# A Correctness Result for Reasoning about One-Dimensional Planning Problems*

**Yuxiao Hu** and **Hector J. Levesque**
Department of Computer Science
University of Toronto
Toronto, Ontario M5S 3G4, Canada
{yuxiao, hector} @cs.toronto.edu

## Abstract

A plan with rich control structures like branches and loops can usually serve as a general solution that solves multiple planning instances in a domain. However, the correctness of such generalized plans is non-trivial to define and verify, especially when it comes to whether or not a plan works for all of the infinitely many instances of the problem. In this paper, we give a precise definition of a generalized plan representation called an FSA plan, with its semantics defined in the situation calculus. Based on this, we identify a class of infinite planning problems, which we call one-dimensional (1d), and prove a correctness result that 1d problems can be verified by finite means. We show that this theoretical result leads to an algorithm that does this verification practically, and a planner based on this verification algorithm efficiently generates provably correct plans for 1d problems.

## Introduction

Planning with rich control structures like branches and loops is drawing increasing attention from the AI community [Manna and Waldinger, 1987; Levesque, 2005; Srivastava *et al.*, 2008]. One advantage of the resulting generalized plan is that the same plan can work in different initial states with very large numbers of objects.

For example, suppose we have the following variant of the logistics problem: There are a number of objects at their source locations (office or home), and the goal is to move them all to their destinations with a truck. The available actions include moving the truck to a location, loading and unloading an object, finding the source and destination locations of an object, and checking whether all objects have been processed. Then, intuitively, the plan in Figure 1 achieves the goal no matter how many objects there are.

Unfortunately, it is not easy to see formally why this plan is *correct* for this problem. We need a formal definition of a plan representation and we must be precise about what it

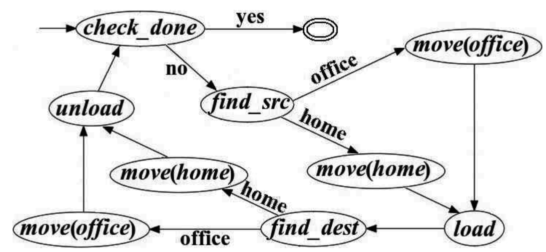


Figure 1: An example plan for the logistic problem

means for such a plan to be correct for all of the infinitely many possible objects and source-destination combinations.

In this paper, we propose a solution to these in terms of a *Finite State Automaton plan (FSA plan)*, a generalized plan representation inspired by and shown more expressive than robot programs [Levesque, 1996]. The semantics of FSA plans is declaratively defined in the situation calculus, so we have a foundation to analyze planning problems, plans, and their correctness. This notion of correctness, although precise and general, has the disadvantage that it uses *second-order logic* and is thus intractable in general. The intractability is inevitable, since FSA plans, like robot programs, are Turing complete in sufficiently expressive action domains [Lin and Levesque, 1998].

However, for restricted action domains, we can do much better. A number of real-world problems involve independently processing an unbounded number of objects: delivering packages according to their shipping labels (as in the logistics example above), pushing the buttons on a safe according to the digits of a combination, keeping or discarding eggs according to their smell, *etc*. In this paper, we identify a class of planning problems, which we call *one-dimensional* or *1d*, that includes these, based on the shape of their action theory. We introduce a verification procedure, which we prove is correct for all 1d problems. We also present empirical evidence that a planner based on this verification procedure efficiently generates provably correct plans on planning problems that appear to be beyond the algorithmic reach of other planners.

## Problem and Plan Representation

In order to represent and reason about the planning problem sketched above, we need a formal action language. In this

*This is a reduced version of the paper with the same name from KR-10. More details and proofs of theorems can be found there.

paper, we appeal to the situation calculus, although the results introduced here could be adapted to other formalisms like $\mathcal{A}$ [Gelfond and Lifschitz, 1993], and the fluent calculus [Thielscher, 1998].

The situation calculus is a first-order, multi-sorted logical language with limited second-order features for representing and reasoning about dynamical environments [McCarthy and Hayes, 1969; Reiter, 2001]. Objects in the domain of the logic are of three disjoint sorts: *situation* for situations, *action* for actions and *object* for everything else. The only *situation* constant $S_0$ denotes the initial situation where no action has yet occurred, and $do(a, s)$ represents the situation after performing action $a$ in situation $s$. We use $do([a_1, \cdots, a_n], s)$ as an abbreviation for $do(a_n, do(\cdots, do(a_1, s)))$. Functions (relations) whose value may vary from situation to situation are called functional (relational) *fluents*, and denoted by a function (relation) whose last argument is a situation term. Functions and relations whose value do not change across situations are called *rigids*. Without loss of generality, we assume that all fluents are functional. The special relation $Poss(a, s)$ states that action $a$ is executable in situation $s$, and the function $\text{SR}(a, s)$ indicates the sensing result of $a$ when performed in $s$. The latter is introduced by Scherl and Levesque (2003) to accommodate knowledge and sensing in the situation calculus. We assume that ordinary actions, not intended for sensing purposes, simply return a fixed value (ok). A formula $\phi$ is *uniform in s* if it does not mention *Poss*, SR, or any situation term other than $s$. We call a fluent formula $\phi$ with all situation arguments eliminated a *situation-suppressed* formula, and use $\phi[s]$ to denote the uniform formula with all situation arguments restored with term $s$.

The dynamics of a planning problem is formalized by a basic action theory (BAT) of the form

$$\mathcal{D} = \mathcal{FA} \cup \Sigma_{pre} \cup \Sigma_{post} \cup \Sigma_{sr} \cup \Sigma_0 \cup \Sigma_{una}$$

where

- $\mathcal{FA}$ is a set of domain-independent axioms defining the legal situations [Reiter, 2001].
- $\Sigma_{pre}$ is a set of action precondition axioms, one for each action symbol of the form $Poss(A(\vec{x}), s) \equiv \Pi_A(\vec{x}, s)$.
- $\Sigma_{post}$ is a set of successor-state axioms, one for each fluent $f$ of the form $f(\vec{x}, do(a, s)) = y \equiv \Phi_f(\vec{x}, a, y, s)$.
- $\Sigma_{sr}$ is a set of sensing result axioms, one for each sensing action of the form $\text{SR}(A(\vec{x}), s) = r \equiv \Theta_A(\vec{x}, r, s)$.
- $\Sigma_0$ is the initial knowledge base stating facts about $S_0$.
- $\Sigma_{una}$ is a set of unique names axioms for actions.

Figure 2 shows a complete specification for the logistic problem above as a BAT.[1] There are four fluents, *loc* (the location of the truck), *loaded* (the loading status of the truck), *parcels_left* (the number of parcels remaining to be delivered), and *misplaced* (whether any processed object has been misplaced). The initial value of *parcels_left* is non-negative but unknown. There is a sensing action *check_done* that tells whether or not all parcels have been processed. In addition to the four fluents, we assume there are two rigid functions, *source* and *dest* that provide the shipping label for each object. For example, $dest(7) = home$ would mean that the destination of the 7th object is home. The values of these functions is not specified, but the sensing action *find_src* returns the source for the current object (according to *parcels_left*), and similarly for *find_dest*.

Given the dynamics, the planning task is to find a plan that is executable in the given environment, and whose execution

**Precondition Axioms:**

$Poss(move(x), s) \;\equiv\; \text{TRUE}$

$Poss(load, s) \;\equiv\; loc(s) = source(parcels\_left(s)) \wedge loaded(s) = \text{FALSE}$

$Poss(unload, s) \;\equiv\; loaded(s) = \text{TRUE}$

$Poss(find\_src, s) \;\equiv\; parcels\_left(s) \neq 0$

$Poss(find\_dest, s) \;\equiv\; parcels\_left(s) \neq 0$

$Poss(check\_done, s) \;\equiv\; \text{TRUE}$

**Successor State Axioms:**

$loc(do(a, s)) = x \;\equiv\; \exists y.\, x = y \wedge a = move(y) \;\vee\; x = loc(s) \wedge a \neq move(y)$

$misplaced(do(a, s)) = x \;\equiv\; x = \text{TRUE} \wedge a = unload \wedge loc(s) \neq dest(parcels\_left(s)) \;\vee\; x = misplaced(s) \wedge (a \neq unload \vee loc(s) = dest(parcels\_left(s)))$

$loaded(do(a, s)) = x \;\equiv\; x = \text{TRUE} \wedge a = load \;\vee\; x = \text{FALSE} \wedge a = unload \;\vee\; x = loaded(s) \wedge a \neq load \wedge a \neq unload$

$parcels\_left(do(a, s)) = x \;\equiv\; x = parcels\_left(s) - 1 \wedge a = unload \;\vee\; x = parcels\_left(s) \wedge a \neq unload$

**Sensing Result Axioms:**

$\text{SR}(find\_src, s) = r \;\equiv\; source(parcels\_left(s)) = r$

$\text{SR}(find\_dest, s) = r \;\equiv\; dest(parcels\_left(s)) = r$

$\text{SR}(check\_done, s) = r \;\equiv\; r = yes \wedge parcels\_left(s) = 0 \;\vee\; r = no \wedge parcels\_left(s) \neq 0$

$\text{SR}(move(x), s) = r \;\equiv\; r = ok$

$\text{SR}(load, s) = r \;\equiv\; r = ok$

$\text{SR}(unload, s) = r \;\equiv\; r = ok$

**Initial Situation Axiom:**

$\forall n.\,(source(n) = home \vee source(n) = office) \wedge \forall n.\,(dest(n) = home \vee dest(n) = office) \wedge loc(S_0) = home \wedge loaded(S_0) = \text{FALSE} \wedge parcels\_left(S_0) \geq 0 \wedge misplaced(S_0) = \text{FALSE}$

**Goal Condition:**

$parcels\_left = 0 \;\wedge\; misplaced = \text{FALSE}$

Figure 2: Axiomatization of logistic in the situation calculus

[1] We omit the foundational axioms $\mathcal{FA}$, the unique names axioms $\Sigma_{una}$, and any domain closure axiom for objects (as will be introduced in Definition 6 below).

achieves some desired goal. Here, we only consider planning problems with final-state goals, defined as follows:

**Definition 1 (The Planning Problem).** A *planning problem* is a pair $\langle \mathcal{D}, G \rangle$, where $\mathcal{D}$ is a basic action theory, and $G$ is a situation-suppressed formula in the situation calculus.

In the case of logistic, the goal $G$ is to make *parcels_left* be 0 while keeping *misplaced* as FALSE. Since the number of parcels and their sources and destinations are left open, this planning problem is not soluble with a sequential plan. We need a more general plan representation with branches and loops to handle the contingencies.

One candidate representation is Levesque's *robot programs* [Levesque, 1996]. Recently, Hu and Levesque (2009) proposed an alternative representation called the *FSA plan*, and showed that it is more general than robot programs, in that all robot programs have an FSA plan representation but not vice versa. Moreover, they also presented a planning algorithm with this representation, which greatly outperforms the robot-program based KPLANNER [Levesque, 2005]. As a result, we appeal to FSA plans in this paper.

**Definition 2 (FSA Plan [Hu and Levesque, 2009]).** An *FSA plan* is a tuple $\langle \mathbf{Q}, \gamma, \delta, Q_0, Q_F \rangle$, where

- $\mathbf{Q}$ is a finite set of program states;
- $Q_0 \in \mathbf{Q}$ is an initial program state;
- $Q_F \in \mathbf{Q}$ is a final program state;
- $\gamma : \mathbf{Q}^- \to \mathbf{A}$ is a function, where $\mathbf{Q}^- = \mathbf{Q} \setminus \{Q_F\}$ and $\mathbf{A}$ is the set of primitive actions;
- $\delta : \mathbf{Q}^- \times \mathbf{R} \to \mathbf{Q}$ is a function, where $\mathbf{R}$ is the set of sensing results, that specifies the program state to transition to for each non-final state and valid sensing result for the associated action.

An example of an FSA plan (shown as a graph) appears in Figure 1. The execution of an FSA plan starts from $q = Q_0$, and executes the action $\gamma(q)$ associated with program state $q$. On observing sensing result $r$, it transitions to the new program state $\delta(q, r)$. This repeats until $Q_F$ is reached.

In order to represent FSA plans in the situation calculus, we assume that there is a sub-sort of *object* called *program-state*, with $Q_0$ and $Q_F$ being two constants of this sort, and two rigid function symbols $\gamma$ and $\delta$. We use a set of sentences *FSA* to axiomatize the plan:

**Definition 3.** *FSA* is a set of axioms consisting of

1. Domain closure axiom for program states
$$(\forall q).\ \{q = Q_0 \vee q = Q_1 \vee \cdots \vee q = Q_n \vee q = Q_F\};$$
2. Unique names axioms for program states
$$Q_i \neq Q_j \text{ for } i \neq j;$$
3. Action association axioms, one for each program state other than $Q_F$, of the form $\gamma(Q) = A$
4. Transition axioms of the form $\delta(Q, R) = Q'$

To capture the desired execution semantics, we introduce a transition relation $T^\star(q_1, s_1, q_2, s_2)$, which intuitively means that from program state $q_1$ and situation $s_1$, the FSA plan will reach $q_2$ and $s_2$ at some point during the execution. The formal definition is given in Definition 4.

**Definition 4.** We use $T^\star(q_1, s_1, q_2, s_2)$ as abbreviation for $(\forall T).\{\ldots \supset T(q_1, s_1, q_2, s_2)\}$, where the ellipsis is the conjunction of the universal closure of the following:

- $T(q, s, q, s)$
- $T(q, s, q'', s'') \wedge T(q'', s'', q', s') \ \supset\ T(q, s, q', s')$
- $\gamma(q) = a \wedge Poss(a, s) \wedge \mathrm{SR}(a, s) = r \wedge \delta(q, r) = q' \ \supset\ T(q, s, q', do(a, s))$

Notice that this definition uses second-order quantification to ensure that $T^\star$ is the least predicate satisfying the three properties above. This essentially constrains the set of tuples satisfying $T^\star$ to be the reflexive transitive closure of the one-step transitions in the FSA plan.

With this transition relation, we can now characterize the correctness of FSA plans as follows.

**Definition 5 (Plan correctness).** Given a planning problem $\langle \mathcal{D}, G \rangle$, a plan axiomatized by *FSA* is correct iff
$$\mathcal{D} \cup FSA \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

The definition essentially says that for an FSA plan to be correct, it must guarantee that for *any* model of $\mathcal{D}$, the execution of the FSA plan will reach the final state $Q_F$, and the goal is satisfied in the corresponding situation $s$. (In the case of logistic, a plan needs to work for any initial value of *parcels_left* and any value for the functions *source* and *dest*.)

This criterion of correctness is general and concise, but its second-order quantification and the potential existence of infinitely many models make it less useful algorithmically. Partly for this reason, existing iterative planners based on a similar representation, like KPLANNER [Levesque, 2005] and FSAPLANNER [Hu and Levesque, 2009], only come with a very weak correctness guarantee: although the generated plan tends to work for all problems in the domain, only certain instances can be *proven* correct. It is thus interesting to ask whether we can generate provably correct plans for restricted classes of planning problems. The rest of this paper gives a positive answer to this question.

## One-Dimensional Planning Problems

The major goal of this paper is to identify a class of planning problems that has a complete procedure to reason about the correctness of solution FSA plans. In this section, we define the class of *1d planning problems*, which is derived from the more restricted *finite problems*.

**Definition 6.** A planning problem $\langle \mathcal{D}, G \rangle$ is *finite* if $\mathcal{D}$ does not contain any predicate symbol other than *Poss* and equality, and the sort *object* has a domain closure axiom
$$\forall x.\ x = o_1 \vee \cdots \vee x = o_l.$$

Intuitively, a finite problem has finitely many objects in the domain. Therefore, the number of ground fluents as well as their range is finite.

A 1d problem is like a finite problem except that there is a special distinguished fluent (called the *planning parameter*) that takes value from a new sort *natural number*, there is a finite set of distinguished actions (called the *decreasing actions*) which decrement the planning parameter, and some of the functions (called *sequence functions*) have an index argument of sort *natural number*.[2] In the case of the logistic example, the planning parameter is *parcels_left*, the decreasing action is *unload*, and the sequence functions are *source* and *dest*. The idea of a 1d planning problem is that the basic action theory is restricted in how it can use the planning parameter and sequence functions, as follows:

**Definition 7.** A planning problem $\langle \mathcal{D}', G' \rangle$ is *1d* with respect to an integer-valued fluent $p$, if there is a finite problem $\langle \mathcal{D}, G \rangle$ whose functions include fluent $f_0$ and rigids $f_1, \cdots, f_m$, and whose actions include $A_1, \cdots, A_d$, such that $\langle \mathcal{D}', G' \rangle$ is derived from $\langle \mathcal{D}, G \rangle$ as follows:

1. Replace the fluent $f_0$ with a planning parameter $p$:
   (a) replace successor-state axiom for $f_0$ by one for $p$:
   $$p(do(a,s)) = x \;\equiv\; x = p(s) - 1 \wedge \mathit{Dec}(a) \vee x = p(s) \wedge \neg \mathit{Dec}(a),$$
   where $\mathit{Dec}(a)$ stands for $(a = A_1 \vee \cdots \vee a = A_d)$;
   (b) replace all atomic formulas involving the term $f_0(s)$ in $\Pi$, $\Phi$, $\Theta$ and $G[s]$ by $p(s) = 0$, where $s$ is the free situation variable in those formulas;
   (c) remove all atomic formulas mentioning $f_0(S_0)$ in $\Sigma_0$, and add $p(S_0) \geq 0$ instead.
2. Replace the rigids $f_1, \cdots, f_m$ with sequence functions $h_1, \cdots, h_m$:
   (a) replace all terms $f_i$ in $\Pi$, $\Phi$, $\Theta$ and $G[s]$ by $h_i(p(s))$, where $s$ is the free variable as above;
   (b) replace all $f_i$ in $\Sigma_0$ with $h_i(n)$, where $n$ is a universally quantified variable of sort *natural number*.

Observe that in a 1d problem, the occurrence of the integer planning parameter is limited to its own successor state axiom, in $\Sigma_0$, and as an argument to a sequence function. Any other use of it is to test whether it is 0. Similarly, we can only apply a sequence function to the current object as determined by the planning parameter (other than in $\Sigma_0$ where we must quantify over all natural numbers). This ensures that the objects can be accessed sequentially in descending order, and that they do not interact with one another. It is not hard to see that logistic conforms to these requirements.

## Main Theorems

Given a planning problem and a candidate plan, an important reasoning task is to decide whether the plan is guaranteed to achieve the goal according to the action theory. In a 1d setting, we need to ensure that the plan achieves the goal no matter what values the planning parameter $p$ and the sequence functions $h_i$ take. Unfortunately, there are infinitely many values that need to be taken into account.

In this section, we consider a correctness result of the following form: if we can prove that a plan is correct under the assumption that $p(S_0) \leq N$ (for a constant $N$ that we calculate), it will follow that the plan is also correct without this assumption. In other words, correctness of the plan for initial values of $p$ up to $N$ will be sufficient.

The first theorem we can prove is the following:

**Theorem 1.** *Suppose $\langle \mathcal{D}, G \rangle$ is a 1d planning problem with planning parameter $p$, and that $\mathcal{D}$ contains FSA axioms for some plan. Let $N_0 = 2 + k_0 \cdot l^m$, where $k_0$ is the number of decreasing program states in the FSA plan, $m$ is the total number of finite and sequence functions, and $l$ is the total number of values that they can take. Then we have:*

*If* $\mathcal{D} \cup \{p(S_0) \leq N_0\} \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s]$,
*then* $\mathcal{D} \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s]$.

What the proof does is to show that despite the fact that the value of the planning parameter is not bounded, the number of situations that can be distinguished in a 1d BAT is bounded. So if a plan were to fail to achieve the goal in a model where $p(S_0) > N_0$, then according to the BAT, it would also fail in a model where $p(S_0) \leq N_0$.

The bound $N_0$ in this theorem is exponential in the number of ground functions, however. It can therefore can be extremely large even for relatively simple action theories.

To obtain a more practical bound in a similar vein, we introduce another theorem, where we do not declaratively specify the bound, but instead only spell out the necessary condition for an integer $N_t$ to be a valid bound.

**Theorem 2.** *Suppose $\langle \mathcal{D}, G \rangle$ is a 1d planning problem with planning parameter $p$, and that $\mathcal{D}$ contains FSA axioms for some plan. Let $\mathit{Seen}(q, s)$ be the abbreviation for*

$$\exists s'.\, T^\star(Q_0, S_0, q, s') \wedge p(s') > 1 \wedge \bigwedge f(s) = f(s') \wedge \bigwedge h(p(s)) = h(p(s'))$$

*where the first conjunction is over the finite fluents $f$, and the second over sequence functions $h$. Suppose $N_t > 0$ satisfies*

$$\mathcal{D} \cup \{p(S_0) = N_t\} \models \forall q, s.\, T^\star(Q_0, S_0, q, s) \wedge p(s) = 1 \supset \mathit{Seen}(q, s)$$

*Then we have the following:*

*If* $\mathcal{D} \cup \{p(S_0) \leq N_t\} \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s]$,
*then* $\mathcal{D} \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s]$.

Intuitively, the $N_t$ here has to be large enough so that a similar situation to the one that decrements the planning parameter from 1 to 0 occurs earlier in the execution trace.

## Experimental Results

Given an FSA plan for a 1d planning problem, Theorems 1 and 2 suggest two algorithms to verify its correctness, which can then be used for plan generation.

### Plan verification

To utilize the idea in Theorem 1, we only need to execute the FSA plan for $p(S_0) = 0, 1, \cdots, N_0$. If the goal is achieved

[2]For simplicity, we assume in this paper that all sequence functions are rigid, but it is not hard to prove that the definitions and theorems work for sequence fluents as well.

in all cases, then the FSA plan is correct in general, according to the theorem, and otherwise, it is incorrect. However, when the bound is large, this algorithm becomes impractical, since the number of possible initial worlds is exponential in the planning parameter. In the logistic example, for instance, each parcel has four possible source-destination combinations, so if we consider a problem containing $514$ parcels (see the bounds for logistic below), the total number of possible combinations would be $4^{514}$.

Fortunately, the bound $N_0$ is a loose, worst-case estimate, and Theorem 2 suggests a better algorithm. We execute the FSA plan starting from $p(S_0) = 0, 1, 2, \cdots$. In each execution, whenever the planning parameter $p$ decreases from 1 to 0, we record the program state, as well as the value of all finite and sequence functions in a table. If for some $N_t$, the execution for $p(S_0) = N_t$ does not add any new row into the table, then this $N_t$ satisfies the criterion of Theorem 2, and thus the plan is guaranteed to be correct in general. If the FSA plan fails before reaching such an $N_t$, then it is incorrect. Notice that when the plan is correct, this algorithm will terminate, since if we reach $N_0$, it is guaranteed correct by Theorem 1.

## Plan generation

With the complete verification algorithms in hand, we can now generate plans that are correct for 1d planning problems. This is done by slightly modifying FSAPLANNER introduced by Hu and Levesque (2009).

The FSAPLANNER works by alternating between a *generation* and a *testing* phase: it generates plans for values of the planning parameter up to a lower bound, and then tests the resulting candidate plans for a higher value of the planning parameter. Although this appears to work for many applications, it has at least two serious problems: (1) the lower and higher bounds must be set by hand and (2) the only formal guarantee is that the plan works for the given values.

The verification algorithms proposed above resolve both of these problems. The idea is to replace the test phase of FSAPLANNER by this verification. Then whenever a plan passes the testing phase, it is guaranteed to be correct. Notice that in both cases, the bounds $N_0$ and $N_t$ can be obtained mechanically from the planning problem itself without manual intervention. The former only depends on the number of fluents and constants that appear in $\Sigma_0$ and $\Sigma_{post}$, whereas the latter is identified by table saturation.

We ran several experiments with variants of FSAPLANNER on four example domains: treechop, variegg, safe and logistic. (The first two are adapted from [Levesque, 2005].)

**treechop:** The goal is to chop down a tree, and put away the axe. The number of chops needed to fell the tree is unknown, but a *look* action tells whether the tree is up or down. Intuitively, a solution involves first *look* and then *chop* whenever *up* is sensed. This repeats until *down* is sensed, in which case we *store* the axe, and are done.

**variegg:** The goal is to get enough good eggs in the bowl from a sequence of eggs, each of which may be either good or bad, in order to make an omelette. A sensing action *check_bowl* tests if there are enough eggs in the bowl, and another *smell_dish* tests whether the egg in the dish is good or bad. Other actions include breaking an egg in the sequence to the dish, moving the egg from dish to bowl and dumping the dish.

**safe:** The goal is to open a safe whose secret combination is written on a piece of paper as a binary string. The action *pick_paper* picks up the paper, and the sensing action *read* reads the first unmarked bit of the combination and return either 0 or 1, or "done" if the end of string is reached. The action *process*$(x)$ crosses the current bit on the paper, and pushes button $x$ on the safe, where $x$ can be 0 or 1. Finally, *open* unlocks the safe if the correct combination is pushed, and jams the safe otherwise.

| Problem | treechop | variegg | safe | logistic |
|---|---|---|---|---|
| $N_{man}$ | 100 | 6 | 4 | 5 |
| Time (secs) | 0.1 | 0.12 | 0.09 | 3.93 |
| $N_0$ | 18 | 345 | 4098 | 514 |
| Time (secs) | 0.03 | $>1$ day | $>1$ day | $>1$ day |
| $N_t$ | 2 | 3 | 2 | 2 |
| Time (secs) | 0.01 | 0.08 | 0.08 | 3.56 |

Figure 3: Comparison of FSAPLANNER using different verification modules

We summarize the parameters/bounds and computation times on the four sample problems in Figure 3. Here, $N_{man}$ is the manually specified test parameter in the original FSAPLANNER, $N_0$ is the exponential bound obtained from Theorem 1, and $N_t$ is the tighter bound based on table-saturation derived from Theorem 2. The corresponding CPU time to generate a correct plan is listed below each parameter/bound. (All runs are in SWI-Prolog under Ubuntu Linux 8.04 on a Intel Core2 3.0GHz CPU machine with 3.2GB memory.)

Comparing the bounds that have guarantees, $N_t$ is much tighter than $N_0$. $N_0$ is impractical for larger planning problems like safe and logistic, whereas $N_t$ is consistently small for all problems. Note that this planner can do even better than the original FSAPLANNER when the manually specified test bound is overestimated. In sum, the table saturation based verification algorithm enables us to efficiently generate correctness-guaranteeing FSA plans for these 1d problems.

# Related Work

The work most similar to ours in this paper is the theorem that "simple problems" can be finitely verified [Levesque, 2005]. However, the definition of simple problems is based on properties of the plan, and thus somewhat *ad hoc*. Our definition of 1d problems, in contrast, is rooted in the situation calculus, and therefore inherits its rigorous proofs.

Another closely related work is Lin's proof technique for goal achievability for rank 1 action theories by model subsumption [Lin, 2008]. His rank 1 action theory is more general than our 1d theory, but the type of plan that can be reasoned about is more restricted: plans with all actions located in a non-nested loop. Efficiently generating iterative plans is also outside of the scope of his work.

The planner Aranda [Srivastava *et al.*, 2008] learns "generalized plans" that involve loops by using abstraction on an example plan. They prove that their planner generates correct plans for problems in "extended-LL" domains. However, it not clear what sort of action theories can or cannot be characterized as extended-LL. It is thus interesting future work to compare the relative expressiveness between extended-LL and 1d problems, and identify a more general class that accommodates both formalisms.

There is also important work on planning in domains where loops are required but correctness in general is not considered at all. The planner loopDistill [Winner and Veloso, 2007] learns from an example partial-order plan. Similarly, the planner introduced by Bonet, Palacios and Geffner (2009) synthesizes finite-state controllers via conformant planning. In both cases, the resulting plans can usually solve problems similar to the examples used to generate them, but under what conditions they will be applicable is not addressed.

Earlier work on deductive synthesis of iterative or recursive plans represents another approach based on theorem proving. For example, Manna and Waldinger (1987) finds recursive procedures to clear blocks in the blocks world, and the resulting plan comes with a strong correctness guarantee. Unfortunately, the price to pay is typically manual intervention (for example, to identify induction hypotheses) and poor performance. Magnusson and Doherty recently proposed to use heuristics to automatically generate induction hypotheses for temporally-extended maintenance goals (2008). However, their planner is incomplete, and for which subclass their approach is complete remains to be investigated.

Finally, there is a separate branch of research in model checking for automatically verifying correctness of computer programs [Clarke *et al.*, 1999]. It is concerned with correctness of programs in predefined computer languages instead of general action domains, and does not aim for program synthesis. However, results and techniques from this community may shed light on our goal of iterative plan verification and generation in the long run.

## Conclusion and Future Work

In this paper, we identified a class of planning problems which we called 1d, and proved that plan correctness for unbounded 1d problems could be checked in a finite and practical way. Based on this theoretical result, we developed a variant of FSAPLANNER, and showed that it efficiently generates provably correct plans for 1d problems.

In the future, we intend to investigate planning problems beyond the 1d class. Consider, for example, the following:

> *We start with a stack A of blocks, with the same number of blue and red ones. We can pick up a block from stack A or B, and put a block on stack B or C. We can also sense when a stack is empty and the color of a block being held. The goal is to get all the blocks onto stack C, alternating in color, with red on the bottom.*

What makes this problem challenging is that we may need to put a block aside (onto stack B) and deal with any number of other blocks before we can finish with it. In a still more general example, consider the Towers of Hanoi. In this case, we spend almost all our time finding a place for disks that are not ready to be moved to their final location. In the future, we hope to develop finite techniques for such problems too.

## References


[Bonet *et al.*, 2009] B. Bonet, H. Palacios, and H. Geffner. Automatic derivation of memoryless policies and finite-state controllers using classical planners. In *Proc. of Intl. Conf. on Automated Planning and Scheduling*, 2009.

[Clarke *et al.*, 1999] Edmund M. Clarke, Orna Grumberg, and Doron A. Peled. *Model Checking*. MIT Press, 1999.

[Gelfond and Lifschitz, 1993] M. Gelfond and V Lifschitz. Representing actions and change by logic programs. *Journal of Logic Programming*, pages 301–323, 1993.

[Hu and Levesque, 2009] Y. Hu and H.J. Levesque. Planning with loops: Some new results. In *ICAPS Workshop on Generalized Planning*, 2009.

[Levesque, 1996] H.J. Levesque. What is planning in the presence of sensing. In *Proceedings of National Conference on Artificial Intelligence*, 1996.

[Levesque, 2005] H.J. Levesque. Planning with loops. In *Proc. of Intl. Joint Conf. on Artificial Intelligence*, 2005.

[Lin and Levesque, 1998] F. Lin and H.J. Levesque. What robots can do: robot programs and effective achievability. *Artificial Intelligence*, 1998.

[Lin, 2008] F. Lin. Proving goal achievability. In *Proceedings of International Conference on the Principles of Knowledge Representation and Reasoning*, 2008.

[Magnusson and Doherty, 2008] M. Magnusson and P. Doherty. Deductive planning with inductive loops. In *Proceedings of National Conference on Artificial Intelligence*, 2008.

[Manna and Waldinger, 1987] Z. Manna and R. Waldinger. How to clear a block: a theory of plans. *Journal of Automated Reasoning*, 3(4):343–377, 1987.

[McCarthy and Hayes, 1969] J. McCarthy and P.J. Hayes. Some philosophical problems from the standpoint of artificial intelligence. *Machine Intelligence*, 1969.

[Reiter, 2001] Raymond Reiter. *Knowledge in Action: Logical Foundations for Specifying and Implementing Dynamical Systems*. MIT Press, 2001.

[Scherl and Levesque, 2003] R. Scherl and H. Levesque. Knowledge, action, and the frame problem. *Artificial Intelligence*, 144, 2003.

[Srivastava *et al.*, 2008] S. Srivastava, N. Immerman, and S. Zilberstein. Learning generalized plans using abstract counting. In *Proceedings of National Conference on Artificial Intelligence*, 2008.

[Thielscher, 1998] M. Thielscher. Introduction to the fluent calculus. *Electronic Transactions on Artificial Intelligence*, 2(3-4):179–192, 1998.

[Winner and Veloso, 2007] E. Winner and M. Veloso. LoopDISTILL: Learning domain-specific planners from example plans. In *Proceedings of ICAPS-07 Workshop on AI Planning and Learning*, 2007.