers or an initial segment of *N.* The elements of the domain, D, of a Layered Model are ordered pairs, $x_i = <x, i>$ where $x \subseteq X$ and $i \in I$. (I will generally use the abbreviation, x_i for $<x, i>$. All variables referring to objects in Layered Models appear in Arial font to distinguish them from the variables of the formal theory.) The second component of each ordered pair determines the layer to which the pair belongs. All pairs of the form $<x, 0>$ (i.e. x_0) belong to a special layer, called the *region layer.*

I require that the domain, *D*, of any Layered Model satisfy the following conditions:

1. For any nonempty subset, x ($\emptyset \neq x \subseteq X$), $x_0 \in D$.
2. For any $i \in I$ and any $Y \subseteq \wp(X)$, if $y_i \in D$ for all $y \in Y$, then $(\cup_{y \in Y}(y))_i \in D$.
3. For any $i \in I$ and any $x_i, y_i \in D$, if $\sim (x \subseteq y)$, then there is a $z_i \in D$ such that $z \subseteq x$ and $z \cap y = \emptyset$.
4. If x_i, $y_i \in D$ and $x \cap y \neq \emptyset$, then $(x \cap y)_i \in D$.

It follows from 2 that for any $i \in I$, the sum of all members of D of the form x_i is also in D. More precisely, for any $i \in I$, let $Y_i = \{x : x \subseteq X \,\&\, x_i \in D\}$. Then, by 2, $L_i = (\cup_{y \in Y_i}(y))_i \in D$. I will call L_i the *Layer i* of *D*. Notice that it follows from condition 1 that Layer 0, the region layer, is X_0. The other layers need not cover that entire topological space. In other words, for $i > 0$, Layer i may be equal to x_i where $x \subset X$.

The parthood relation, **P**, is defined on a Layered Model, *D*, as follows:

$Px_iy_j =: x \subseteq y \,\&\, i = j$.

Notice that it follows from this definition of parthood that any $x_i \in D$ is part of Layer i. Also, when overlap, **O**, is defined in terms of **P** in the usual way,

$O(x_i, y_j) =: \exists z(P(z, x_i) \,\&\, P(z, y_j))$,

then it follows that only objects which are parts of the same layer can overlap: $O(x_i, y_j)$ iff $x \cap y \neq \emptyset$ and $i = j$.

If underlap, U, is defined in the usual way,

$U(x_i, y_j) =: \exists z(P(x_i, z) \,\&\, P(y_j, z))$

then it follows that, since all objects are parts of their layer, objects underlap if and only if they are parts of the same layer: $U(x_i, y_j)$ iff $i = j$.

A connection relation, C, is defined on *D* as:

$Cx_iy_j =: cl(x) \cap y \neq \emptyset$ or $x \cap cl(y) \neq \emptyset$ and $i = j$.

As with parthood, only objects residing on the same layer can be connected to each other.

Other mereotopological relations, such as external connection and tangential parthood, can be defined in terms of P and C in the usual way. For example, an external connection relation, EC, can be defined as follows:

$EC(x_i, y_j) =: \sim O(x_i, y_j)$ and $C(x_i, y_j)$.

In contrast to parthood and connection relations, coincidence relations can hold between objects from different layers. Three different coincidence relations are defined on *D.*

$Coin(x_i, y_j) =: x \cap y \neq \emptyset$	(coincidence)
$CCoin(x_i, y_j) =: x = y$	(complete coincidence)
$Cov(x_i, y_j) =: x \subseteq y$	(covering)

Another relation that can hold among objects from different layers is abutment, A.

$A(x_i, y_j) =: x \cap y = \emptyset \,\&\, (cl(x) \cap y \neq \emptyset$ or $x \cap cl(y) \neq \emptyset)$

Finally, I add the function, r, from *D* to Layer 0 which assigns each member of the domain to its representative on the region layer:

$r(x_0) = x_0$

Notice that, when restricted to Layer 0, r is the identity function: for any $x_0 \in D$, $r(x_0) = x_0$. Also notice that two objects abut if and only if their regions are externally connected.

I will give a simple example of a Layered Model designed to illustrate the way in which the models can be used to represent spatial relations among regions, material objects, and immaterial entities such as holes. The background topological space for this model is K^3 with its standard topology. The region layer in this model has as parts the members of the set $\{x_0 : \emptyset \neq x \subseteq \Re^3\}$.

Suppose that we wish to represent relations holding among a vase, a portion of water in the vase, a flower standing in the vase, and the interior of the vase. The vase, water, and flower are represented, respectively, by parts, V_1, W_1, and f_1, of Layer 1 where the subsets, v, w, and f, of $\Re^3$ are disjoint. The interior of the vase, a hole, is represented by h_2 on Layer 2 where h and v are disjoint, but connected, w is a proper subset of h $(w \subset h)$ and h and f have a nonempty intersection $(h \cap f \neq \emptyset)$. According to this representation, the water is not part of the hole, but the fact that the water is contained in the interior of the vase is represented in the model by the covering relation, $Cov(W_1, h_2)$, holding between W_1 and h_2. Similarly, the flower does not overlap the interior of the vase, but the fact that the flower is partially contained in the interior of the vase is represented by the coincidence relation: $Coin(f_1, h_2)$. The vase and its interior do not coincide. Nor are they connected. But the fact that the vase touches its interior is represented by the abutment relation: $A(v_1, h_2)$. Finally, every object is exactly co-located with its spatial region. This is represented in the model by the complete coincidence of the object and its region. For example, $CCoin(v_1, v_0)$.

3 Layered Mereology

A mereology is a formal theory of the binary parthood relation. My aim in this section is to develop a mereology, called *Layered Mereology,* that is satisfied by the parthood relation, P, defined on Layered Models. Layered Mereology is extended in §4 and §5 to Layered Mereotopology, a theory that also includes coincidence and connection relations.

Layered Mereology (and also its extension, Layered Mereotopology) is formulated in first-order logic. Relations and functions are represented in the formal theory with plain text letters to distinguish them from the relations and functions defined in Layered Models. Layered Mereology assumes one primitive, the binary relation P, which, on the intended interpretation, represents parthood.

The following relations are defined in the formal theory in terms of P:

(D1)PPxy =: Pxy & ~ Pyx (x is a proper part of y)
(D2)Oxy =: ∃z (Pzx & Pzy) (x and y overlap)
(D3)Uxy =: ∃z (Pxz & Pyz) (x an y underlap)

It is trivial to see that, if P is interpreted as P in Layered Models, the defined relations O and U will be interpreted as U and O, respectively.

The axioms of Layered Mereology will be somewhat nonstandard. For example, they cannot require that any pair of objects, x and y, have a mereological sum. My goal is to axiomatize P in such a way that, when restricted to a single layer, it satisfies the axioms (and axiom schema) of General Extensional Mereology (GEM):[3]

(P1) Pxx
(P2) Pxy & Pyx → x = y
(P3) Pxy & Pyz → Pxz
(P4) ~Pxy → ∃z(Pzx & ~Ozy)
(GEM5) ∃x ϕ → ∃z∀w (Owz <-> ∃x (ϕ & Owx))

The ϕ in (GEM5) stands for any first-order formula in which z does not occur free. (GEM5) states that if any member of the domain satisfies the formula ϕ, then there must be a sum of all objects satisfying ϕ in the domain. (GEM5) must be altered for Layered Mereology because Layered Models only include the sums of objects that arc parts of the same layer.

I will discuss summation in Layered Models in more detail shortly. For now, notice that the relation P in Layered Models satisfies each of the first four axioms of GEM. They are therefore used in their original forms as axioms for Layered Mereology.[4]

The first three axioms require that P is a partial ordering. (PI) says that P is reflexive. (P2) says that P is antisymmetric. (P3) says that P is transitive. That (P1)-(P3) are satisfied by P in Layered Models follows immediately from the fact that P is just the subset relation restricted to the separate layers.

(P4) says that, if x is not a part of y, then there is some part, z, of x that does not overlap y. (P4) is satisfied in Layered Models by virtue of condition 3 of §2.

It follows from (PI) - (P4) that overlap, O, is extensional. This means that any two members of the domain that overlap the same objects are identical.

(PT1) x = y ↔ ∀z (Oxz ↔ Oyz)

Because O is extensional, for any formula ϕ in which z does not occur free, if we can assign z to a member of the domain that satisfies

∀w (Owz <-> ∃x (ϕ & Owx)) (*)

then this object is the unique sum of all ϕ-ers. However, for any such formula ϕ, there need not be an object satisfying (*) even if some member of the domain satisfies ϕ. For example, let ϕ be x = x and let *D* be the domain of a Layered Model. Then every member of *D* satisfies ϕ. But if there are $x_i, y_j \in D$ with $i \neq j$, then no member of *D* satisfies (*). This is because such an object would have to overlap every member of *D* and there can be no member of *D* that overlaps both x_i and y_j for $i \neq j$.

Thus, we need a restricted version of (GEM5) that requires sums to exist only if all summands are part of the same layer. Such an axiom schema will be satisfied in all Layered Models by virtue of condition 2 of §2. Given that two objects in a Layered Model are parts of the same layer if and only if they underlap, the restricted summation axiom schema can read as follows:

(P5) (∃x ϕ & ∀x,y(ϕ & ϕ/y → Uxy)) → ∃z ∀w(Owz <-> ∃x (ϕ & Owx))

Here, ϕ/y is the formula ϕ with all free instances of x replaced by y (and where variable substitution is performed as necessary so y is free in ϕ/y where x is free in ϕ). (P5) says that if there is some object that satisfies ϕ and any two objects that satisfy ϕ underlap, then there is a sum of all objects satisfying ϕ.

For convenience, when ϕ is any formula in which z does not occur free, I will use the abbreviation zSMx[ϕ] for the substitution instance of the formula (*).

We would like to be able to say more things about layers within the mereology. So far, we can only say that two objects are on the same layer. We would like to be able to say that a certain object is a layer or is the layer of a particular object. (D4) defines a relation holding between y and z when z is the sum of all objects that y underlaps (i.e. y's layer).

(D4) Lyz =: zSMx[Uxy] (z is y's layer)

[3] Throughout this paper, initial universal quantifiers arc suppressed unless they are needed for clarity.

[4] All axioms of Layered Mereology are labeled with a "P". "PT" is used for theorems of Layered Mereology.

It is easy to see that when P is interpreted as P in Layered Models, Lx_iy_j holds if and only if y_j = Layer i. However, axioms (P1)-(P5) do not allow us to infer that for any object, y, there is some object that is the sum of all objects that underlap y. In other words, our axioms so far do not allow us to infer that every object has a layer. This would follow from (P5) if we knew that any two objects that underlap y must underlap each other (i.e. that U is transitive). Notice that the underlap relation for Layered Models, U, is an equivalence relation (reflexive, symmetric, and transitive) and that the sets consisting of all objects from a single layer are the equivalence classes determined by the U relation. It follows from (P1)-(P5) that U is reflexive and symmetric, but not that U is transitive.

I, therefore, add a final axiom to Layered Mereology:

(P6) (Uxy & Uyz) → Uxz (underlap is transitive)

It follows immediately that every object has a layer:

(PT2) $\forall y \exists z$ Lyz (every object has a layer)

Thus the relation, L, is a function. I will use the functional term l(x) to stand for the layer of x.

The following theorems can also be derived:

(PT3) Pxl(x) (every object is part of its layer)

(PT4) Uxy <-> l(x) = l(y)
(two objects underlap iff they have the same layer)

(PT5) Uxy <-> Pyl(x)
(x underlaps y iff y is part of x's layer)

The unary predicate, LY, distinguishes certain members of the domain as layers.

(D5) LYz =: $\exists$x Lxz (z is a layer)

It follows easily from (PT3), (PT4), and (D5) that:

(PT6) LYz ↔ Lzz (z is a layer iff z is its own layer)

(PT6) tells us that layers are those members of the domain which are the mereological sums of all objects that they underlap.

When P is interpreted as P in Layered Models, LYXj if and only if Xi = Layer i.

We can use the formula schema zSMx[>] to introduce more useful relations. (D6) - (D8) are the standard definitions of the sum, product, and difference relations. (D9) defines a *relative* complement relation.

(D6) +(v, y, z) =: zSMx[Pxv v Pxy]
(z is the binary sum of v and y)

(D7) x(v, y, z) =: zSMx(Pxv & Pxy)
(z is the binary product of v and y)

(D8) -(v, y, z) =: zSMx(Pxy & ~Oxv)
(z is the difference of v in y)

(D9) -(v, z) =: zSMx(Uxv & ~Oxv)
(z is the relative complement of v)

The following theorems concerning the relative complement can be derived:

(PT7) $\exists$z(-(x, z)) ↔ ~LYx (x has a relative complement iff x is not a layer)

(PT8) -(x, z) ↔ -(x, l(x), z)(z is x's relative complement iff z is the difference of x in x's layer)

Using (PT4) it is easy to prove that, when they exist, sums, products, and relative complements belong to the same layer as the original object(s).

It is straightforward to show that any layer of a model of Layered Mereology is a model of General Extensional Mereology (GEM). More precisely, we can prove the following:

(Meta-Theorem)Let M be any model of Layered Mereology with domain, D. Note that M need not belong to the class of Layered Models defined in §2. Let w $\in$ D and let D_w = {y : y $\in$ D & l(y) = l(w)}. Let M_w be the structure whose domain is D_w with P interpreted as in M (i.e. Pyz holds in M_w iff y, z $\in$ D_w and Pyz holds in M). Then M_w satisfies axioms (PI) - (P4) and axiom schema (GEM5).

4 The Region Function

In Layered Mereology, we have no way of stating that two objects coincide. Layered Mereology lets us describe the parthood relations between objects. It does not let us describe the (relative) locations of objects. To do this, I extend Layered Mereology by adding the unary function, r. On the intended interpretation r assigns each object, x, to the region, r(x), at which x is exactly located. In Layered Models, r is interpreted as the function r.

Using r, we can define a one-place predicate, R, which distinguishes the sub-domain of regions.

(D10) Ry =: $\exists$x(r(x) = y) (y is a region)

When r is interpreted as r in Layered Models, Rx, if and only if i = 0.

The axioms for r are added to axioms (PI) - (P6).[5] It is easy to check that they are satisfied in Layered Models.

(RI) Ry & Rz -> Uyz (all regions are located in the same layer)

(R2) Ry & Uyz -> r(z) = z (every member of the region layer is its own region)

The theorems below are immediate consequences of (RI)and(R2).

(RT1) Ry -> r(y) = y (every region is located at itself)

(RT2) Ry & Uyz -> Rz (every member of a region's layer is a region)

(RT3) Ex ϕ & $\forall$x (ϕ -> Rx) & zSMx[ϕ]> Rz
(every sum of regions is a region)

Additional axioms relate the region function to parthood.

(R3) Pxy -> Pr(x)r(y) (if x is part of y, then x's region is part of y's region)

Notice that the converse of (R3) is not generally satisfied in Layered Models. r(Xj) may be part of r(yj) even though Xj is not part of yj. This will be the case whenever x $\subsetneq$ y, but i # j.

On the other hand, in Layered Models, if 0(r(Xj), r(yj)) and i = j, then 0(x„ yj must also hold.

[5] All axioms specific to the r function are marked with "R". Theorems specific to the r function are marked with "RT".

(R4) Uxy & 0(r(x), r(y)) -> Oxy (if x and y are on the same layer and x's region overlaps y's region, then x overlaps y)

The coincidence relations are defined in terms of the region function, r.

(DI 1) CCoin(x, y) =: r(x) = r(y)(x and y completely coincide)

(D12) Cov(x, y) =: P(r(x), r(y)) (y covers x)

(D13) Coin(x, y) =: 0(r(x), r(y)) (x and y coincide)

It is easy to check that when P is interpreted as P and r as r in Layered Models, CCoin, Cov, and Coin are, respectively, CCoin, Cov, and Coin.

The following theorems can be derived from (P1)-(P6) and(RI)-(R4).

(RT4) Cov(x, y) & Cov(y, x) <-> CCoin(x, y) (y covers x and x covers y iff x and y completely coincide)

(RT5) CCoin(x, y) & CCoin(x, z) & Uyz -> y = z (any object can completely coincide with at most one object in any layer)

(RT6) CCoin(x, y) & Uxy -> x = y (if x and y completely coincide and are on the same layer, then x = y)

(RT7) Cov(x, y) & Uxy —> Pxy (if y covers x and x and y are on the same layer, then x is part of y)

(RT8) Coin(x, y) & Uxy —> Oxy (if x and y coincide and are on the same layer, then x and y overlap)

In addition, the implications illustrated in the diagram below can be derived. The arrow indicates that the atomic formula at the start of the arrow implies the atomic formula at the end of the arrow.

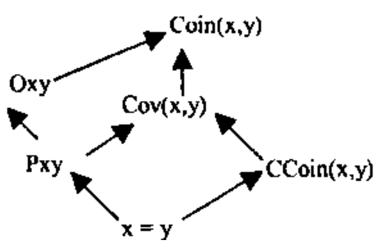

Figure 1: Implication hierarchy

5 Layered Mereotopology

The base theory can be extended to Layered Mereotopology in a straightforward way by adding a connection relation, C, where Cxy means "x is connected to y". C is interpreted as C in Layered Models. [6]

(CI) Cxx (connection is reflexive)

(C2) Cxy -> Cyx (connection is symmetric)

(C3) Pxy -> ∀z(Czx -> Czy) (if x is part of y, everything connected to x is connected to y)

(C4) Cxy -> Uxy (if x and y are connected, then they are parts of the same layer)

(C5) Cxy -> C(r(x), r(y)) (if x and y are connected, their regions are also connected)

[6] Axioms specific to Layered Mereotopology are marked with a "C". Theorems are marked with "CT"

(C6) Uxy & C(r(x), r(y)) -> Cxy (if x and y are members of the same layer and their regions are connected, then x and y are connected)

External Connection and Abutment are defined as follows.

(D14) ECxy =: Cxy & ~ Oxy (x and y are externally connected)

(D15) Axy =: EC(r(x), r(y)) (x and y abut)

When C is interpreted as C in Layered Models, EC is the relation, EC and A is the relation, A.

The following theorems are easily derived:

(CT1) ECxy —> Axy (if x and y are externally connected, then x and y abut)

(CT2) Uxy -> (Axy <-> ECxy) (if x and y are on the same layer, then they abut iff they are externally connected)

(CT3) Axy —> ~Coin(x, y) (if x and y abut, then they do not coincide)

The tangential part relation, TP, is usually defined in terms of external connection as follows:

TPxy =: Pxy & ∃z(ECzx & ECzy) (**)

This definition is not appropriate for layered models. To see why, consider the Layered Model whose underlying topological space is K with its standard topology and whose layers have the members of the following sets as parts:

Layer 0: $\{x_0 : \varnothing \neq x \subseteq \Re\}$

Layer 1: $\{x_1 : \varnothing \neq x \subseteq [0, 1]\}$

It would follow from the standard definition of TP that $[0, l]_1$ has no tangential parts, since it is not externally connected to any member of the domain. For example, it would follow that $[0, 0]_1$ and $[1, l]_1$ are not tangential parts of [0, l]1.

I will therefore use the following definition of tangential part:

(D16) TPxy =: Pxy & ∃z(Azx & Azy) (x is a tangential part of y)

Applying this definition to the previous model, it turns out that any part of $[0, l]^1$ that contains either $[0, 0]_1$ or $[1, l]_1$ is a tangential part of $[0, l]_1$. More generally, it follows from (D16) that, for objects x and y on the same layer, x is a tangential part of y if and only if x's region is a tangential part of y's region.

(CT5) Uxy -> (TPxy <> TP(r(x), r(y)))

Interior parthood is then defined as usual.

(D17) IPxy =: Pxy & -TPxy (x is an interior part of y)

Relational counterparts of topological operators can now be defined in the usual way. For instance, an object's interior can be defined as the sum of its interior parts:

(D18) INT(y, z)=: zSMx[IPxy] (z is the interior of y)

Since each layer of any model of Layered Mereology is a model of GEM, it is easy to show that each layer of any model of Layered Mereotopology is a model of the standard mereotopology which uses (**) as the definition of tangential parthood and includes axioms (C1)-(C3) in addition to those of GEM.

6 Conclusions and Further Work

The goal of this paper was to construct a mereotopology for domains that include coincident but non-overlapping entities. The result is an extension of mereotopology that includes coincidence relations and a region function in addition to mereotopological relations.

These additional elements of the theory make it particularly appropriate for applications that involve reasoning about objects that are located in holes. Reasoning about holes is crucial in a wide variety of domains, including medicine (body cavities and orifices) and mechanics (valves, pathways formed by piping). For more examples, see [Casati and Varzi, 1994].

The particular theory presented in this paper allows the same mereotopological relations to apply directly to all spatial entities, including regions, material objects, and holes. This approach is an alternative to that of [Cohn, 2001] in which spatial relations apply to material objects only indirectly, via the spatial regions at which they are located. One advantage of allowing direct descriptions of the spatial properties of material objects is that this leaves open the possibility of attributing different spatial structures to material objects and the regions at which they are located. For example, we may wish to represent material objects as having only closed, regular, divisible parts, but represent spatial regions as sums of points. Slight changes in the conditions on the domains of Layered Models (specifically, condition 4) and in the axioms of Layered Mereotopology (specifically, (R4)) would allow models in which the parts of different layers are restricted to different granularities.

One other project for further work is to relax the restrictions on the underlap relation so that members of the domain that properly coincide (i.e. coincide without overlapping) can be parts of the same object. An organism has both material parts, such as a heart and a liver, and immaterial parts, such as an abdominal cavity. It has been argued in [Schulz and Hahn, 2001] that material and immaterial parts of the body may properly coincide: the brain is located in the cranial cavity, but it is not part of the cranial cavity. It is possible to define models similar Layered Models in which properly coincident members of the domain may underlap. It remains to develop a formal theory of coincidence and mereotopological relations for these kinds of models.

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