A Simulated Annealing Approach to the Travelling Tournament Problem

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Abstract

Automating the scheduling of sport leagues has received considerable attention in recent years. This paper considers the traveling tournament problem (TTP) proposed in [8; 4] to abstract the salient features of major league baseball (MLB) in the United States. It proposes a simulated annealing algorithm (TTSA) for the TTP that produces significant improvements over previous approaches.

Introduction The scheduling of sport leagues has become an important class of combinatorial optimization applications in recent years since they represent significant sources of revenue and generate extremely challenging optimization problems. This paper considers the travelling tournament problem *(TIP)* proposed in [4] to abstract the salient features of Major League Baseball (MLB) in the United States. The key to the MLB schedule is a conflict between minimizing travel distances and feasibility constraints on the home/away patterns. Travel distances are a major issue in MLB because of the number of teams and the fact that teams go on "road trips" to visit several opponents before returning home. The feasibility constraints in the MLB restricts the number of successive games that can be played at home or away.

The TTP is an abstraction of the MLB intended to stimulate research in sport scheduling. A solution to the TTP is a double round-robin tournament which satisfies sophisticated feasibility constraints (e.g., no more than three away games in a row) and minimizes the total travel distances of the teams. [4] argues that, without an approach to the TTP, it is unlikely that suitable schedules can be obtained for the MLB. The TTP has raised significant interest in recent years since the challenge instances were proposed. [4] describes both constraint and integer programming approaches to the TTP which generate high-quality solutions. [1] explores a Lagrangian relaxation approach (together with constraint programming techniques) which improves some of the results. Other lower and upper bounds are given in [8], although the details of how they are obtained does not seem to be available.

Problem Description The problem was introduced by Easton, Nemhauser and Trick [8; 4], which contains many interesting discussion on sport scheduling. An input consists of n teams (n even) and an n x n symmetric matrix d,, such that d_{ij} represents the distance between the homes of teams T\ and T_J. A solution is a schedule in which each team plays with each other twice, once in each team's home. Such a schedule is called a *double round-robin tournament.* It should be clear that a double round-robin tournament has 2n - 2 rounds. For a given schedule 5, the cost of a team as the total distance that it has to travel starting from its home, playing the scheduled games in 5, and returning back home. The cost of a solution is defined as the sum of the cost of every team.

The goal is to find a schedule with minimum cost satisfying the following two constraints: (1) At most Constraints: No more than three consecutive home or away games are allowed for any team. (2) Norepeat Constraints: A game of T_i at T_j's home cannot be followed by a game of T_j at T_i's home. As a consequence, a double round-robin schedule is feasible if it satisfies the atmost and norepeat constraints and is infeasible otherwise. A schedule is conveniently represented by a table indicating the opponents of the teams. Each line corresponds to a team and each column corresponds to a round. The opponent of team T_i at round r_k is given by the absolute value of element *(i, k).* If (I, *k)* is positive, the game takes place at T_i's home, otherwise at TVs opponent home. Consider for instance the schedule *S* for 6 teams (and thus 10 rounds).

T\R	1	2	3	4	5	6	7	8	9	10
1	6	-2	4	3	-5	-4	-3	5	2	-6
2	5	1	-3	-6	4	3	6	-4	-1	-5
3	-4	5	2	-1	6	-2	1	-6	-5	4
4	3	6	-1	-5	-2	1	5	2	-6	-3
5	-2	-3	6	4	1	-6	-4	-1	3	2
6	-1	-4	-5	2	-3	5	-2	3	4	1

Schedule *S* specifies that team T_1 has the following schedule. It successively plays against teams T_6 at home, T_2 away, T_4 at home, T_3 at home, T_5 away, T_4 away, T_3 away, T_5 at home, T_2 at home, T_6 away. The travel cost of team T\ is

$$d_{12} + d_{21} + d_{15} + d_{54} + d_{43} + d_{31} + d_{16} + d_{61}.$$

Observe that long stretches of games at home do not contribute to the travel cost but are limited by the atmost constraints. This kind of tension is precisely why this problem is hard to solve in practice.

The Local Search This paper proposes an advanced simulated annealing algorithm (TTSA) for the TTP. As usual, the algorithm starts from an initial configuration. Its basic step moves from the current configuration *c* to a configuration in the neighborhood of c. TTSA is based on four main design decisions. (1) Constraints are separated into two groups: hard constraints, which are always satisfied by the configurations, and soft constraints, which may or may not be violated. The hard constraints are the round-robin constraints, while the soft constraints are the no-repeat and atmost constraints. In other words, all configurations in the search represents a double round-robin tournament, which may or may not violate the no-repeat and atmost constraints. Exploring the infeasible region seems to be particularly important for this problem. Obviously, to drive the search toward feasible solutions, TTSA modifies the original objective function to include a penalty term. (2) TTSA is based on a large neighborhood of size $O(n^3)$, where *n* is the number of teams. Some of these moves can be regarded as a form of ejection chains which is often used in tabu search [6; 7]. (3) TTSA dynamically adjusts the objective function to balance the time spent in the feasible and infeasible regions. This adjustments resembles the strategic oscillation idea 15] successfully in tabu search to sohe generalized assignment problems [3], although the details differ since simulated annealing is used as the meta-heuristics. (4) TTSA also uses reheats (e.g., [21) to escape local minima at low temperatures. The "reheats" increase the temperature again and divide the search in several phases.

The Neighborhood The neighborhood consists of four different moves. Among them, partial swaps of rounds and teams are critical to find high-quality solutions. Consider the move *PartialSwapRounds(S,* $T_t, r_k,$ r_1). This move considers team T_i and swaps its games at rounds r_k and r_1. Then the rest of the schedule for rounds r_k and r_1 is updated (in a deterministic way) to produce a double round-robin tournament. Consider the schedule *S*

[T\R	1	2	3	4	5	6	7	8	9	10
1 1	6	-2	2	3	-5	-4	-3	5	4	-6
2	5	1	-1	-5	4	3	6	-4	-6:	-3
3	-4	5	4	-1	6	-2	1	-6	-5	2
4	3	6	-3	-6	-2	1	5	2	-1	-5
5	-2	-3	6	2	1	-6	-4	-1	3	4
6	-1	-4	-5	4	-3	5	-2	3	2	1

and the move *PartialSwapRounds(S,T2,r2,rg).* Obviously swapping the game in rounds r_2 and r_9 would not lead to a round-robin tournament. It is also necessary to swap the games of team 1,4, and 6 in order to obtain:

T\R	1	2	3	4	5	6	7	8	9	10
1	6	4	2	3	-5	-4	-3	5	-2	-6
2	5	-6	-1	-5	4	3	6	-4		-3
3	-4	5	4	-1	6	-2	1	-6	-5	2
4	3		-3	-6	-2	1	5	2	6	
5	-2	-3	6	2	1	-6	-4	-1	3	
6	-1	2	-5	4	-3	5	-2	3	-4	1

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n	Old Best	min(D)	max(D)	mean(D)	std(D)
8	39721	39721	39721	39721	0
10	61608	59583	59806	59605.96	53.36
12	118955	112800	114946	113853.00	467.91
14	205894	190368	195456	192931.86	1188.08
16	281660	267194	280925	275015.88	2488.02

Table 1: Solution Quality of TTSA on the TTP

This move can thus be regarded as a form of ejection chain [6; 7]. Finding which games to swap is not difficult: it suffices to find the connected component which contains the games of T_i in rounds r_2 and r_l in the graph where the vertices are the teams and where an edge contains two teams if they play against each other in rounds r_k and r_l. All the teams in this component must have their games swapped. Note that there are $O(n^3)$ such moves.

Experiments TTSA was applied to the National League benchmark described in 14; 8]. Over the course of this research, TTSA was able to match the best solutions for NL4, NL6, and NL8, and to improve the solutions for NL10, NL12, NL14, and NL16 significantly. The best existing solutions were those in [8] and were the best to our knowledge at the time of writing. The last update was on November 1, 2002. TTSA is the first algorithm to go lower than 200,000 for 14 teams and 280,000 for 16 teams. The results arc summarized Table 1. Observe that even our worst results on these instances improve the previous best solutions.

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