A Unified Framework of Propositional Knowledge Base Revision and Update Based on State Transition Models

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Abstract

Belief revision and belief update are two of the most basic types of belief change operations. We need to select either revision or update when we accept new information into the current belief, however, such decision making has not been considered. In this paper, we propose a unified framework of revision and update based on state transition models that enable us to do such decision making. This framework provides a hybrid operation of revision and update, called acceptance.

1 Introduction

Belief revision [Alchourrón *et al.*, 1985] and belief update [Katsuno and Mendelzon, 1992] are two of the most basic types of belief change operations. When we accept new information into the current belief, we need to estimate whether the new information represents more reliable information about a static world, or it reports some (unspecified) change occurred in a dynamic world. This estimation causes decision making to select either revision to fix some errors in the current belief, or update to reflect some change into the belief.

In this paper, we introduce a *state transition model* as a unified framework of belief revision and belief update. The state transition model illustrates *prior knowledge* for estimation about the background of the new information, and also provides a *selection mechanism* for the decision making. Using the state transition model, we also propose a hybrid operation, called *acceptance*, of revision and update.

2 Knowledge Base Revision and Update

Katsuno and Mendelzon [Katsuno and Mendelzon, 1991] have rephrased the AGM postulates for revision [Alchourrón *et al.*, 1985], and have provided a possible worlds characterization of revision. For a given propositional sentence KB that represents the current knowledge base, and a propositional sentence α that represents new information about a static world, $KB \circ \alpha$ denotes a *revision* of KB by α. Revision operators are characterized by postulates (R1) – (R6).

(R1) $KB \circ \alpha \models \alpha$.

(R2) If $KB \wedge \alpha$ is satisfiable, then $KB \circ \alpha \equiv KB \wedge \alpha$.

(R3) If α is satisfiable, then $KB \circ \alpha$ is also satisfiable.

(R4) If $\models \alpha \leftrightarrow \beta$, then $KB \circ \alpha \equiv KB \circ \beta$.

(R5) $(KB \circ \alpha) \wedge \beta \models KB \circ (\alpha \wedge \beta)$.

(R6) If $(KB \circ \alpha) \wedge \beta$ is satisfiable, then $KB \circ (\alpha \wedge \beta) \models (KB \circ \alpha) \wedge \beta$.

Katsuno and Mendelzon [Katsuno and Mendelzon, 1992] have proposed a general characterization of belief update. For a given knowledge base KB and a sentence α that represents new information by some (unspecified) change in a dynamic world, $KB \diamond \alpha$ denotes an *update* of KB by α. Update operators are characterized by postulates (U1) – (U8).

(U1) $KB \diamond \alpha \models \alpha$.

(U2) If $KB \models \alpha$, then $KB \diamond \alpha \equiv KB$.

(U3) If both KB and α are satisfiable, then $KB \diamond \alpha$ is also satisfiable.

(U4) If $\models \alpha \leftrightarrow \beta$, then $KB \diamond \alpha \equiv KB \diamond \beta$.

(U5) $(KB \diamond \alpha) \wedge \beta \models KB \diamond (\alpha \wedge \beta)$.

(U6) If $KB \diamond \alpha \models \beta$ and $KB \diamond \beta \models \alpha$, then $KB \diamond \alpha \equiv KB \diamond \beta$.

(U7) If KB is complete, then $(KB \diamond \alpha) \wedge (KB \diamond \beta) \models KB \diamond (\alpha \vee \beta)$.

(U8) $(KB_1 \vee KB_2) \diamond \alpha \equiv (KB_1 \diamond \alpha) \vee (KB_2 \diamond \alpha)$.

3 State Transition Models

We introduce a *state transition model* as prior knowledge for estimation about the background of the new information. State transition models are closely related to *event models* for abductive characterization of belief update [Boutilier, 1996].

Definition 1 *A state transition model is a triple $\langle W, T, \preceq \rangle$, where W is a non-empty set of possible worlds, $T \subseteq W \times W$ is a non-empty set of state transitions, and $\preceq$ is a total preorder on T.*

$(x, y) \in T$ is a state transition from the *starting point* x to the *terminal* y. The intuitive meaning of $(x, y) \in T$ is "we *know* that some (unspecified) change at x may cause the situation y". The total preorder $\preceq$ illustrates relative plausibility of state transitions. If we have $(x, y) \preceq (u, v)$, we interpret that (x, y) is at least as plausible as is (u, v). Using the state transition model, we can represent the following two criteria about relative plausibility of possible worlds:

- Comparing (x, y) and (u, v) such that $x \neq u$: the relative plausibility of x and u as the actual world.
- Comparing (x, y) and (x, v) with the same starting point x: the relative plausibility of y and v as the result of some change at x.

Definition 2 *For any $R \subseteq T$, we define $Sp(R) \subseteq W$ and $Ter(R) \subseteq W$ as follows, respectively:*

$$Sp(R) = \{x \in W \mid \exists(x, y) \in R\}, \quad (1)$$
$$Ter(R) = \{y \in W \mid \exists(x, y) \in R\}. \quad (2)$$

If $R = \emptyset$, we define $Sp(R) = Ter(R) = \emptyset$.

For any $X \subseteq W$ and $Y \subseteq W$, we define $(X, Y) \subseteq T$ by:

$$(X, Y) = \{(x, y) \in T \mid x \in X, y \in Y\}. \quad (3)$$

If either $X = \emptyset$ or $Y = \emptyset$, we define $(X, Y) = \emptyset$. For any singleton $\{w\} \subseteq W$, we abbreviate $(\{w\}, X)$ and $(X, \{w\})$ as (w, X) and (X, w), respectively.

Definition 3 *Let STM be a state transition model. STM is called centered iff the following two conditions hold:*

1. *For each world $w \in W$, $(w, w) \in T$.*
2. *For any non-empty subset $X \subseteq W$ and any $w \in X$, the loop (w, w) is the minimum element in (w, X), that is, if $(w, x) \in (w, X)$ and $x \neq w$, then $(w, w) \prec (w, x)$.*

Using the given state transition model, the current knowledge base KB is semantically characterized by starting points of the most plausible state transitions in T.

Definition 4 *Let STM be a state transition model. A knowledge base KB induced by STM is a propositional sentence such that*

$$\|KB\| = Sp\left(\min_{\preceq} T\right). \quad (4)$$

4 Acceptance: A Hybrid Operation of Revision and Update

Using the given state transition model that illustrates prior knowledge for estimation, we provide a selection mechanism to decide we use either revision or update when we accept new information. *Explainability* of the new information we define below is the key concept of such decision making.

Definition 5 *Let STM be a state transition model, and KB is a knowledge base induced by STM. A sentence α is explainable by STM iff $(w, \|\alpha\|) \neq \emptyset$ for all $w \in \|KB\|$.*

We have the following simple selection strategy by explainability of the new information: Let $CSTM = \langle W, T, \preceq \rangle$ be a *centered* state transition model, and KB be the current knowledge base induced by $CSTM$. For any observation α,

1. If α is explainable by $CSTM$, we regard α as the new information by some change, and *update* KB by α.
2. Otherwise, we regard α as more reliable information about a static world, and *revise* KB by α.

According to the selection strategy, we introduce a hybrid operation of revision and update, called *acceptance*, based on the given centered state transition model $CSTM$. We use $KB \lhd \alpha$ to denote the result of acceptance of α into KB. The symbol $\lhd$ is called an *acceptance operator*. We intend to have either $KB \lhd \alpha \equiv KB \circ \alpha$ or $KB \lhd \alpha \equiv KB \diamond \alpha$ based on explainability of α by $CSTM$.

Theorem 1 *Let $CSTM$ be a centered state transition model, KB be a knowledge base induced by $CSTM$, and $\lhd$ be an acceptance operator defined by the following equation:*

$$\|KB \lhd \alpha\| = \begin{cases} \bigcup_{w \in \|KB\|} \left\{ Ter\left(\min_{\preceq}(w, \|\alpha\|)\right)\right\} & \text{if } \alpha \text{ is explainable by } CSTM, \\ Sp\left(\min_{\preceq}(\|\alpha\|, W)\right) & \text{otherwise.} \end{cases} \quad (5)$$

Then, for any sentence α,

1. *If α is explainable by $CSTM$, then $\lhd$ satisfies postulates (U1) – (U4), the following weakened (U5):*

   (U5w) *If $\alpha \wedge \beta$ is explainable, then $(KB \diamond \alpha) \wedge \beta \models KB \diamond (\alpha \wedge \beta)$,*

   postulates (U6), (U7), and a postulate (U9) proposed by Boutilier [Boutilier, 1996]:

   (U9) *If KB is complete, $(KB \diamond \alpha) \not\models \neg\beta$ and $KB \diamond \alpha \models \gamma$, then $KB \diamond (\alpha \wedge \beta) \models \gamma$.*
2. *If α is not explainable by $CSTM$, then $\lhd$ satisfies postulates (R1) – (R6).*

The acceptance operator $\lhd$ is well-defined as a revision operator $\circ$ when α is not explainable by $CSTM$. On the other hand, $\lhd$ does not satisfy (U8) when α is explainable. However, in equation (5), α is independently evaluated in each possible world $w \in \|KB\|$, therefore the idea of semantic characterization of KM update [Katsuno and Mendelzon, 1992] is illustrated in this framework.

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