Repairing Preference-Based Argumentation Frameworks

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Abstract

Argumentation is a reasoning model based on the construction and evaluation of arguments. Dung has proposed an abstract argumentation framework in which arguments are assumed to have the same strength. This assumption is unfortunately not realistic. Consequently, three main extensions of the framework have been proposed in the literature. The basic idea is that if an argument is stronger than its attacker, the attack fails.

The aim of the paper is twofold: First, it shows that the three extensions of Dung framework may lead to unintended results. Second, it proposes a new approach that takes into account the strengths of arguments, and that ensures sound results. We start by presenting two minimal requirements that any preference-based argumentation framework should satisfy, namely the conflict-freeness of arguments extensions and the generalization of Dung's framework. Inspired from works on handling inconsistency in knowledge bases, the proposed approach defines a binary relation on the powerset of arguments. The maximal elements of this relation represent the extensions of the new framework.

1 Introduction

Preferences are used in most models that have been developed to solve conflicts (e.g. [Benferhat *et al.*, 1993; Brewka, 1989; Cayrol *et al.*, 1993; Gelfond and Son, 1997]). Conflicts have a better chance to be solved in presence of such information. Preferences have also been introduced into *argumentation* theory. Argumentation is a reasoning model based on the construction and the evaluation of arguments. An argument gives a reason to believe a statement, to perform an action, or to choose an option, etc. Due to its explanatory power, argumentation is gaining an increasing interest in Artificial Intelligence, namely for handling inconsistency (e.g. [Amgoud and Cayrol, 2002a; Besnard and Hunter, 2008; Simari and Loui, 1992]), and decision making (e.g. [Amgoud and Prade, 2009]). Most of the models that treat the cited applications are instantiations of an abstract framework developed in [Dung, 1995]. This framework consists of a set of arguments and a binary relation that captures attacks among arguments. Arguments are assumed to have all the same strength. This assumption is unfortunately not realistic since it may be the case that an argument relies on certain information, while another argument is built from less certain ones. The former is clearly stronger than the latter. In [Benferhat *et al.*, 1993; Cayrol *et al.*, 1993; Prakken and Sartor, 1997; Simari and Loui, 1992] different preference relations between arguments have been defined. A preference relation captures differences in arguments' strengths.

In [Amgoud and Cayrol, 2002b], a first extension of Dung's framework has been proposed. It takes as input a set of arguments, an attack relation, and a preference relation between arguments. This relation is abstract and can be instantiated in different ways. This proposal has recently been generalized in [Modgil, 2009] in order to reason even about preferences. Thus, arguments may support preferences about arguments. The last extension of Dung's framework has been proposed in [Bench-Capon, 2003]. It assumes that each argument promotes a value, and a preference between two arguments comes from the importance of the respective values that are promoted by the two arguments. Whatever the source of the preference relation is, the idea behind the three extensions is that an attack from an argument a to an argument b fails if b is stronger than a.

The aim of the paper is twofold: First, it shows that while the above idea is interesting and seems meaningful, the three extensions return unintended results. Second, it proposes a new preference-based argumentation framework (PAF) that ensures sound results. We start by defining two basic requirements that any PAF should satisfy. Namely, the extensions should be conflict-free w.r.t. the attack relation. The second requirement consists of recovering Dung's acceptability extensions in case preferences are not available. We then propose a new approach which defines a binary relation on the powerset of arguments. The maximal elements of this relation are the extensions of the new framework. Three relations are particularly proposed in the paper. They capture respectively stable extensions, preferred extensions and grounded extensions of Dung's framework.

The paper is organized as follows: Section 2 recalls briefly Dung's framework as well as its extensions with preferences. Section 3 presents their limits through a simple example. Section 4 develops the new approach, and Section 5 concludes.

2 Dung's Framework And Its Extensions

In the seminal paper [Dung, 1995], an *argumentation framework* is a pair $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a set of arguments and $\mathcal{R}$ is a binary relation between arguments, representing attacks among them ($\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$). The notation $(a, b) \in \mathcal{R}$ or $a\mathcal{R}b$ means that the argument a attacks the argument b. Different acceptability semantics for evaluating arguments have been proposed. Before recalling them, let us first define the notions of conflict-free and defense.

Definition 1 (Conflict-free, Defense) *Let $\mathcal{B} \subseteq \mathcal{A}$ and $a \in \mathcal{A}$.*

- $\mathcal{B}$ *is* conflict-free *iff* $\nexists$ *a, b* $\in \mathcal{B}$ *s.t.* $a\mathcal{R}b$.
- $\mathcal{B}$ defends a *iff* $\forall b \in \mathcal{A}$ *if* $b\mathcal{R}a$, *then* $\exists c \in \mathcal{B}$ *s.t.* $c\mathcal{R}b$.

The main semantics introduced by Dung are recalled in the following definition.

Definition 2 (Acceptability semantics) *Let $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, and $\mathcal{B}$ be a conflict-free set of arguments.*

- $\mathcal{B}$ *is a* admissible *iff it defends all its elements.*
- $\mathcal{B}$ *is a* preferred extension *iff it is a maximal (w.r.t. set $\subseteq$) admissible set.*
- $\mathcal{B}$ *is a* stable extension *iff it is a preferred extension that attacks any argument in* $\mathcal{A} \setminus \mathcal{B}$.
- $\mathcal{B}$ *is a* grounded extension, *denoted* GE, *iff* $\mathcal{B}$ *is the least fixpoint of a function* $\mathcal{F}$ *where* $\mathcal{F}(S) = \{a \in \mathcal{A} \mid S$ *defends* $a\}$, *for* $S \subseteq \mathcal{A}$.

In [Amgoud and Cayrol, 2002b], a first extension of Dung's framework has been proposed. It takes as input a set $\mathcal{A}$ of arguments, an attack relation $\mathcal{R}$, and a (partial or total) preorder[1] $\geq$ on $\mathcal{A}$. This preorder is a preference relation between arguments. The expression $(a, b) \in \geq$ or $a \geq b$ means that the argument a is at least as strong as b. The symbol $>$ denotes the strict relation associated with $\geq$. Indeed, $a > b$ iff $a \geq b$ and not $(b \geq a)$. From the two relations $\mathcal{R}$ and $\geq$, a new binary relation, Def, is defined as follows: a Def b iff $a\mathcal{R}b$ and not $(b > a)$. This means that among all the attacks in $\mathcal{R}$, only the ones that hold between incomparable and indifferent arguments and the ones that agree with the preference relation are kept. In order to evaluate the acceptability of the arguments, Dung's acceptability semantics are applied to the framework $\langle \mathcal{A}, \texttt{Def} \rangle$.

In [Modgil, 2009], the preference relation $\geq$ is given by arguments.The idea is that an argument may support a preference between two other arguments. Two attack relations are assumed: a classical one denoted by $\mathcal{R}$, and another relation, $\mathcal{D}$, that ranges from an argument of $\mathcal{A}$ to an element of $\mathcal{R}$. An expression $(a, (b, c))$ means that the argument a supports a preference of c over b. This preference conflicts with the fact that b attacks c. A new relation, Def, is defined as follows: a $\texttt{Def}_{\mathcal{S}}$ b iff $a\mathcal{R}b$ and $\exists c \in \mathcal{S}$ such that $(c, (a, b)) \in \mathcal{D}$, where $\mathcal{S} \subseteq \mathcal{A}$.

[1] A binary relation is a *preorder* iff it is *reflexive* and *transitive*.

The extension proposed in [Bench-Capon, 2003], called value-based framework, assumes that a set $\mathcal{V}$ of values is available. Each argument in $\mathcal{A}$ promotes one value given by a function val (i.e. val : $\mathcal{A} \mapsto \mathcal{V}$). The values may not have the same importance and this is captured by a binary relation Pref. This latter is assumed to be irreflexive, asymmetric and transitive. Like in [Amgoud and Cayrol, 2002a], a new relation, called defeats, is defined as follows: $(a, b) \in \texttt{defeats}$ iff $(a, b) \in \mathcal{R} \wedge (\texttt{val}(b), \texttt{val}(a)) \notin \texttt{Pref}$. Dung's acceptability semantics are applied to the framework $\langle \mathcal{A}, \texttt{defeats} \rangle$.

3 Critical Examples

This section shows through a simple example that the above two extensions may lead to counter-intuitive results.

Let us consider a case of an agent who wants to buy a given violin. An expert says that the violin in question is produced by Stradivari (s), that's why it is expensive ($s \rightarrow e$). This agent has thus an argument a_1 whose conclusion is "the violin is expensive". Suppose now that the 3-years old son of this agent says that the violin was not produced by Stradivari ($\neg s$). Thus, an argument a_2 which attacks a_1 is given. In sum, $\mathcal{A} = \{a_1, a_2\}$ and $\mathcal{R} = \{(a_2, a_1)\}$. According to Dung's framework, argument a_2 wins. This is inadmissible, especially since it is clear that an argument of an expert is stronger than an argument given by a 3-years old child. In the framework presented in [Amgoud and Cayrol, 2002a], the fact that a_1 is stronger than a_2 is taken into account. Thus, the relation $\geq = \{(a_1, a_1), (a_2, a_2), (a_1, a_2)\}$ is available. However, in this framework the relation Def is empty. Consequently, both arguments are in the unique preferred extension of the framework $\langle \mathcal{A}, \texttt{Def} \rangle$. This means that this extension is not conflict-free. Moreover, both s and $\neg s$ are deduced.

According to [Modgil, 2009], there are three arguments: a_1, a_2 and a_3 where a_3 expresses the fact that a_1 is strictly preferred to a_2. Thus, $\mathcal{A} = \{a_1, a_2, a_3\}$, $\mathcal{R} = \{(a_2, a_1)\}$, and $\mathcal{D} = \{(a_3, (a_2, a_1))\}$. The set $\{a_1, a_2, a_3\}$ is a preferred extension which is not conflict-free w.r.t. $\mathcal{R}$.

The same problem holds in the value-based framework of [Bench-Capon, 2003]. Assume that the set of values is $V = \{expert, child\}$ and, of course, $(expert, child) \in \texttt{Pref}$. The value of a_1 is expert while the value of a_2 is child. The new relation defeats is empty. So, like with the previous preference-based framework, the two arguments appear in the same extension which is not conflict-free.

4 A New Approach

The previous section has highlighted the limits of existing preference-based argumentation frameworks. Even if the idea pursued by these frameworks is intuitive and meaningful, their results are not satisfactory and violate a key property. This property concerns the conflict-freeness of their extensions w.r.t. the attack relation $\mathcal{R}$. In this section, we propose a new preference-based argumentation framework. It takes as input three elements: a set $\mathcal{A}$ of arguments, an attack relation

$\mathcal{R}$, and a (partial or total) preorder $\geq$. It returns *extensions* that are subsets of $\mathcal{A}$. These extensions satisfy the two following basic requirements:

Conflict-freeness: If $\mathcal{E}$ is an extension of $(\mathcal{A}, \mathcal{R}, \geq)$, then $\mathcal{E}$ is conflict free w.r.t. $\mathcal{R}$.

Generalization: If $(\nexists a, b \in \mathcal{A})$ s.t. $(a, b) \in \mathcal{R}$ and $(b, a) \in >$, then any extension of $(\mathcal{A}, \mathcal{R}, \geq)$ is also an extension of Dung's framework $\langle \mathcal{A}, \mathcal{R} \rangle$ and vice versa.

The first requirement ensures that the extensions returned by the new framework are conflict-free. This is important since it ensures safe results in the sense that inconsistent conclusions are avoided. The second one captures the idea that an attack fails in case the attacker is weaker than its target. Moreover, it states that the proposed approach extends Dung's framework, i.e. it refines its acceptability semantics.

In what follows, we show how extensions of a PAF are computed. We follow the same reasoning as in some approaches developed for handling inconsistency in knowledge bases, namely the coherence-based ones. In [Cayrol *et al.*, 1993], for instance, an inconsistent knowledge base Σ is equipped with a partial or total preorder, meaning that formulas of Σ have not the same priority. Then, preference relations among consistent sub-bases of Σ are defined. The maximal elements w.r.t. those preference relations represent the preferred ones. We apply the same idea in an argumentation context. Indeed, by analogy, the inconsistent base represents our conflicting set of arguments, and the priorities between formulas of Σ are the preferences between arguments. We define preference relations, denoted by $\succeq$, between the different conflict-free sets of arguments. Thus, $\succeq \subseteq 2^{\mathcal{A}} \times 2^{\mathcal{A}}$. The relation $\succ$ is the strict version of $\succeq$, that is for $\mathcal{E}, \mathcal{E}' \subseteq \mathcal{A}$, $\mathcal{E} \succ \mathcal{E}'$ iff $\mathcal{E} \succeq \mathcal{E}'$ and not $(\mathcal{E}' \succeq \mathcal{E})$. The maximal elements of $\succeq$ are the extensions of a PAF. This notion of maximality is defined as follows.

Definition 3 (Maximal elements) *Let $\mathcal{E}$ be a conflict-free set of arguments. $\mathcal{E}$ is* maximal *w.r.t. $\succeq$ iff:*

1. $(\forall \mathcal{E}' \subseteq \mathcal{A})$ $((\mathcal{E}'$ *is conflict-free*$) \Rightarrow (\mathcal{E} \succeq \mathcal{E}'))$
2. *No strict superset of $\mathcal{E}$ is conflict-free and verifies* (1)

Let $\succeq_{max}$ denote the set of maximal sets w.r.t. $\succeq$.

The above definition privileges maximal (for set inclusion) sets of arguments among the conflict-free ones. It is worth mentioning that different relations $\succeq$ can be defined, and may lead to different sets of extensions. Moreover, those extensions are not necessarily the ones got by Dung's acceptability semantics. The new preference-based argumentation framework is defined as follows.

Definition 4 (PAF) *A* PAF *is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is an attack relation, and $\geq$ is a (partial or total) preorder on $\mathcal{A}$. Extensions of $(\mathcal{A}, \mathcal{R}, \geq)$ are the maximal elements of a relation $\succeq \subseteq 2^{\mathcal{A}} \times 2^{\mathcal{A}}$ that satisfies the two basic requirements.*

In what follows, we propose three relations which generalize respectively stable, preferred and grounded semantics. Note that, without loss of generality, we assume that there are no self-attacking arguments, i.e., $(\nexists x \in \mathcal{A})$ s.t. $(x, x) \in \mathcal{R}$. Moreover, the set $\mathcal{A}$ is assumed to be finite.

4.1 Generalization Of Stable Semantics

This section presents a relation $\succeq$ that generalizes stable semantics. The idea behind this relation is the following: given two conflict-free sets of arguments, $\mathcal{E}$ and $\mathcal{E}'$, we say that $\mathcal{E}'$ is better then $\mathcal{E}$ iff any argument in $\mathcal{E} \setminus \mathcal{E}'$ is weaker than at least one argument in $\mathcal{E}' \setminus \mathcal{E}$ or is attacked by it. Formally:

Definition 5 *Let $\mathcal{E}, \mathcal{E}'$ be two conflict-free sets of arguments. $\mathcal{E}' \succeq \mathcal{E}$ iff $(\forall x \in \mathcal{E} \setminus \mathcal{E}')$ $(\exists x' \in \mathcal{E}' \setminus \mathcal{E})$ s.t. $(((x', x) \in \mathcal{R} \wedge (x, x') \notin >) \vee (x', x) \in >)$.*

Let us illustrate this definition through the following simple example.

Example 1 *Let $\mathcal{A} = \{a, b, c\}$, $\geq = \{(a, a), (b, b), (a, b)\}$ and $\mathcal{R} = \{(a, b), (b, a), (b, c), (c, b)\}$. The conflict-free sets of arguments are: $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{a\}$, $\mathcal{E}_3 = \{b\}$, $\mathcal{E}_4 = \{c\}$, and $\mathcal{E}_5 = \{a, c\}$. It can be checked that the following relations hold: $\mathcal{E}_2 \succeq \mathcal{E}_1$, $\mathcal{E}_3 \succeq \mathcal{E}_1$, $\mathcal{E}_4 \succeq \mathcal{E}_1$, $\mathcal{E}_5 \succeq \mathcal{E}_1$, $\mathcal{E}_5 \succeq \mathcal{E}_4$, $\mathcal{E}_5 \succeq \mathcal{E}_2$, $\mathcal{E}_5 \succeq \mathcal{E}_3$, $\mathcal{E}_4 \succeq \mathcal{E}_3$, $\mathcal{E}_3 \succeq \mathcal{E}_4$, and $\mathcal{E}_2 \succeq \mathcal{E}_3$. It can be checked that $\succeq_{max} = \{\mathcal{E}_5\}$.*

Note that relation $\succeq$ is not transitive. However, the following property states that this relation privileges maximal for set inclusion elements.

Property 1 *Let $\mathcal{E}, \mathcal{E}'$ be conflict-free sets of arguments. If $\mathcal{E} \subsetneq \mathcal{E}'$ then $\mathcal{E}' \succ \mathcal{E}$.*

Proof *Let us prove that $\mathcal{E}' \succeq \mathcal{E}$. We have that $\mathcal{E} \setminus \mathcal{E}' = \emptyset$, and, consequently, there are no arguments in $\mathcal{E} \setminus \mathcal{E}'$. Let us now see why $\neg(\mathcal{E} \succeq \mathcal{E}')$. Since $\mathcal{E} \subsetneq \mathcal{E}'$, then $\exists x' \in \mathcal{E}' \setminus \mathcal{E}$. But, from the fact that $\mathcal{E} \setminus \mathcal{E}'$ is empty, we conclude that $\nexists x \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(x, x') \in >$ or $((x, x') \in \mathcal{R} \wedge (x', x) \notin >)$.* ■

With the above relation, Definition 3 can be simplified.

Property 2 *Let $\mathcal{E}'$ be a conflict-free set of arguments. It holds that $\mathcal{E}' \in \succeq_{max}$ iff $(\forall \mathcal{E} \subseteq \mathcal{A})$ $((\mathcal{E}$ is conflict-free$) \Rightarrow (\mathcal{E}' \succeq \mathcal{E}))$.*

Proof *$\Rightarrow$ Trivial, according to Definition 3.*
$\Leftarrow$ Let $(\forall \mathcal{E} \subseteq \mathcal{A})$ $((\mathcal{E}$ is conflict-free$) \Rightarrow (\mathcal{E}' \succeq \mathcal{E}))$. We will prove that $(\nexists \mathcal{E}'' \subseteq \mathcal{A})$ s.t. $\mathcal{E}''$ is conflict-free $\wedge$ $\mathcal{E}' \subsetneq \mathcal{E}''$ $\wedge$ $(\forall \mathcal{E}''' \subseteq \mathcal{A})$ $(\mathcal{E}'''$ conflict-free $\Rightarrow \mathcal{E}'' \succeq \mathcal{E}''')$. Suppose the contrary. Since $\mathcal{E}' \subsetneq \mathcal{E}''$ then Property 1 implies that $\neg(\mathcal{E}' \succeq \mathcal{E}'')$. Contradiction. ■

The following property shows that the maximal sets of arguments w.r.t. the relation $\succeq$ given in Definition 5 are maximal conflict-free subsets of $\mathcal{A}$.

Property 3 *Let $\mathcal{E}$ be a conflict-free set of arguments. If $\mathcal{E} \in \succeq_{max}$, then $\mathcal{E}$ is a maximal conflict-free set.*

Proof *Suppose the contrary, i.e., that $\mathcal{E} \in \succeq_{max}$ and that $\mathcal{E}$ is not a maximal conflict-free set. This means that $(\exists x \in \mathcal{A})$ s.t. $x \notin \mathcal{E}$ and $\mathcal{E} \cup \{x\}$ is conflict-free. According to Property 1, $(\mathcal{E} \cup \{x\}) \succ \mathcal{E}$. Contradiction with the fact $\mathcal{E} \in \succeq_{max}$.* ■

The converse is not true, as illustrated by the next example.

Example 2 (Ex. 1 Cont.) *The set $\mathcal{E}_3$ is maximal conflict-free but does not belong to $\succeq_{max}$.*

We can show that the proposed framework handles correctly the example discussed in Section 3.

Example 3 *Recall that* $\mathcal{A} = \{a_1, a_2\}$, $\mathcal{R} = \{(a_2, a_1)\}$ *and* $\geq = \{(a_1, a_1), (a_2, a_2), (a_1, a_2)\}$. *Conflict-free sets are:* $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{a_1\}$, $\mathcal{E}_3 = \{a_2\}$. *It can easily be checked that* $\succeq_{max} = \{\mathcal{E}_2\}$. *Thus, the new PAF has a unique extension which is* $\{a_1\}$.

The extensions of the new PAF are conflict-free w.r.t the attack relation $\mathcal{R}$.

Property 4 *Let* $(\mathcal{A}, \mathcal{R}, \geq)$ *be a preference-based argumentation framework. The extensions of this framework w.r.t. the relation* $\succeq$ *given in Definition 5 are conflict-free w.r.t.* $\mathcal{R}$.

Proof *This follows from the definition of* $\succeq$. ■

Regarding the second requirement, the following theorem proves that the extensions of our preference-based argumentation framework coincide with stable extensions in case preferences are not available and when any attacked argument is not stronger than its attacker.

Theorem 1 *Let* $(\mathcal{A}, \mathcal{R}, \geq)$ *be a preference-based argumentation framework, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *denote its extensions w.r.t.* $\succeq$. *If* $(\nexists x, y \in \mathcal{A})$ *s.t.* $(x, y) \in \mathcal{R} \wedge (y, x) \in >$, *then each* $\mathcal{E}_i$ *is a stable extension of* $\langle \mathcal{A}, \mathcal{R} \rangle$ *and vice versa.*

Proof $\Rightarrow$ *Let* $\mathcal{E}' \in \succeq_{max}$.

- *Since* $\mathcal{E}' \in \succeq_{max}$ *then it is conflict-free.*
- *We will now prove that* $\mathcal{E}'$ *defends all its elements. Let us suppose that* $(\exists a \in \mathcal{E}')$ $(\exists x \in \mathcal{A})$ *s.t.* $(x, a) \in \mathcal{R}$ $\wedge$ $(\nexists y \in \mathcal{E}')$ $(y, x) \in \mathcal{R}$. *Since* $\mathcal{E}'$ *is conflict-free, then* $x \notin \mathcal{E}'$. *Let* $\mathcal{E} = \{x\} \cup \{t \in \mathcal{E}' \mid (x, t) \notin \mathcal{R} \wedge (t, x) \notin \mathcal{R}\}$. *It is clear the* $\mathcal{E}$ *is conflict-free since* $\mathcal{E}$ *is union of two conflict-free sets which do not attack one another. Since* $\mathcal{E}' \in \succeq_{max}$ *then* $\mathcal{E}' \succeq \mathcal{E}$. *In particular, since* $x \in \mathcal{E} \setminus \mathcal{E}'$, *then* $(\exists x' \in \mathcal{E}' \setminus \mathcal{E})$ *s.t.* $((x', x) \in \mathcal{R} \wedge (x, x') \notin >) \vee (x', x) \in >$. *Since* $(\nexists y \in \mathcal{E}')$ $(y, x) \in \mathcal{R}$, *then it must be the case that* $(x', x) \notin \mathcal{R}$ *and* $(x', x) \in >$. *Since* $x' \in \mathcal{E}'$ *and* $x' \notin \mathcal{E}$ *then, with respect to definition of* $\mathcal{E}$, *from* $x' \notin \mathcal{E}$ *we have that* $(x, x') \in \mathcal{R}$ *or* $(x', x) \in \mathcal{R}$. *Since we have just seen that* $(x', x) \notin \mathcal{R}$, *it must be that* $(x, x') \in \mathcal{R}$. *Recall that we have* $(x', x) \in >$. *But we supposed that* $(\nexists z, z' \in \mathcal{A})$ *s.t.* $(z, z') \in \mathcal{R}$ *and* $(z', z) \in >$. *Contradiction. Thus,* $\mathcal{E}'$ *defends its arguments.*
- *We have just shown that* $\mathcal{E}'$ *is admissible, i.e., it is conflict-free and it defends all its arguments. We will now prove that* $\mathcal{E}'$ *attacks all arguments in* $\mathcal{A} \setminus \mathcal{E}'$. *Let* $x \notin \mathcal{E}'$ *be an argument and suppose that* $(\nexists y \in \mathcal{E}')$ $(y, x) \in \mathcal{R}$. *Either* x *attacks some argument of* $\mathcal{E}'$ *or not. If it is the case, i.e.,* $(\exists a \in \mathcal{E}')$ *s.t.* $(x, a) \in \mathcal{R}$ *then, since* $\mathcal{E}'$ *defends all its elements, it holds that* $(\exists y \in \mathcal{E}')$ *s.t.* $(y, x) \in \mathcal{R}$. *Contradiction. So, it must be that* $(\nexists a \in \mathcal{E}')$ *s.t.* $(x, a) \in \mathcal{R}$. *This means that* $\mathcal{E} = \mathcal{E}' \cup \{x\}$ *is conflict-free. According to Property 1, it holds that* $\neg(\mathcal{E}' \succeq \mathcal{E})$. *Contradiction with the fact that* $\mathcal{E}' \in \succeq_{max}$.

  So, $\mathcal{E}$ *is conflict-free and it attacks all arguments in* $\mathcal{A} \setminus \mathcal{E}$. *This means that* $\mathcal{E}$ *is a stable extension of framework* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$.

$\Leftarrow$ *Let* $\mathcal{E}'$ *be a stable extension of the framework* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *and let us prove that* $\mathcal{E}' \in \succeq_{max}$.

- *Since* $\mathcal{E}'$ *is stable then it is conflict-free.*
- *We will prove that for an arbitrary conflict-free set of arguments* $\mathcal{E}$ *it holds that* $\mathcal{E}' \succeq \mathcal{E}$. *Let* $\mathcal{E} \subseteq \mathcal{A}$ *be a conflict-free set. If* $\mathcal{E} \setminus \mathcal{E}' = \emptyset$ *the proof is over. If it is not the case, let* $x \in \mathcal{E} \setminus \mathcal{E}'$. *Since* $x \notin \mathcal{E}'$ *and* $\mathcal{E}'$ *is a stable extension, then* $(\exists x' \in \mathcal{E}')$ *s.t.* $(x', x) \in \mathcal{R}$. *We supposed that* $(\nexists z, z' \in \mathcal{A})$ *s.t.* $(z, z') \in \mathcal{R}$ *and* $(z', z) \in >$. *Thus,* $(x, x') \notin >$. *Since* $x \in \mathcal{E} \setminus \mathcal{E}'$ *was arbitrary, it holds that* $\mathcal{E}' \succeq \mathcal{E}$.
- *Using Property 2, we conclude that* $\mathcal{E}' \in \succeq_{max}$.

■

From this result it follows that when preferences are not available, stable extensions are retrieved.

Corollary 1 *Let* $(\mathcal{A}, \mathcal{R}, \geq)$ *be a preference-based argumentation framework, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *denote its extensions w.r.t.* $\succeq$. *If* $\geq = \{(x, x) \mid x \in \mathcal{A}\}$, *then each* $\mathcal{E}_i$ *is a stable extension of* $\langle \mathcal{A}, \mathcal{R} \rangle$ *and vice versa.*

Proof *Since* $\geq = \{(x, x) \mid x \in \mathcal{A}\}$ *then* $(\nexists x, y \in \mathcal{A})$ *s.t.* $x \neq y \wedge (x, y) \in \mathcal{R} \wedge (y, x) \in >$. *Since we supposed that* $(\nexists x \in \mathcal{A})$ *s.t.* $(x, x) \in \mathcal{R}$ *then* $(\nexists x, y \in \mathcal{A})$ *s.t.* $(x, y) \in \mathcal{R} \wedge (y, x) \in >$. *Thus, Theorem 1 implies that extensions of* $(\mathcal{A}, \mathcal{R}, \geq)$ *are exactly the stable extensions of* $\langle \mathcal{A}, \mathcal{R} \rangle$. ■

Note that the relation $\succeq$ gives more information than Dung's acceptability semantics. Indeed, even when preferences are not available, the relation $\succeq$ compares conflict-free sets of arguments, as can be shown on the following example.

Example 4 *Let* $\mathcal{A} = \{a, b, c\}$, $\geq = \{(a, a), (b, b), (c, c)\}$ *and* $\mathcal{R} = \{(a, b), (b, c)\}$. *Note that in this case, the preference relation* $\geq$ *is useless. Thus, the only stable extension of* $\langle \mathcal{A}, \mathcal{R} \langle$ *is* $\{a, c\}$. *This is also the only maximal element of the relation* $\succeq$. *However, in the new PAF, it is also possible to compare the two sets:* $\{a\}$ *and* $\{b\}$. *It can be checked that* $\{a\} \succeq \{b\}$.

4.2 Generalization Of Preferred Semantics

In this section, we define a relation $\succeq$ that allows to retrieve preferred extensions in case preferences between arguments are not available or are not important. The basic idea behind this relation is that a set $\mathcal{E}'$ is better than $\mathcal{E}$ iff for every attack from $\mathcal{E}$ to $\mathcal{E}'$ which does not fail $\mathcal{E}'$ is capable to defend the attacked argument and that for every attack from $\mathcal{E}'$ to $\mathcal{E}$ which fails, there is another attack from $\mathcal{E}'$ which defends the argument which failed in its attack.

Definition 6 *Let* $\mathcal{E}, \mathcal{E}'$ *be conflict-free sets of arguments.* $\mathcal{E}' \succeq \mathcal{E}$ *iff* $(\forall x' \in \mathcal{E}')(\forall x \in \mathcal{E})$ *if* $(((x, x') \in \mathcal{R} \wedge (x', x) \notin >)$ *or* $((x', x) \in \mathcal{R} \wedge (x, x') \in >))$ *then* $((\exists y' \in \mathcal{E}')$ *s.t.* $((y', x) \in \mathcal{R} \wedge (x, y') \notin >))$.

Let us illustrate this definition through the next example.

Example 5 (Ex. 1 Cont.) *One can easily see that it holds that* $\mathcal{E}_2 \succ \mathcal{E}_3$, $\mathcal{E}_3 \succeq \mathcal{E}_4$, $\mathcal{E}_4 \succeq \mathcal{E}_3$, $\mathcal{E}_5 \succ \mathcal{E}_3, \ldots$ *It can also be checked that* $\succeq_{max} = \{\mathcal{E}_5\}$.

Note that this relation is not transitive. It is also clear from the above definition that the corresponding framework satisfies the conflict-freeness requirement.

Property 5 *Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a preference-based argumentation framework. The extensions of this framework w.r.t. $\succeq$ given in Definition 6 are conflict-free w.r.t. $\mathcal{R}$.*

Proof *This follows from the definition of $\succeq$.* ■

Regarding the second requirement, the following theorem shows that the preference-based framework that uses this relation generalizes preferred extensions.

Theorem 2 *Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a preference-based argumentation framework, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ denote its extensions w.r.t. $\succeq$. If $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$, then each $\mathcal{E}_i$ is a preferred extension of $\langle \mathcal{A}, \mathcal{R} \rangle$ and vice versa.*

Proof *Since we supposed that $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$ then $\mathcal{E}' \succeq \mathcal{E}$ iff $(\forall x' \in \mathcal{E}')$ $(\forall x \in \mathcal{E})$ if $(x, x') \in \mathcal{R}$ then $(\exists y' \in \mathcal{E}')$ s.t. $(y, x) \in \mathcal{R}$.*

$\Leftarrow$ Let $\mathcal{E}'$ be a preferred extension of $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$.

- *Since $\mathcal{E}'$ is a preferred extension then it is conflict-free.*
- *Let us prove that $\mathcal{E}' \in \succeq_{max}$. Suppose the contrary. This means that one of the following is true:*
  1. *$(\exists \mathcal{E} \subseteq \mathcal{A})$ s.t. $\mathcal{E}$ is conflict-free and $\neg(\mathcal{E}' \succeq \mathcal{E})$*
  2. *$(\exists \mathcal{E} \subseteq \mathcal{A})$ s.t. $\mathcal{E}$ is conflict-free $\wedge\, \mathcal{E}' \subsetneq \mathcal{E} \wedge (\forall \mathcal{E}'' \subseteq \mathcal{A})$ if $\mathcal{E}''$ is conflict-free then $\mathcal{E} \succeq \mathcal{E}''$*

  Let (1) be the case. Since $\neg(\mathcal{E}' \succeq \mathcal{E})$ then $(\exists x' \in \mathcal{E}')(\exists x \in \mathcal{E})$ s.t. $(x, x') \in \mathcal{R} \wedge (\nexists y' \in \mathcal{E}')$ s.t. $(y', x) \in \mathcal{R}$. This leads to the conclusion that $\mathcal{E}'$ does not defend its arguments, thus it cannot be a preferred extension. Contradiction. So, it must be that (2) holds. Since $\mathcal{E}'$ is preferred and $\mathcal{E}' \subsetneq \mathcal{E}$ then $\mathcal{E}$ is not admissible. From the fact that $\mathcal{E}$ is conflict-free, one concludes that it does not defend its arguments. Thus, $(\exists x'' \in \mathcal{E}'' \setminus \mathcal{E}')$ s.t. $(\exists y \in \mathcal{A})$ s.t. $(y, x'') \in \mathcal{R} \wedge (\nexists z'' \in \mathcal{E}'')$ s.t. $(z'', y) \in \mathcal{R}$. Hence, $\neg(\mathcal{E}'' \succeq \{y\})$. Contradiction.

$\Rightarrow$ Let $\mathcal{E}' \in \succeq_{max}$. We will prove that $\mathcal{E}'$ is a preferred extension of Dung's argumentation framework $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$.

- *Since $\mathcal{E}'$ is then it is conflict-free.*
- *Let us prove that $\mathcal{E}'$ defends all its arguments. Suppose not. This means that $(\exists y \in \mathcal{A})$ s.t. $(y, x') \in \mathcal{R} \wedge (\nexists z' \in \mathcal{E}')$ s.t. $(z', y) \in \mathcal{R}$. This means that $\neg(\mathcal{E}' \succeq \{y\})$. Contradiction.*
- *We have just seen that $\mathcal{E}'$ is admissible. Let us prove that $\mathcal{E}'$ is a preferred extension of $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$. Suppose the contrary, i.e., $(\exists \mathcal{E} \subseteq \mathcal{A})$ s.t. $\mathcal{E}$ is a preferred extension and $\mathcal{E}' \subsetneq \mathcal{E}$. Since $\mathcal{E}' \in \succeq_{max}$ then $\mathcal{E} \notin \succeq_{max}$. On the other hand, since $\mathcal{E}$ is a preferred extension, then $\mathcal{E} \in \succeq_{max}$, as we have proved in the first part of this theorem. Contradiction.*

■

When preferences are not available, the framework that uses the relation $\succeq$ retrieves preferred extensions.

Corollary 2 *Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a preference-based argumentation framework, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ denote its extensions w.r.t. $\succeq$. If $\geq = \{(x, x) \mid x \in \mathcal{A}\}$, then each $\mathcal{E}_i$ is a preferred extension of $\langle \mathcal{A}, \mathcal{R} \rangle$ and vice versa.*

Proof *Since $\geq = \{(x, x) \mid x \in \mathcal{A}\}$ then $(\nexists x, y \in \mathcal{A})$ s.t. $x \neq y \wedge (x, y) \in \mathcal{R} \wedge (y, x) \in >$. Since we supposed that $(\nexists x \in \mathcal{A})$ s.t. $(x, x) \in \mathcal{R}$ then $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$. Theorem 2 now implies that extensions of $(\mathcal{A}, \mathcal{R}, \geq)$ are exactly the preferred extensions of $\langle \mathcal{A}, \mathcal{R} \rangle$.* ■

4.3 Generalization Of Grounded Semantics

In this section, we define a relation $\succeq$ that allows to retrieve the grounded extension in case preferences between arguments are not available or are not important. The basic idea behind this relation is that a set is not worse than another if it can strongly defend all its arguments against all attacks that come from another set.

We first generalize the notion of strong defense by taking into account preferences between arguments. The idea is that an argument has either to be preferred to its attacker or has to be defended by arguments that themselves can be strongly defended without using the argument in question.

Definition 7 (Strong defense) *Let $\mathcal{E}' \subseteq \mathcal{A}$. $\mathcal{E}'$* strongly defends *an argument x from attacks of a set $\mathcal{E}$, denoted by $sd(x, \mathcal{E}', \mathcal{E})$ iff $(\forall y \in \mathcal{E})$ if $(((y, x) \in \mathcal{R} \wedge (x, y) \notin >)$ or $((x, y) \in \mathcal{R} \wedge (y, x) \in >))$ then $((\exists z \in \mathcal{E}' \setminus \{x\})$ s.t. $((z, y) \in \mathcal{R} \wedge (y, z) \notin > \wedge sd(z, \mathcal{E}' \setminus \{x\}, \mathcal{E})))$.*
If the third argument of sd is not specified, then $sd(x, \mathcal{E}) \equiv sd(x, \mathcal{E}, \mathcal{A})$.

Let us illustrate this notion through the following example.

Example 6 (Ex. 1 Cont.) *It holds that $sd(a, \{a\}, \{b\})$ since a is strictly preferred to b thus it can defend itself. However, we have $\neg sd(b, \{b\}, \{c\})$ since b cannot defend itself against c. On the other hand, it does hold that $sd(c, \{a, c\}, \{b\})$ since a can defend c against b and a is protected from b since it is strictly preferred to it.*

This relation amounts to prefer the subsets that strongly defend all their arguments. In particular, $\mathcal{E}' \succeq \mathcal{E}$ iff $\mathcal{E}'$ strongly defends all its arguments against all attacks of $\mathcal{E}$.

Definition 8 *Let $\mathcal{E}, \mathcal{E}'$ be conflict-free sets of arguments. We say that $\mathcal{E}' \succeq \mathcal{E}$ iff $(\forall x' \in \mathcal{E}')$ $sd(x', \mathcal{E}', \mathcal{E})$.*

Example 7 *Let $\mathcal{A} = \{a, b, c\}$, $\geq = \{(a, a), (b, b), (b, a)\}$ and $\mathcal{R} = \{(a, b), (b, a), (b, c), (c, b)\}$. One can check that there is exactly one subset of $\mathcal{A}$ which is preferred to all other subsets of arguments. This set is the empty one. While we do have $\{b\} \succeq \{a\}$, we have $\neg(\{b\} \succeq \{c\})$, so $\{b\}$ is not an extension of $(\mathcal{A}, \mathcal{R}, \geq)$. We have also $\neg(\{a\} \succeq \{b\})$, $\neg(\{c\} \succeq \{b\})$ and $\neg(\{a, c\} \succeq \{b\})$. This is expected and a natural output since neither b nor c are capable to defend strongly themselves and, on the other hand, it can be said that a is the worst argument in this framework, thus not strong enough to be better than b.*

Theorem 3 *Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a preference-based argumentation framework. If $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$, then $\succeq_{max}$ contains exactly one element which coincides with the grounded extension of $\langle \mathcal{A}, \mathcal{R} \rangle$.*

Proof *Since we supposed that $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$ then we can simplify Definition 7 which becomes: $sd(x, \mathcal{E}', \mathcal{E})$ iff $(\forall y \in \mathcal{E})$ (if $(y, x) \in \mathcal{R}$ then $(\exists z \in \mathcal{E}' \setminus \{x\})$ s.t. $((z, y) \in \mathcal{R} \wedge sd(z, \mathcal{E}' \setminus \{x\}, \mathcal{E})))$. In this particular case when no attacked argument is strictly preferred to its attacker,* *our definition of $sd(x, \mathcal{E})$ becomes exactly the same as Definition 13 in [Baroni and Giacomin, 2007]. Thus, using Proposition 50 and Proposition 51 of the same paper, we conclude that $x \in \texttt{GE}$ iff $sd(x, \texttt{GE})$, where GE is the grounded extension of the framework $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$.*

$\Leftarrow$ Let $\mathcal{E}'$ be the grounded extension of $\langle \mathcal{A}, \mathcal{R} \rangle$.

- *Since $\mathcal{E}'$ is the grounded extension then it is conflict-free.*
- *We will prove that for an arbitrary conflict-free set $\mathcal{E} \subseteq \mathcal{A}$ it holds that $\mathcal{E}' \succeq \mathcal{E}$. Let $\mathcal{E} \subseteq \mathcal{A}$ be conflict-free. Since $\mathcal{E}'$ is the grounded extension then $x \in \mathcal{E}' \Rightarrow sd(x, \mathcal{E}')$. On the other hand, $(\forall x \in \mathcal{E}')$ $sd(x, \mathcal{E}')$ implies that $sd(x, \mathcal{E}', \mathcal{E})$. Thus, $\mathcal{E}' \succeq \mathcal{E}$. Since $\mathcal{E}$ was arbitrary, then $(\forall \mathcal{E} \subseteq \mathcal{A})$ (($\mathcal{E}$ is conflict-free) $\Rightarrow (\mathcal{E}' \succeq \mathcal{E})$).*
- *We will now prove that $(\nexists \mathcal{E} \subseteq \mathcal{A})$ s.t. $\mathcal{E}$ is conflict-free and $\mathcal{E}' \subsetneq \mathcal{E}$ and $((\forall \mathcal{E}'' \subseteq \mathcal{A})$ ($\mathcal{E}''$ conflict-free) $\Rightarrow (\mathcal{E} \succeq \mathcal{E}''))$. Suppose the contrary. Suppose also that $(\forall x \in \mathcal{E})$ $sd(x, \mathcal{E})$. If this is the case, according to Proposition 51 in [Baroni and Giacomin, 2007], $\mathcal{E} \subseteq \texttt{GE}$. Contradiction. So, it must be that $(\exists x \in \mathcal{E})$ s.t. $\neg sd(x, \mathcal{E})$. Thus, $(\exists y \in \mathcal{A})$ s.t. $\neg sd(x, \mathcal{E}, \{y\})$. Consequently, $\neg(\mathcal{E} \succeq \{y\})$. Contradiction. So, we have proved that $\mathcal{E}' \in \succeq_{max}$.*

$\Rightarrow$ Let $\mathcal{E}' \in \succeq_{max}$ and let us prove that $\mathcal{E}' = \texttt{GE}$. Since $(\forall x \in \mathcal{A})$ $\mathcal{E}' \succeq \{x\}$ then $(\forall x' \in \mathcal{E}')$ $sd(x', \mathcal{E}')$. From the fact that $(\forall x' \in \mathcal{E}')$ $sd(x, \mathcal{E}')$ and Proposition 51 of [Baroni and Giacomin, 2007] we have that $\mathcal{E}' \subseteq \texttt{GE}$. Let us now prove that $\mathcal{E}' = \texttt{GE}$. Suppose not, i.e., suppose that $\mathcal{E}' \subsetneq \texttt{GE}$. We have proved in the first part of this theorem that $\texttt{GE} \in \succeq_{max}$. Contradiction, since we have supposed that $\mathcal{E}' \in \succeq_{max}$ and we have $\mathcal{E}' \subsetneq \texttt{GE}$. ■

When preferences are not available, the framework that uses the relation $\succeq$ retrieves exactly grounded extension.

Corollary 3 *Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a preference-based argumentation framework. If $\geq = \{(x, x) \mid x \in \mathcal{A}\}$, then $\succeq_{max}$ contains exactly one element which coincides with the grounded extension of $\langle \mathcal{A}, \mathcal{R} \rangle$.*

Proof *Since $\geq = \{(x, x) \mid x \in \mathcal{A}\}$ then $(\nexists x, y \in \mathcal{A})$ s.t. $x \neq y \wedge (x, y) \in \mathcal{R} \wedge (y, x) \in >$. Since we supposed that $(\nexists x \in \mathcal{A})$ s.t. $(x, x) \in \mathcal{R}$ then $(\nexists x, y \in \mathcal{A})$ s.t. $(x, y) \in \mathcal{R} \wedge (y, x) \in >$. Theorem 3 now implies that $(\mathcal{A}, \mathcal{R}, \geq)$ has exactly one extension which is the grounded extension of $\langle \mathcal{A}, \mathcal{R} \rangle$.* ■

5 Conclusion

This paper has shown through a simple example that existing preference-based argumentation frameworks may lead to undesirable results. This means that the way preferences between arguments are taken into account is not appropriate. We have then proposed an alternative approach that satisfies two basic requirements: conflict-freeness of extensions, and recovering Dung's acceptability semantics when preferences are not available. The approach amounts to define a relation on the powerset of arguments. In other words, it compares pairs of conflict-free subsets of arguments. The best elements w.r.t. this relation are the extensions of the new framework. The approach has three main advantages: i) it is general since different relations can be defined, ii) it enforces the new framework to satisfy key properties, namely conflict-freeness of the extensions and recovering Dung's semantics, iii) it allows to compare any pair of subsets of arguments, contrary to Dung's approach in which there are only two categories of sets: the ones that are considered as extensions and all the remaining ones. The results presented in this paper show also how to characterize Dung's semantics in terms of a relation between subsets of arguments. To the best of our knowledge this is the first work in this direction. It allows to better understand the underpinning of those semantics.

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