Quantum Computation and Image Processing: New Trends in Artificial Intelligence

S.E. Venegas-Andraca and S. Bose
Centre for Quantum Computation, Department of Physics, University of Oxford
Parks Road, Oxford 0X1 3PU, U.K.
{svenegas, s.bose}@qubit.org
http://www.qubit.org

Abstract

The analysis and use of visual information is a first order task for AI researchers. Due to the architecture of classical computers and to the computational complexity of state-of-the-art algorithms, it is required to find better ways to store, process and retrieve information for image processing. One plausible and exciting approach is Quantum Information Processing (QIP). In this poster we present an initial step towards the definition of an emerging field, Quantum Image Processing, by showing how to store an image using a quantum system as well as some of the unique properties of that storage process derived from quantum mechanics laws.

1 Introduction

In 1985 Deutsch [Deutsch, 1985] developed a theoretical machine named Universal Quantum Turing Machine (UQTM), that is, a machine based on quantum theory capable of performing computations, and showed that such machine was a generalization of a universal Turing machine. It is also shown in [Deutsch, 1985] that a quantum computer (QC), that is, a *physical* system capable of performing computations according to the rules of quantum mechanics, can perform certain tasks faster than its classical counterpart. Deutsch and Jozsa [Deutsch and Jozsa, 1992] and Shor [Shor, 1994] showed concrete problems where such speed-up is possible.

Among QC properties, we find:

1. Superposition of states. The basic component of a QC is a qubit, that is, a physical entity (such as an electron or a photon) that can be mathematically represented as a vector in a 2D Hilbert space $\mathcal{H}^2$. The general form of a qubit is
$$|\psi\rangle = \alpha\,|x\rangle + \beta\,|y\rangle$$
where α, β are complex numbers constrained by $|\alpha|^2 + |\beta|^2 = 1$ and $\{|x\rangle, |y\rangle\}$ is an arbitrary basis of $\mathcal{H}^2$ [Nielsen and Chuang, 2000]. Thus, $|\psi\rangle$ is a superposition of states $|x\rangle$ and $|y\rangle$, and therefore $|\psi\rangle$ can be prepared in an infinite number of ways by varying the values of α and β. See figure 1.a. In contrast, classical computers measure bit values using only one basis, {0,1} and the only two possible states are those that correspond to the measurement outcomes, 0 or 1.

2. Entanglement. Entanglement is a special correlation among quantum systems that has no paragon in classical systems. Entanglement is seen to be at the heart of QIP unique properties, and an example of it is its role in Quantum Teleportation [Nielsen and Chuang, 2000].

The previous properties may have deep implications in several fields of Computer Science, in particular in Artificial Intelligence (AI) both on theoretical (e.g. faster algorithms and secure tranmissions) and technological spheres (current technology is now being built taking into account quantum effects due to component size).

The purpose of this poster is to propose an initial step towards the integration of QIP techniques in image processing and, as a future work, in related fields as Computer Vision and Pattern Recognition.

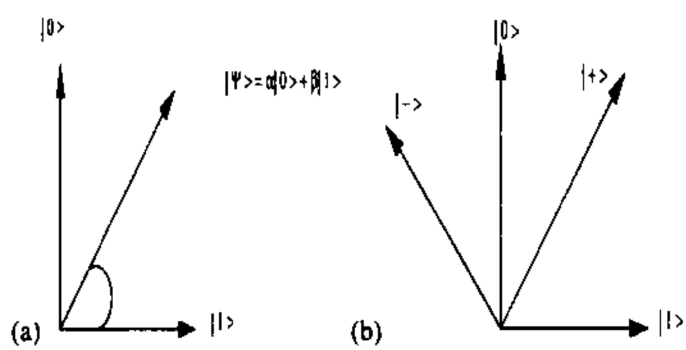

Figure 1: . Mathematical Representation for a qubit

a) The canonical counterpart of classical bits {0, 1} in QIP is $\{|0\rangle, |1\rangle\}$. An arbitrary qubit can be written as $|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, where $|\alpha|^2 + |\beta|^2$ — 1.

b) Note that $|\Psi\rangle$ can be written as a linear combination of an infinite number of bases, in particular as a combination of either B_1 — $\{|0\rangle, |1\rangle\}$ or B_2 - $\{|+\rangle, |-\rangle\}$, where $|+\rangle = [|0\rangle + |1\rangle]/\sqrt{2}$ and $|-\rangle = [|0\rangle - |1\rangle]/\sqrt{2}$

2 Storing an image in a qubit array

2.1 Visual Information Storage in a Multi Particle Quantum System

Let $Q = \{|q_1\rangle_1, |q_2\rangle_2, \ldots, |q_n\rangle_n\}$ be a set of qubits. Our goal is to store visual information in Q. In order to take advantage of $\mathcal{H}^2$ properties, allowed quantum states used to store visual information in a qubit $|q_i\rangle$ are M — $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$.

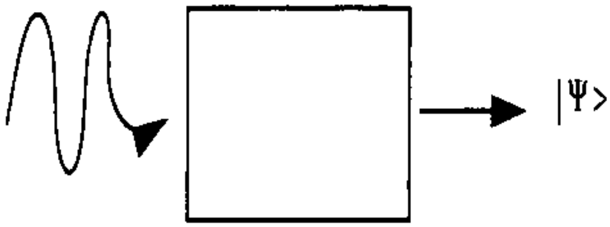

Figure 2: . Frequency to quantum state apparatus
Schematics of an apparatus capable of detecting electromagnetic frequencies and producing a set of quantum states as output. |i/') being an overall state of that set of qubits

We define a machine *B* that divides the whole frequency range of the visible spectra into a partition $F_1, F_2, \ldots, F_M$ and assigns a qubit array (that is, a set of spatially ordered qubit states) Q_t to each partition subset *Fi* (see figure 2). Note that *B* must act as a bijective function between frequency partition subsets F_i and sets of qubit states Q_t, i.e. it is not allowed to have the same qubit state distribution for two different frequency subsets. So, for each pixel from a given image, *B* produces a particular qubit array from the set $\{Q_i\}$.

Let us suppose that machine *B* produces only qubit states $|0\rangle$ and $|1\rangle$. Thus, the number of colors that can be represented in an array of n qubits is 2^n, just like in the classical case (for gray scale or primary color values, for instance) but, if we include qubit states $|+\rangle$ and $|-\rangle$, then the number of different colors that can be stored is 2^{ln}.

However, quantum mechanics has a constraint: reading (that is, measuring) non-orthogonal states (as $|0\rangle, |1\rangle, |+\rangle$ and $|-\rangle$ are) is a probabilistic process. Thus, if only one basis is used to measure all states in M, information retrieval will not be accurate. In addition, the postulates of quantum mechanics state that the post-measurement quantum state is, in general, different from the pre-measurement quantum state (they are equal if and only if the pre-measurement quantum state is equal to one of the orthogonal vectors of the measurement basis).

In order to perform accurate measurements, we now introduce per each pixel, a set of control qubits C - $\{|p_1\rangle_1, |p_2\rangle_2, \ldots, |p_n\rangle_n\}$. $|0\rangle$ and $|1\rangle$ are the only allowed quantum states for elements in C. The purpose of this new element is to use qubit $|p_i\rangle$ to know which basis has to be used to perform a measurement on $|q_i\rangle$ (a generalization of this scheme is to allow the quantum programmer to choose only one storing basis for each pixel). So, by measuring qubit $|p_i\rangle$ in the canonical basis, it is possible to perform accurate measurements. Formally speaking, the observable used to measure control qubits $|p_i\rangle$ is $\hat{A}_1 = \alpha_1 |0\rangle\langle 0| + \beta_1 |1\rangle\langle 1|$. The observables used to measure qubits $|q_i\rangle$ are, in the case the outcome in measuring *A* was *a1* then $\hat{A}_2 = \alpha_2 |0\rangle\langle 0| + \beta_2 |1\rangle\langle 1|$, while in the case the outcome was β_1, then we measure $\hat{A}_2' = \alpha_2' |+\rangle\langle +| + \beta_2' |-\rangle\langle -|$.

2.2 Quantum secrecy vs Eavesdropping

It is relevant to point out the fact that having no *a priori* knowledge of which measurement basis has to be used to read information in qubits $|q_i\rangle$ introduces two very convenient properties: secrecy and eavesdropping detection. Indeed, if an eavesdropper tries to read the information in $|q_i\rangle$ without reading $|p_i\rangle$, two main advantages with no counterpart in classical computers are found: a) the eavesdropper has an only 50% chance of reading accurate information (if he/she fails to choose the right measurement basis, he/she will get only a random outcome), and b) the post-measurement state of a measurement performed with a wrong basis allows us to detect eavesdropping [Bouwmeester *et ai,* 2001]. This means that in order to provide secure transmission of information, it is sufficient to send only control qubits $|p_i\rangle$ via secure cryptographic scheme.

3 Future Work

Next steps include the use of entanglement for gray level and color image segmentation. The basic idea is to entangle qubits with similar colors and to develop quantum algorithms for image segmentation.

Acknowledgments

S.E. Venegas-Andraca gratefully acknowledges the receipt of a CONACyT scholarship from the Mexican National Council of Science and Technology.

The authors want to thank N. Paunkovic for his very stimulating and wise comments about this work.

References

[Deutsch, 1985] David Deutsch. Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer. In *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences,* Volume 400, Issue 1818, pages 97-117, July 8, 1985.

[Deutsch and Jozsa, 1992] David Deutsch and Richard Jozsa Rapid Solution of Problems by Quantum Computation. In *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences,* Volume 439, Issue 1907, pages 553-558,"December 8, 1992.

[Shor, 1994] Peter Shor. Polynomial-Time Algorithms for Prime Factorization and Discrete Algorithms on a Quantum Computer. In *Proceedings of the 35th Annual Symposium on Foundations of Computer Science,* pages 124—134, Santa Fe, NM, Nov. 20-22, 1994. IEEE Computer Society Press.

[Nielsen and Chuang, 2000] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information.* Cambridge University Press, Cambridge, United Kingdom, 2000.

[Gonzalez and Woods, 1993] R. Gonzalez and R. Woods. *Digital Image Processing.* Addison-Wesley Co., 1993

[Bouwmeester *et al.*, 2001] Dirk Bouwmeester, Artur Ekert and Anton Zeilinger, editors. *The Physics of Quantum Information.* Springer-Verlag, Berlin, Germany, 2001.