# Revision of Partially Ordered Information: Axiomatization, Semantics and Iteration

**Salem Benferhat** and **Sylvain Lagrue**
CRIL-CNRS, Université d'Artois
rue Jean Souvraz
62307 Lens Cedex. France.
{benferhat,lagrue}@cril.univ-artois.fr

**Odile Papini**
LSIS-CNRS, Université du Sud Toulon-Var.
BP 132.
83957 La Garde Cedex France.
papini@univ-tln.fr

## Abstract

This paper deals with iterated revision of partially ordered information. The first part of this paper concerns the Katsuno-Mendelzon's postulates: we first point out that these postulates are not fully satisfactory since only a class of partially ordered information can be revised. We then propose a suitable definition of faithful assignment, followed by a new set of postulates and a representation theorem. The second part of this paper investigates additional postulates dedicated to iterated revision operators of partially ordered information. Three extensions of well-known iterated belief revision operations for dealing with partially ordered information are briefly presented.

## 1 Introduction

Belief revision is a central topic in databases, philosophy and artificial intelligence. The revision of a database consists of the insertion of some new information while preserving consistency.

This process has been discussed in different frameworks (probability theory, Spohn's ordinal conditional functions, Grove's systems of spheres, possibility theory, etc.) and from different points of view (axiomatization [Alchourrón *et al.*, 1985; Katsuno and Mendelzon, 1991], computational issue, etc.). The proposal of Alchourrón, Gärdenfors and Makinson (AGM) [Alchourrón *et al.*, 1985] represents one of the most influential work. It concerns revision operations that revise a theory (representing agent's current beliefs) with a formula (representing new pieces of information). The AGM framework consists in a set of natural postulates that any revision operation should satisfy.

An important issue is that any revision operator satisfying AGM's postulates is implicitly based on a priority ordering. This result suggests that an agent's epistemic state, denoted by $\Psi$, is something more complex than a simple representation of agent's current beliefs (called belief set and denoted by $Bel(\Psi)$). It also contains strategies to revise his beliefs. The AGM's postulates have been largely studied and/or adapted in the literature. For instance, additional postulates have been proposed to manage the concept of iterated belief change in [Darwiche and Pearl, 1997] (see also [Rott, 2001; Lehmann, 1995; Nayak, 1994; Konieczny and Pino Pérez, 2000]). More generally, iterated revision is applied to an epistemic state, instead of a belief set, and the result is also an epistemic state.

Epistemic states are often represented by total pre-orders on possible worlds (or interpretations), denoted by $\leq_\Psi$. The set of models of $Bel(\Psi)$ are identified to be minimal (i.e, preferred) elements with respect to $\leq_\Psi$.

There is another issue in belief revision and which has been less investigated in the literature: the iterated revision of epistemic states represented by partial pre-orders (instead of total pre-orders) [Katsuno and Mendelzon, 1991]. Partial pre-orders offer more flexibility than total pre-orders to represent incomplete knowledge.

Katsuno and Mendelzon proposed in section 5 of [Katsuno and Mendelzon, 1991], a set of postulates and a representation theorem that characterize revision operations based on partial pre-orders. Unfortunately, the proposed approach is not satisfactory since only one class of partial pre-orders can be revised. For instance, assume that we have a partial pre-order $\preceq_\Psi$ on four possible interpretations $\{\omega_0, \omega_1, \omega_2, \omega_3\}$, such that $\omega_0$ is "strictly preferred" to $\omega_1$ and $\omega_1$ is "strictly preferred" to $\omega_2$ but $\omega_3$ is incomparable with $\omega_0$, $\omega_1$ and $\omega_2$ (cf. figure 1, for sake of clarity, reflexivity and transitivity are not represented). Clearly, $\omega_0$ and $\omega_3$ characterize (are models of) agent's currents beliefs $Bel(\Psi)$ (no interpretation is strictly preferred to $\omega_0$ or to $\omega_3$), but $\preceq_\Psi$ is not a faithful assignment in the sense of [Katsuno and Mendelzon, 1991].

More generally, there is no consistent propositional formula that admits $\preceq_\Psi$ of Figure 1 as a faithful assignment. In fact, faithful assignment requires that each model of agent's current beliefs (namely $Bel(\Psi)$) should be strictly preferred to each counter-model of agent's current beliefs. This requirement is appropriate for total pre-orders, but is too strong for partial pre-orders.

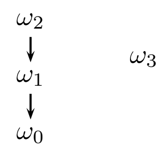


Figure 1: Example of $\preceq_\Psi$ which is not a faithful assignment.

This paper investigates iterated belief revision of partially ordered information. Its main contributions can be summarized as follows:

- We propose an alternative definition of faithful assignment and an alternative set of natural postulates for revising partially ordered information.
- We provide a representation theorem for iterated revision operators satisfying this new set of postulates.
- We investigate additional postulates dedicated to iterated belief revision. In particular, we discuss the extension of Darwiche and Pearl's postulates, Boutilier's postulate and we propose two alternative postulates for capturing other forms of iterated belief revision.
- Lastly, we illustrate our results with three well-known operators (revision with memory, possibilistic revision and natural belief revision) for revising partially ordered information.

The rest of this paper is organized as follows. Section 2 presents the Katsuno and Mendelzon's postulates for belief revision. Section 3 contains a modification of these postulates. Section 4 characterizes Darwiche and Pearl's postulates and Boutilier's postulate for iterated belief revision with partially ordered information. Lastly, Section 5 proposes two alternative additional postulates and provides three examples of operators that satisfy the proposed postulates.

# 2 Notations and KM postulates

## 2.1 Notations

In this paper we use propositional calculus, denoted by $\mathcal{L}$, as knowledge representation language with usual connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\equiv$ (logical equivalence). The lower case letters $a$, $b$, $c$, $\cdots$, are used to denote propositional variables and lower case Greeks letters $\varphi$, $\psi$, $\cdots$, are used to denote formulas. We denote by $\Omega$ the set of interpretations of $\mathcal{L}$ and by $Mod(\psi)$ the set of models of a formula $\psi$, that is $Mod(\psi) = \{\omega \in \Omega, \omega \models \psi\}$. Upper case Greeks letters $\Psi$, $\Phi \cdots$, are used to denote epistemic states.

A partial pre-order, denoted by $\preceq$, on a set $A$ is a reflexive and transitive binary relation. Let $x, y \in A$, we define the equality $x = y$ iff $x \preceq y$ and $y \preceq x$. The corresponding strict partial pre-order associated with $\preceq$, denoted by $\prec$, is defined as usual: $x \prec y$ iff $x \preceq y$ holds but $x \preceq y$ does not hold. We denote by $\sim$ the incomparability relation, namely $x \sim y$ iff neither $x \preceq y$ nor $y \preceq x$ holds.

Given $\preceq$ on a set $A$, the set of the minimal elements of $A$, denoted by $Min(A, \preceq)$, is defined by $Min(A, \preceq) = \{x : x \in A, \nexists y \in A, y \prec x\}$.

## 2.2 The KM postulates for belief revision

Katsuno and Mendelzon proposed in [Katsuno and Mendelzon, 1991] a reformalization of the well-known AGM postulates [Alchourrón *et al.*, 1985]. In this framework, the epistemic state of an agent is represented by a formula $\psi$. A revision operator $\circ$ is a function from a formula $\psi$ and produces a new formula denoted by $\psi \circ \mu$. The operator $\circ$ should satisfy the following postulates.

$(R_1)$ $\psi \circ \mu \models \mu$,

$(R_2)$ if $\psi \wedge \mu$ is satisfiable, then $\psi \circ \mu \equiv \psi \wedge \mu$,

$(R_3)$ if $\mu$ is satisfiable, then $\psi \circ \mu$ is satisfiable,

$(R_4)$ if $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$,

$(R_5)$ $(\psi \circ \mu) \wedge \varphi \models \psi \circ (\mu \wedge \varphi)$,

$(R_6)$ if $(\psi \circ \mu) \wedge \varphi$ is satisfiable, then $\psi \circ (\mu \wedge \varphi) \models (\psi \circ \mu) \wedge \varphi$.

An important result is that any revision operator that satisfies $(R_1) - (R_6)$ is implicitly based a total pre-order $\leq_\psi$ on $\Omega$.

In order to deal with partial pre-orders, Katsuno and Mendelzon proposed in section 5 of [Katsuno and Mendelzon, 1991] to replace $(R_6)$ by the two following postulates:

$(R_7)$ if $\psi \circ \mu_1 \models \mu_2$ and $\psi \circ \mu_2 \models \mu_1$ then $\psi \circ \mu_1 \equiv \psi \circ \mu_2$,

$(R_8)$ $(\psi \circ \mu_1) \wedge (\psi \circ \mu_2) \models \psi \circ (\mu_1 \vee \mu_2)$.

The following representation theorem has been proposed with these postulates for partial pre-orders.

**Theorem 1 ([Katsuno and Mendelzon, 1991])** *A revision operator $\circ$ satisfies the postulates $(R_1)$ to $(R_5)$, $(R_7)$ and $(R_8)$ iff there exists a faithful assignment that maps $\psi$ to a partial pre-order $\preceq_\psi$ such that: $Mod(\psi \circ \mu) = Min(Mod(\mu), \preceq_\psi)$.*

Where the faithful assignment is defined as follows.

**Definition 1 (faithful assignment)** *Let $\psi$ be a propositional formula. A partial pre-order on $\Omega$, associated with $\psi$, denoted by $\preceq_\psi$, is said to be a faithful assignment if the three following conditions hold :*

*(1) if $\omega, \omega' \in Mod(\psi)$ then $\omega <_\psi \omega'$ does not hold,*

*(2) if $\omega \in Mod(\psi)$ and $\omega' \notin Mod(\psi)$, then $\omega <_\psi \omega'$ holds,*

*(3) if $\psi \equiv \varphi$, then $\leq_\psi = \leq_\varphi$.*

In the following, by KM postulates, we refer to $(R_1) - (R_5)$, $(R_7)$ and $(R_8)$.

# 3 KM postulates revisited

In the introduction, we pointed out that KM postulates are not fully satisfactory for iterated belief revision of partially ordered information. This section justifies this statement and proposes a revisited set of KM postulates.

## 3.1 Faithful assignment revisited

This subsection proposes a modified definition of faithful assignment. The one given by definition 1 is not appropriate. Indeed, if $\preceq_\Psi$ is a partial pre-order on the set of interpretations $\Omega$ representing agent's epistemic state, and if we define $Mod(Bel(\Psi)) = Min(\Omega, \preceq_\Psi)$ to be the current agent's beliefs, then there is no guarantee that $\preceq_\Psi$ will be a faithful assignment.

For instance, the partial pre-order given by figure 1 is not a faithful assignment, since condition (2) of Definition 1 is not satisfied. Indeed $Mod(Bel(\Psi)) = \{\omega_0, \omega_3\}$ and $\omega_3 \prec_\Psi \omega_2$ does not hold. Of course, one may simply suggest to enforce minimal elements of $\preceq_\Psi$ to be equal. In our example, we enforce $\omega_0$ to be equal to $\omega_3$ (as it is illustrated by Figure 2).

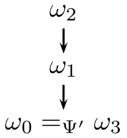


Figure 2: Example of $\preceq_{\Psi'}$.

But this is not satisfactory. Indeed, if $\preceq_{\Psi'}$ denotes the partial pre-order obtained from $\preceq_\Psi$ by enforcing minimal elements of $\preceq_\Psi$ to be equal then, in general, $Bel(\Psi' \circ \mu) \not\equiv Bel(\Psi \circ \mu)$. In our example, if we consider a new piece of information $\mu$ such that $Mod(\mu) = \{\omega_1, \omega_3\}$, then, it can easily be checked that $Bel(\Psi \circ \mu) \not\equiv Bel(\Psi' \circ \mu)$. Indeed $Mod(Bel(\Psi \circ \mu)) = Min(\{\omega_1, \omega_3\}, \preceq_\Psi) = \{\omega_1, \omega_3\}$ and $Mod(Bel(\Psi' \circ \mu)) = Min(\{\omega_1, \omega_3\}, \preceq_{\Psi'}) = \{\omega_3\}$, where $\preceq_\Psi$ and $\preceq_{\Psi'}$ are respectively represented by Figure 1 and Figure 2.

The problem with Definition 1 is the condition (2), which requires that each model of agent's current beliefs is preferred to all counter-models of agent's current beliefs. We propose to weaken this condition by only requiring that each counter-model of agent's current beliefs is strictly less preferred than at least one model of agent's current beliefs.

In order to deal with iterated belief revision, which applies to epistemic states instead of formulas, we also modify condition (3) exactly as it is proposed in [Darwiche and Pearl, 1997]. These two changes lead to the following modified definition of faithful assignment.

**Definition 2 (P-faithful assignment)** *Let $\Psi$ be an epistemic state. A partial pre-order on $\Omega$, associated to $\Psi$ and denoted by $\preceq_\Psi$ is said to be a P-faithful assignment if the three following conditions hold :*

*(1) if $\omega, \omega' \models Bel(\Psi)$ then $\omega \prec_\Psi \omega'$ does not hold,*

*(2) if $\omega' \not\models Bel(\Psi)$, then there exists $\omega$ such that $\omega \models Bel(\Psi)$ and $\omega \prec_\Psi \omega'$,*

*(3) if $\Psi = \Phi$, then $\preceq_\Psi = \preceq_\Phi$.*

Clearly, if $\preceq_\Psi$ is a faithful assignment, then it is also a P-faithful assignment. Note that if $\preceq_\Psi$ is a total pre-order, then this definition is equivalent to the definition of a faithful assignment of Darwiche and Pearl [Darwiche and Pearl, 1997].

### 3.2 Revisited postulates for partially ordered information

Let us now present the "revision" of KM postulates. In the whole paper, we assume that the new information $\mu$ is consistent and the initial beliefs $Mod(Bel(\Psi)$ is not empty. Basically, there is a need of two changes. The first change is the same as the one proposed by Darwiche and Pearl in [Darwiche and Pearl, 1997] (see also [Friedman and Halpern, 1999]). It consists in a simple adaptation of KM postulates in order to make belief revision operation a function on epistemic states instead on set of beliefs. More precisely, we propose to replace postulate $(R_4)$ by the one proposed in [Darwiche and Pearl, 1997].

$(P_4)$ if $\Psi_1 = \Psi_2$ and $\mu_1 \equiv \mu_2$ then $Bel(\Psi_1 \circ \mu_1) \equiv Bel(\Psi_2 \circ \mu_2)$.

Postulate $(P_4)$ requires the equality between epistemic states while $R_4$ requires the equivalence between beliefs sets.

The second change concerns postulate $(R_2)$. We argue that $(R_2)$ is too strong when dealing with partial pre-orders.

Let us consider again the example represented by Figure 1. Let $\mu$ be such that $Mod(\mu) = \{\omega_1, \omega_3\}$. Since $Min(Mod(\mu), \preceq_\Psi) = \{\omega_1, \omega_3\}$, we expect to have $Bel(\Psi \circ \mu) \equiv \{\omega_1, \omega_3\}$. But $R_2$ leads to have $Bel(\Psi \circ \mu) \equiv \{\omega_3\}$. Indeed, $(R_2)$ says that if $Bel(\Psi) \wedge \mu$ is consistent, then $Bel(\Psi) \circ \mu \equiv Bel(\Psi) \wedge \mu$. This equivalence makes sense if for each $\omega, \omega'$ models of $Bel(\Psi)$, $\omega =_\Psi \omega'$ holds. However, when models of $Bel(\Psi)$ may be incomparable, then $(R_2)$ becomes questionable.

We suggest to replace $R_2$ by the following postulate. It concerns the situation where the new piece of information is a tautology. In this case, agent's current beliefs should not change. Namely:

$(P_2)$ $Bel(\Psi \circ \top) \equiv Bel(\Psi)$.

### 3.3 Representation theorem

The new set of postulates that we propose for characterizing the revision of partially ordered beliefs are the following ones.

$(P_1)$ $Bel(\Psi \circ \mu) \models \mu$,

$(P_2)$ $Bel(\Psi \circ \top) \equiv Bel(\Psi)$,

$(P_3)$ if $\mu$ is satisfiable, then $Bel(\Psi \circ \mu)$ is satisfiable,

$(P_4)$ if $\Psi_1 = \Psi_2$ and $\mu_1 \equiv \mu_2$ then $Bel(\Psi_1 \circ \mu_1) \equiv Bel(\Psi_2 \circ \mu_2)$,

$(P_5)$ $Bel(\Psi \circ \mu) \wedge \varphi \models Bel(\Psi \circ (\mu \wedge \varphi))$,

$(P_6)$ if $Bel(\Psi \circ \mu_1) \models \mu_2$ and $Bel(\Psi \circ \mu_2) \models \mu_1$ then $Bel(\Psi \circ \mu_1) \equiv Bel(\Psi \circ \mu_2)$,

$(P_7)$ $Bel(\Psi \circ \mu_1) \wedge Bel(\Psi \circ \mu_2) \models Bel(\Psi \circ (\mu_1 \vee \mu_2))$.

The postulates $(P_1)$, $(P_3)$, $(P_4)$, $(P_5)$, correspond respectively to the postulates $(R_1)$,$(R_3)$, $(R_4)$, $(R_5)$. The postulates $(P_6)$ and $(P_7)$ are respectively a reformalization of the postulates $(R_7)$ and $(R_8)$ proposed in [Katsuno and Mendelzon, 1991] for epistemic states.

Two weakenings of $(R_2)$ can be derived from this set of postulates:

$(P_2')$ $Bel(\Psi) \wedge \mu \models Bel(\Psi \circ \mu)$,

$(P_{2w})$ if $Bel(\Psi) \models \mu$ then $Bel(\Psi \circ \mu) \equiv Bel(\Psi) \wedge \mu$.

The first one means that joint of initial beliefs $Bel(\Psi)$ and the new information are contained in the set of new belief's models. The second one means that if we revise with some pieces of information that can be inferred from agent's current beliefs, then the agent's beliefs should not change.

Given this set of postulates and the definition of P-faithful assignment, we provide the following representation theorem:

**Theorem 2** *A revision operator $\circ$ satisfies the postulates $(P_1)$ to $(P_7)$ iff there exists a P-faithful assignment that maps each $\Psi$ to a partial pre-order $\preceq_\Psi$ such that $Mod(Bel(\Psi \circ \mu)) = Min(Mod(\mu), \preceq_\Psi)$.*

## 4 Characterization of iterated revision postulates

Several authors (e.g. [Darwiche and Pearl, 1997]) argue that, in order to have a genuine iterated belief revision operator, we should augment basic AGM postulates with some additional postulates. Indeed, these basic postulates constrain the relationships between $Bel(\Psi)$ and $Bel(\Psi \circ \mu)$. The aim of these additional postulates is to provide more constraints between $\Psi$ and $\Psi \circ \mu$ by, for instance, expliciting the relationships between $\preceq_\Psi$ and $\preceq_{\Psi\circ\mu}$. Given a formula $\mu$, there are three questions that need to be addressed:

- How the order between models of $\mu$ does evolve?
- How the order between counter-models of $\mu$ does evolve?
- How the order between models and counter-models of $\mu$ does evolve?

Darwiche and Pearl [Darwiche and Pearl, 1997] gave four natural postulates for iterated belief revision that provide answers for these questions.

$(C_1)$ if $\alpha \models \mu$, then $Bel((\Psi \circ \mu) \circ \alpha) \equiv Bel(\Psi \circ \alpha)$,

$(C_2)$ if $\alpha \models \neg\mu$, then $Bel((\Psi \circ \mu) \circ \alpha) \equiv Bel(\Psi \circ \alpha)$,

$(C_3)$ if $Bel(\Psi \circ \alpha) \models \mu$, then $Bel((\Psi \circ \mu) \circ \alpha) \models \mu$,

$(C_4)$ if $Bel(\Psi \circ \alpha) \not\models \neg\mu$, then $Bel((\Psi \circ \mu) \circ \alpha) \not\models \neg\mu$.

Another postulate, which has also been discussed in the literature, is the one which characterizes Boutilier's natural belief revision [Boutilier, 1993] (and hinted in [Spohn, 1988]) for totally ordered beliefs:

$(C_B)$ If $Bel(\Psi \circ \mu) \models \neg\alpha$ then $Bel((\Psi \circ \mu) \circ \alpha) \equiv Bel(\Psi \circ \alpha)$.

This section provides a characterization of the Darwiche and Pearl's postulates and the Boutilier's postulate, when epistemic states are represented by partial pre-orders.

In the following, we restrict ourselves to iterated belief revision operators that satisfy $(P_1) - (P_7)$. Let $\circ$ be a such revision operator and $\preceq_\Psi$ be its associated P-faithful assignment. In order to characterize $(C_1) - (C_4)$ and $(C_B)$, we define five properties on relationships between $\preceq_\Psi$ and $\preceq_{\Psi\circ\mu}$.

$(Ip_1)$ if $\omega \models \mu$ and $\omega' \models \mu$ then $\omega \prec_\Psi \omega'$ iff $\omega \prec_{\Psi\circ\mu} \omega'$,

$(Ip_2)$ if $\omega \not\models \mu$ and $\omega' \not\models \mu$ then $\omega \prec_\Psi \omega'$ iff $\omega \prec_{\Psi\circ\mu} \omega'$,

$(Ip_3)$ if $\omega \models \mu$ and $\omega' \not\models \mu$ then $\omega \prec_\Psi \omega'$ implies $\omega \prec_{\Psi\circ\mu} \omega'$,

$(Ip_4)$ if $\omega \models \mu$ and $\omega' \not\models \mu$ then $\omega \preceq_\Psi \omega'$ implies $\omega \preceq_{\Psi\circ\mu} \omega'$,

$(Ip_5)$ if $\omega, \omega' \not\models Bel(\Psi \circ \mu)$ then $\omega \prec_\Psi \omega'$ implies $\omega \prec_{\Psi\circ\mu} \omega'$.

$(Ip_1)$ (resp. $(Ip_2)$) means that the strict order between models (resp. counter-models) of the new piece of information $\mu$ is preserved. $(Ip_3)$ means that if a model of $\mu$ is strictly preferred to a counter-models then this strict preference is preserved. $(Ip_4)$ is similar with $(Ip_3)$ for non-strict preference. Finally, $(Ip_5)$ means that the strict order between the counter-models of the agent's current beliefs is preserved.

The following representation theorem provides a characterization of $(C_1) - (C_4)$.

**Theorem 3** *Let $\circ$ be a revision operator satisfying the postulates $(P1) - (P_7)$. Then:*

*(1) $\circ$ satisfies $(C_1)$ iff its associated faithful assignment satisfies $(Ip_1)$,*

*(2) $\circ$ satisfies $(C_2)$ iff its associated faithful assignment satisfies $(Ip_2)$,*

*(3) $\circ$ satisfies $(C_3)$ iff its associated faithful assignment satisfies $(Ip_3)$,*

Note that only the characterization of $(C_3)$ is identical to the one provided by Darwiche and Pearl [Darwiche and Pearl, 1997]. The others are different.

More precisely, there are two main differences with the DP representation theorem. First, we have only

(4) if the faithful assignment associated with $\circ$ satisfies $(Ip_4)$ then $\circ$ satisfies $(C_4)$.

The converse does not hold.

The second main difference concerns $(C_1)$ and $(C_2)$. Indeed, Darwiche and Pearl have shown that when $\preceq_\Psi$ is a total pre-order, then $(C_1)$ (resp. $(C_2)$) guarantees that $\preceq_\Psi$ and $\preceq_{\Psi\circ\mu}$ to be identical on their subdomain $Mod(\mu) \times Mod(\mu)$ (resp. $Mod(\neg\mu) \times Mod(\neg\mu)$). However, when $\preceq_\Psi$ is a partial pre-order, then only the strict relations $\prec_\Psi$ and $\prec_{\Psi\circ\mu}$ are identical. Namely, it may happen that $\omega =_\Psi \omega'$ and $\omega \sim_{\Psi\circ\mu} \omega'$ both hold for revision operator satisfying $(P_1) - (P_7)$ and $(C_1)$ (resp. $(C_2)$).

A similar remark also holds for Boutilier's postulate $(C_B)$.

**Theorem 4** *Let $\circ$ be a revision operator satisfying the postulates $(P1) - (P_7)$. Then:*

*$\circ$ satisfies $(C_B)$ iff $\circ$ satisfies $(Ip_5)$.*

When $\preceq_\Psi$ is a total pre-order, then $\prec_{\Psi\circ\mu}$ is uniquely defined for satisfying $(C_B)$. Theorem 4 shows that the unicity of the result is no longer valid and hence $\preceq_{\Psi\circ\mu}$ is not uniquely defined. Again, only the strict relation associated with $\preceq_{\Psi\circ\mu}$ is uniquely defined.

An example of revision operator satisfying $(C_B)$ is an extension to "natural revision" for partial pre-orders:

**Definition 3** *Let $\Psi$ be an epistemic state and $\mu$ be a propositional formula, the revised epistemic state $\Psi \circ_n \mu$ corresponds to the following partial pre-order:*

- *if $\omega, \omega' \in Min(Mod(\mu), \preceq_\Psi)$ then $\omega \preceq_{\Psi\circ_n\mu} \omega'$ iff $\omega \preceq_\Psi \omega'$,*
- *if $\omega, \omega' \notin Min(Mod(\mu), \preceq_\Psi)$ then $\omega \preceq_{\Psi\circ_n\mu} \omega'$ iff $\omega \preceq_\Psi \omega'$,*

- *if* $\omega \in Min(Mod(\mu), \preceq_\Psi)$ *and* $\omega' \notin Min(Mod(\mu), \preceq_\Psi)$ *then* $\omega \prec_{\Psi \circ_n \mu} \omega'$.

The first condition imposes that the order between the preferred models of $\mu$ is preserved. The second condition preserves the ordering between non-preferred models of $\mu$. Finally, the third condition imposes that any minimal models of $\mu$ is preferred to any other model.

**Example 1** *Let us consider the epistemic state* $\Psi$ *represented by Figure 3, where* $\omega_0 = \{\neg a, \neg b\}$, $\omega_1 = \{a, \neg b\}$, $\omega_2 = \{\neg a, b\}$, $\omega_3 = \{a, b\}$. *Let* $\mu$ *be a new piece of information such that* $Mod(\mu) = \{\omega_1, \omega_3\}$. *Then the revision of* $\Psi$ *by the new piece of information* $\mu$ *is represented by the partial pre-order* $\preceq_{\Psi \circ_n \mu}$ *described by Figure 4.*

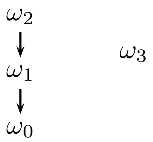

Figure 3: $\preceq_\Psi$

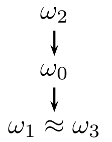

Figure 4: $\preceq_{\Psi \circ_n \mu}$

According to this definition it can be checked that the revision operator $\circ_n$ provided by definition 3 satisfies the postulates $(P_1) - (P_7)$ and $(C_B)$. Moreover, it can be noticed that $\circ_n$ does not satisfy $(R_2)$ since $Mod(Bel(\Psi)) = \{\omega_0, \omega_3\}$, $Mod(Bel(\Psi) \circ_n \mu) = \{\omega_1, \omega_3\}$ and $Mod(Bel(\Psi) \wedge \mu) = \{\omega_3\}$.

## 5 Two alternative postulates

Among additional postulates $(C_1) - (C_4)$ proposed in [Darwiche and Pearl, 1997] for iterated belief revision, $(C_1)$ seems to be very acceptable. However it is less obvious with $(C_2)$ (for more detailed discussion, see, for instance, [Lehmann, 1995; Friedman and Halpern, 1999]). Moreover, $(C_3)$ may appear to be too cautious. We now propose to analyze two alternative postulates which capture two other forms of iterated belief revision.

The first one departs from the idea of $(C_2)$, and it is close to the idea of conditioning with a completely sure information used in uncertainty theories. In probability theory (and similarly in possibility theory), if we revise a probability distribution by some completely sure formula, then all counter-models of these formulas are considered as impossible. This properties is captured by the following postulate.

$(C_{cond})$ $Bel((\Psi \circ \mu) \circ \neg\mu) \equiv \neg\mu$.

It expresses that if $\Psi$ is revised by $\mu$, and again by its contrary $\neg\mu$, then piece of initial beliefs are ignored, and only last pieces of information are retained. The characterization of this postulate is as follows.

**Theorem 5** *A revision operator* $\circ$ *satisfies the postulates* $(P1) - (P_7)$ *and* $(C_{cond})$ *iff its corresponding P-faithful assignment leads to* $\preceq_{\Psi \circ \mu}$, *which is such that if* $\omega \models \neg\mu$ *and* $\omega' \models \neg\mu$ *then neither* $\omega \prec_{\Psi \circ \mu} \omega'$, *nor* $\omega' \prec_{\Psi \circ \mu} \omega$ *holds.*

We illustrate this postulate by the following definition of the extension of qualitative possibilistic revision [Dubois and Prade, 1992].

**Definition 4** *Let* $\Psi$ *be an epistemic state such that its corresponding P-faithful assignment leads to* $\preceq_\Psi$. *Let* $\mu$ *be a propositional formula, The revised epistemic state* $\Psi \circ_\pi \mu$ *is defined by:*

- *if* $\omega, \omega' \in Mod(\mu)$ *then* $\omega \preceq_{\Psi \circ_\pi \mu} \omega'$ *iff* $\omega \preceq_\Psi \omega'$,
- *if* $\omega, \omega' \notin Mod(\mu)$ *then* $\omega =_{\Psi \circ_\pi \mu} \omega'$,
- *if* $\omega \in Mod(\mu)$ *and* $\omega' \notin Mod(\mu)$ *then* $\omega \prec_{\Psi \circ_\pi \mu} \omega'$.

Namely, $\preceq_\Psi$ and $\preceq_{\Psi \circ_\pi \mu}$ are identical on subdomain $Mod(\mu) \times Mod(\mu)$. Each models of $\mu$ are preferred to each counter-models of $\mu$. And lastly, the counter-models of $\mu$ are considered equally preferred.

**Example 2** *Let us again consider the epistemic state* $\Psi$ *represented by Figure 3. Let* $\mu$ *be a new information such that* $Mod(\mu) = \{\omega_1, \omega_3\}$. *Then, the revision of* $\Psi$ *by the new information* $\mu$ *is represented by the partial pre-order* $\preceq_{\Psi \circ_\pi}$ *described by figure 5*

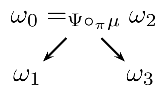

Figure 5: $\preceq_{\Psi \circ_\pi \mu}$

According to this definition it can be checked that the revision operator $\circ_\pi$ provided by definition 4 satisfies the postulates $(P_1) - (P_7)$, $(C_1)$, $(C_3) - (C_4)$ and $(C_{cond})$.

The second postulate concerns relationships between models and counter-models of the new piece of information $\mu$. The postulate $(C_3)$ only requires if some strict preference in $\preceq_\Psi$ holds, then this preference should also hold in $\preceq_{\Psi \circ \mu}$. We hence propose a stronger version of $(C_3)$, called $(C_{lex})$, and characterized by:

$(C_{lex})$ If $\alpha$ is not contradictory, $\alpha \models \mu$ and $\beta \models \neg\mu$ then $Bel((\Psi \circ \mu) \circ (\alpha \vee \beta)) \equiv Bel(\Psi \circ \alpha)$.

Note also that $(C_{lex})$ implies $(C_1)$, by replacing $\beta$ by the contradiction $\bot$.

**Theorem 6** *A revision operator* $\circ$ *satisfies the postulates* $(P1) - (P_7)$ *and* $(C_{lex})$ *iff its associated P-faithful assignments leading to* $\preceq_\Psi$ *and* $\preceq_{\Psi \circ \mu}$ *are such that*

- *if* $\omega, \omega' \models \mu$ *then* $\omega \prec_\Psi \omega'$ *iff* $\omega \prec_{\Psi \circ \mu} \omega'$,
- *if* $\omega \models \mu$ *and* $\omega' \not\models \mu$ *then* $\omega \prec_{\Psi \circ \mu} \omega'$.

Revision operator $\circ$ satisfying $(C_{lex})$ constraints each models to be strictly preferred to all counter-models of the new piece of information $\mu$. Moreover, it preservers the order between the models of $\mu$. An example of revision operator satisfying $(C_{lex})$ is the revision with memory proposed in [Papini, 2001] and hinted by Spohn [Spohn, 1988] (see also [Nayak, 1994; Konieczny and Pino Pérez, 2000]).

**Definition 5** *Let $\Psi$ be an epistemic state and $\mu$ be a propositional formula, the revised epistemic state $\Psi \circ_{\triangleright} \mu$ corresponds to the following partial pre-order:*

- *if $\omega$, $\omega' \in Mod(\mu)$ then $\omega \preceq_{\Psi \circ_{\triangleright} \mu} \omega'$ iff $\omega \preceq_{\Psi} \omega'$,*
- *if $\omega$, $\omega' \notin Mod(\mu)$ then $\omega \preceq_{\Psi \circ_{\triangleright} \mu} \omega'$ iff $\omega \preceq_{\Psi} \omega'$,*
- *if $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega \prec_{\Psi \circ_{\triangleright} \mu} \omega'$.*

**Example 3** *Let us again consider the epistemic state $\Psi$ represented by Figure 3. Let $\mu$ be a new information such that $\mu = a$. We have $Mod(\mu) = \{\omega_1, \omega_3\}$. Then, the revision of $\Psi$ by the new information $\mu$ is represented by the partial pre-order $\preceq_{\Psi \circ_{\triangleright}}$ described by figure 6*

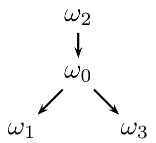


Figure 6: $\preceq_{\Psi \circ_{\triangleright} \mu}$

## 6 Conclusion

This paper has first proposed a new definition of faithful assignment suitable for revising partially ordered information. We then propose a revised version of Katsuno and Mendelzon's postulates. A representation theorem has been proposed for any revision operator satisfying this new set of postulates.

The paper also proposed a characterization of Darwiche and Pearl's postulates and a characterization of Boutilier's postulate when information is partially ordered. Two new postulates with their representation theorem have been proposed for capturing two other forms of belief revision.

When dealing with partial pre-orders, we have shown that revision operators satisfying iterated revision postulates such $C_1$ does not allow to guarantee that $\preceq_{\Psi}$ and $\preceq_{\Psi \circ \mu}$ to be identical on some subdomains (for instance on $Mod(\mu) \times Mod(\mu)$ if $C_1$ is satisfied). A future work will be to investigate additional postulates that preserve equality on some subdomains. For instance, it would be interesting to consider a stronger version of $P_2$, by requiring equality between epistemic states instead of logical equivalence between belief sets.

Even if results of this paper are oriented belief revision, it can be interesting to discuss this work in the framework of updating [Katsuno and Mendelzon, 1992] (since updating has also questioned $(R_2)$, even if representation theorems are clearly different).

Another future work is to extend results to this paper to the revision of a partial pre-order by a partial pre-order and extends, for instance, postulate proposed in [Benferhat *et al.*, 2000].

## References

[Alchourrón *et al.*, 1985] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet functions for contraction and revision. *Journal of Symbolic Logic*, 50:510–530, 1985.

[Benferhat *et al.*, 2000] Salem Benferhat, Sébastien Konieczny, Odile Papini, and Ramón A. Pino Pérez. Iterated revision by epistemic states : Axioms, semantics and syntax. In *Proceedings of European Conference on Artificial Intelligence (ECAI'2000)*, pages 13–17, 2000.

[Boutilier, 1993] Craig Boutilier. Revision sequences and nested conditionals. In *Proceedings of the 13th International Joint Conference on Artificial Intelligence (IJCAI'93)*, pages 519–531, 1993.

[Darwiche and Pearl, 1997] Adnan Darwiche and Judea Pearl. On the logic of iterated belief revision. *Artificial intelligence*, 89:1–29, 1997.

[Dubois and Prade, 1992] Didier Dubois and Henri Prade. Belief change and possibility theory. In Peter Gärdenfors, editor, *Belief Revision*, pages 142–182. Cambridge University Press, 1992.

[Friedman and Halpern, 1999] Nir Friedman and Joseph Y. Halpern. Belief revision: a critique. *Journal of Logic, Language, and Information*, 8:401–420, 1999.

[Katsuno and Mendelzon, 1991] Hirofumi Katsuno and Alberto O. Mendelzon. Propositional knowledge base revision and minimal change. *Artificial Intelligence*, 52:263–294, 1991.

[Katsuno and Mendelzon, 1992] Hirofumi Katsuno and Alberto O. Mendelzon. On the difference between updating a knowledge base and revising it. In *Belief Revision*, pages 183–203. Cambridge University Press, 1992.

[Konieczny and Pino Pérez, 2000] Sébastien Konieczny and Ramón Pino Pérez. A framework for iterated revision. *Journal of Applied Non-Classical Logics*, 10(3-4):339–367, 2000.

[Lehmann, 1995] Daniel Lehmann. Belief revision, revised. In *Proceedings of 14th Int. Joint Conf. on Artificial Intelligence*, pages 1534–1539, 1995.

[Nayak, 1994] Abhaya C. Nayak. Iterated belief change based on epistemic entrenchment. *Erkenntnis*, 41(1994):353–390, 1994.

[Papini, 2001] Odile Papini. Iterated revision operations stemming from the history of an agent's observations. In Hans Rott and Mary-Anne Williams, editors, *Frontiers of Belief Revision*, pages 279–301. Kluwer Academic, 2001.

[Rott, 2001] Hans Rott. *Change, Choice and Inference: A Study of Belief Revision and Nonmonotonic Reasoning*. Oxford University Press, 2001.

[Spohn, 1988] Wolfgang Spohn. Ordinal conditional functions: a dynamic theory of epistemic state. *Causation in Decision, Belief Change and Statistics*, pages 105–134, 1988.