## Query-Driven Repairing of Inconsistent DL-Lite Knowledge Bases

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### Abstract

We consider the problem of query-driven repairing of inconsistent DL-Lite knowledge bases: query answers are computed under inconsistency-tolerant semantics, and the user provides feedback about which answers are erroneous or missing. The aim is to find a set of ABox modifications (deletions and additions), called a repair plan, that addresses as many of the defects as possible. After formalizing this problem and introducing different notions of optimality, we investigate the computational complexity of reasoning about optimal repair plans and propose interactive algorithms for computing such plans. For deletion-only repair plans, we also present a prototype implementation of the core components of the algorithm.

### 1 Introduction

Ontology-mediated query answering (OMQA) is a promising recent approach to data access in which conceptual knowledge provided by an ontology is exploited when querying incomplete data (see [Bienvenu and Ortiz, 2015] for a survey). As efficiency is a primary concern, significant research efforts have been devoted to identifying ontology languages with favorable computational properties. The DL-Lite family of description logics (DLs) [Calvanese et al., 2007], which underlies the OWL 2 QL profile [Motik et al., 2012], has garnered significant interest as it allows OMQA to be reduced, via query rewriting, to standard database query evaluation.

Beyond efficiency, it is important for OMQA systems to be robust to inconsistencies stemming from errors in the data. Inspired by work on consistent query answering in databases [Bertossi, 2011], several inconsistency-tolerant semantics have been developed for OMQA, with the aim of providing meaningful answers in the presence of inconsistencies. Of particular relevance to the present paper are the brave semantics [Bienvenu and Rosati, 2013], which returns all query answers that are supported by some internally consistent set of facts, and the more conservative IAR semantics [Lembo et al., 2010] that requires that facts in the support not belong to any minimal inconsistent subset. Both semantics have appealing computational properties: for most DL-Lite dialects, including the dialect DL-Lite$_{\mathcal{R}}$ considered in this paper, conjunctive query answering is tractable in data complexity and can be implemented using query rewriting techniques [Lembo et al., 2011; Bienvenu and Rosati, 2013].

While inconsistency-tolerant semantics are essential for returning useful results when consistency cannot be achieved, they by no means replace the need for tools for improving data quality. That is why in this paper we propose a complementary approach that exploits user feedback about query results to identify and correct errors. We consider the following scenario: a user interacts with an OMQA system, posing conjunctive queries and receiving the results, which are sorted into the possible answers (i.e., those holding under the weaker brave semantics) and the (almost) sure answers (holding under IAR semantics). When reviewing the results, the user can indicate that some of the retrieved answer tuples are erroneous, whereas other tuples should definitely be considered answers. Ideally, the unwanted tuples should not be returned as possible (brave) answers, and all of the desired tuples should be found among the sure (IAR) answers. The aim is thus to construct a set of atomic changes (deletions and additions of facts), called a repair plan, that achieves as many of these objectives as possible, subject to the constraint that the changes must be validated by the user.

There are several reasons to use queries to guide the repairing process. First, we note that it is typically impossible (for lack of time or information) to clean the entire dataset, and therefore reasonable to focus the effort on the parts of the data that are most relevant to users’ needs. In the database arena, this observation has inspired work on integrating entity resolution into the querying process [Altwaijry et al., 2013]. Second, expert users may have a good idea of which answers are expected for queries concerning their area of expertise, and thus queries provide a natural way of identifying flaws. Indeed, Kontokostas et al. (2014) recently proposed to use queries to search for errors and help evaluate linked data quality. Finally, even non-expert users may notice anomalies when examining query results, and it would be a shame not to capitalize on this information, and in this way, help distribute the costly and time-consuming task of improving data quality, as argued in [Bergman et al., 2015].

The contributions of this paper are as follows. In Section 3, we formalize query-driven repairing problems and illustrate the main challenges, in particular, the fact that there may not
exist any repair plan that resolves all identified errors. This leads us to introduce in Section 4 different notions of optimal repair plan. Adopting DL-LiteR as the ontology language, we study the complexity of reasoning about the different kinds of optimal repair plan and provide interactive algorithms for constructing such plans. In Section 5, we focus on the important special case of deletion-only repair plans, for which all of the optimality notions coincide. We take advantage of the more restricted search space to improve the general approach, and we analyze the complexity of the decision problems used in our algorithm. Finally, in Section 6, we present preliminary experiments about our implementation of the core components of the algorithm for the deletion-only case. We conclude with a discussion of related and future work.

Omitted proofs and additional information about the experiments can be found in [Bienvenu et al., 2016b].

## 2 Preliminaries

Following the presentation of [Bienvenu et al., 2016a], we recall the basics of DLs and inconsistency-tolerant semantics.

### Syntax

A DL knowledge base (KB) consists of an ABox and a TBox, both constructed from a set $N_C$ of concept names (unary predicates), a set $N_R$ of role names (binary predicates), and a set $N_I$ of individuals (constants). The ABox (dataset) is a finite set of concept assertions $A(a)$ and role assertions $R(a, b)$, where $a \in N_C$, $R \in N_R$, $a, b \in N_I$. The TBox (ontology) is a finite set of axioms whose form depends on the particular DL. In DL-LiteR, TBox axioms are either concept inclusions $B \subseteq C$ or role inclusions $P \subseteq S$ built according to the following syntax (where $A \in N_C$ and $R \in N_R$):

$$B ::= A \mid \exists P . C \mid \neg B , P ::= R \mid R^{-} , S ::= P \mid \neg P$$

### Semantics

An interpretation has the form $\mathcal{I} = (\Delta^I, \cdot^I)$, where $\Delta^I$ is a non-empty set and $\cdot^I$ maps each $a \in N_I$ to $a^I \in \Delta^I$, each $A \in N_C$ to $A^I \subseteq \Delta^I$, and each $R \in N_R$ to $R^I \subseteq \Delta^I \times \Delta^I$. The function $\cdot^I$ is extended to general concepts and roles in the standard way, e.g. $(R \circ C)^I = \{ (d, e) \mid (d, e) \in R^I \land (e, d) \in C^I \}$ and $(\exists R\cdot)^I = \{ d \mid \exists e : (d, e) \in R^I \}$. An interpretation $\mathcal{I}$ satisfies an inclusion $G \subseteq H$ if $G^I \subseteq H^I$; it satisfies $A(a)$ (resp. $R(a, b)$) if $a^I \in A^I$ (resp. $(a^I, b^I) \in R^I$). We call $\mathcal{I}$ a model of $K = (\mathcal{T}, \mathcal{A})$ if $\mathcal{I}$ satisfies all axioms in $\mathcal{T}$ and assertions in $\mathcal{A}$. A KB is consistent if it has a model, and an ABox $\mathcal{A}$ is $\mathcal{T}$-consistent if the KB $(\mathcal{T}, \mathcal{A})$ is consistent.

### Example 1

As a running example, we consider a simple KB $\mathcal{K}_{ex} = (\mathcal{T}_{ex}, \mathcal{A}_{ex})$ about the university domain that contains concepts for postdoctoral researchers (Postdoc), professors (Pr), Ph.D holders (PhD), and graduate courses (GradC), as well as roles to link advisors to their students (Adv), instructors to their courses (Teach) and student to the courses they attend (TakeC). The ABox $\mathcal{A}_{ex}$ provides information about an individual $a$:

$$\mathcal{T}_{ex} = \{ \text{Postdoc} \subseteq \text{PhD}, \text{Pr} \subseteq \text{PhD}, \text{Postdoc} \subseteq \neg \text{Pr}, \text{FPr} \subseteq \text{Pr}, \text{APr} \subseteq \text{Pr}, \text{APr} \subseteq \neg \text{FPr}, \exists \text{Adv} \subseteq \text{Pr} \}$$

$$\mathcal{A}_{ex} = \{ \text{Postdoc}(a), \text{APr}(a), \text{Adv}(a, b), \text{Teach}(a, c) \}$$

Observe that $\mathcal{A}_{ex}$ is $\mathcal{T}_{ex}$-inconsistent.

### Queries

We focus on conjunctive queries (CQs) which take the form $q(\vec{x}) = \exists y \psi(\vec{x}, y)$, where $\psi$ is a conjunction of atoms of the forms $A(t)$ or $R(t, t')$, with $t, t'$ individuals or variables from $\vec{x} \cup y$. A CQ is called Boolean (BCQ) if it has no free variables (i.e. $\vec{x} = \emptyset$). Given a CQ $q$ with free variables $\vec{x} = \{x_1, \ldots, x_k\}$ and a tuple of individuals $\vec{a} = \{a_1, \ldots, a_k\}$, we use $q(\vec{a})$ to denote the BCQ resulting from replacing each $x_i$ by $a_i$. A tuple $\vec{a}$ is a certain answer to $q$ over $\mathcal{K}$, written $\mathcal{K} \models q(\vec{a})$, iff $q(\vec{a})$ holds in every model of $\mathcal{K}$. When we use the generic term query, we mean a CQ.

### Causes and Conflicts

A cause for a BCQ $q$ w.r.t. $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ is a minimal $\mathcal{T}$-consistent subset $\mathcal{C} \subseteq \mathcal{A}$ such that $(\mathcal{T}, \mathcal{C}) \models q$. We use $\text{causes}(q, \mathcal{K})$ to refer to the set of causes for $q$. A conflict for $\mathcal{K}$ is a minimal $\mathcal{T}$-inconsistent subset of $\mathcal{A}$, and $\text{conf}(\mathcal{K})$ denotes the set of conflicts for $\mathcal{K}$.

When $\mathcal{K}$ is a DL-LiteR KB, every conflict for $\mathcal{K}$ has at most two assertions. We can thus define the set of conflicts of a set of assertions $\mathcal{C} \subseteq \mathcal{A}$ as follows:

$$\text{conf}(\mathcal{C}, \mathcal{K}) = \{ \beta \mid \exists \alpha \in \mathcal{C}, \{ \alpha, \beta \} \in \text{conf}(\mathcal{K}) \}.$$
A first way of repairing the data is to delete assertions from the ABox that lead to undesirable consequences, either because they contribute to the derivation of an unwanted answer or because they conflict with causes of some wanted answer.

**Example 3.** Reconsider the KB $\mathcal{K} = (\mathcal{T}_{\text{ex}}, \mathcal{A}_{\text{ex}})$, and suppose $a$ is an unwanted answer for $\mathcal{P}(x)$ but a wanted answer for $\mathcal{P}(x)$. Deleting the assertions $\mathcal{A}(a)$ and $\mathcal{D}(a, b)$ achieve the objectives since $(\mathcal{T}_{\text{ex}}, \{\mathcal{P}(a), \mathcal{D}(a, c)\}) \not\models_{\text{brave}} \mathcal{P}(a)$ and $(\mathcal{T}_{\text{ex}}, \{\mathcal{P}(a), \mathcal{D}(a, c)\}) \models_{\text{iar}} \mathcal{D}(a)$.  

The next example shows that, due to data incompleteness, it can also be necessary to add new assertions.

**Example 4.** Consider $\mathcal{K} = (\mathcal{T}_{\text{ex}}, \{\mathcal{A}(a)\})$ with the same wanted and unwanted answers as in Ex. 3. The assertion $\mathcal{A}(a)$ has to be removed to satisfy the unwanted answer, but then there remains no cause for the wanted answer. This is due to the fact that the only cause of $\mathcal{A}(a)$ in $\mathcal{K}$ contains an erroneous assertion: there is no ‘good’ reason in the initial ABox for $\mathcal{D}(a)$ to hold. A solution is for the user to add a cause he knows for $\mathcal{D}(a)$, such as $\mathcal{P}(a)$.

We now provide a formal definition of the query-driven repairing problem investigated in this paper.

**Definition 1.** A query-driven repairing problem (QRP) consists of a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ to repair and two sets $\mathcal{W}, \mathcal{U}$ of BCQs that $\mathcal{K}$ should entail ($\mathcal{W}$) or not entail ($\mathcal{U}$). A repair plan (for $\mathcal{A}$) is a pair $\mathcal{R} = (\mathcal{E}_-, \mathcal{E}_+)$ such that $\mathcal{E}_- \subseteq \mathcal{A}$ and $\mathcal{E}_+ \cap \mathcal{A} = \emptyset$; if $\mathcal{E}_+ = \emptyset$, we say that $\mathcal{R}$ is deletion-only.

The sets $\mathcal{U}$ and $\mathcal{V}$ correspond to the unwanted and wanted answers in our scenario: $q(\bar{a}) \in \mathcal{U}$ (resp. $\mathcal{V}$) means that $\bar{a}$ is an unwanted (resp. wanted) answer for $q$. Slightly abusing terminology, we will use the term unwanted (resp. wanted) answers to refer to the BCQs in $\mathcal{U}$ (resp. $\mathcal{V}$). We say that a repair plan $(\mathcal{E}_-, \mathcal{E}_+)$ addresses all defects of a QRP $(\mathcal{K}, \mathcal{W}, \mathcal{U})$ if the KB $\mathcal{K}' = (\mathcal{T}, (\mathcal{A}, \mathcal{E}_-) \cup \mathcal{E}_+)$ is such that $\mathcal{K}' \models_{\text{iar}} q$ for every $q \in \mathcal{W}$, and $\mathcal{K}' \not\models_{\text{brave}} q$ for every $q \in \mathcal{U}$.

The next example shows that by considering several answers at the same time, we can exploit the interaction between the different answers to reduce the search space.

**Example 5.** Consider the KB $\mathcal{K} = (\mathcal{T}_{\text{ex}}, \mathcal{A})$ with ABox $\mathcal{A} = \{\mathcal{P}(a), \mathcal{A}(b), \mathcal{F}(b), \mathcal{D}(b, c), \mathcal{G}(c), \mathcal{T}(s, c), \mathcal{T}(c, b)\}$. It is easy to see that $\mathcal{K}$ is inconsistent, and its two repairs are obtained by removing either $\mathcal{P}(b)$ or $\mathcal{F}(b)$. Evaluating the queries $q_1(x) = \mathcal{P}(x)$ and $q_2(x) = \exists y \mathcal{F}(x, y) \land \mathcal{G}(y) \land \mathcal{D}(z, v)$ over this KB yields:

\[
\mathcal{K} \models_{\text{brave}} q_1(b) \quad \mathcal{K} \models_{\text{brave}} q_2(b) \quad \mathcal{K} \models_{\text{iar}} q_2(a).
\]

We consider the QRP $(\mathcal{K}, \mathcal{W}, \mathcal{U})$ with wanted answers $\mathcal{W} = \{q_1(b), q_2(a)\}$ and unwanted answers $\mathcal{U} = \{q_2(b)\}$.

Two deletion-only repair plans address all defects: $\{\mathcal{P}(b), \mathcal{F}(b), \mathcal{D}(b, c)\}$ and $\{\mathcal{P}(b), \mathcal{D}(b, c)\}$. Indeed, we must delete exactly one of $\mathcal{P}(b)$ and $\mathcal{F}(b)$ for $q_1(b)$ to be entailed under IAR semantics, and we cannot remove $\mathcal{G}(c)$ or $\mathcal{T}(c, b)$ without losing the wanted answer $q_2(a)$. Thus, the only way to get rid of $q_2(b)$ is to delete $\mathcal{T}(b, c)$.

If we consider only $\mathcal{U}$ (i.e. $\mathcal{W} = \emptyset$), there are additional possibilities such as $\{\mathcal{G}(c)\}$ and $\{\mathcal{T}(s, c)\}$, and there is no evidence that $\mathcal{T}(b, c)$ has to be deleted.  

If we want to avoid introducing new errors, a fully automated repairing process is impossible: we need the user to validate every assertion that is removed or added in order to remove (resp. add) only assertions that are false (resp. true).

**Example 6.** Reconsider the problem from Ex. 5, and suppose that the user knows that $\mathcal{T}(s, c)$ is false and every other assertion in $\mathcal{A}$ is true. An automatic repairing will remove the true assertion $\mathcal{T}(b, c)$. The problem is due to the absence of a ‘good’ cause for the wanted answer $q_2(a)$.

Since we will be studying an interactive repairing process, in which users must validate changes, we will also need to formalize the user’s knowledge. For the purposes of this paper, we assume that the user’s knowledge is consistent with the considered TBox $\mathcal{T}$, and so can be captured as a set $\mathcal{M}_{\text{user}}$ of models of $\mathcal{T}$. Instead of using $\mathcal{M}_{\text{user}}$ directly, it will be more convenient to work with the function $\mathcal{F}(\mathcal{U})$ induced from $\mathcal{M}_{\text{user}}$ that assigns truth values to BCQs in the obvious way: $\mathcal{F}(\mathcal{U}) = q$ if $q$ is true in every $\mathcal{M} \in \mathcal{M}_{\text{user}}$, $\mathcal{F}(\mathcal{U}) = \bot$ if $q$ is false in every $\mathcal{M} \in \mathcal{M}_{\text{user}}$, and $\mathcal{F}(\mathcal{U}) = \emptyset$ otherwise. We will assume throughout the paper the following truthfulness condition: $\mathcal{F}(\mathcal{U}) = \bot$ for every $q \in \mathcal{U}$, and $\mathcal{F}(\mathcal{U}) = \emptyset$ for every $q \in \mathcal{W}$.

We now formalize the requirement that repair plans only contain changes that are sanctioned by the user.

**Definition 2.** A repair plan $(\mathcal{E}_-, \mathcal{E}_+)$ is valid w.r.t. user $\mathcal{U}$ just in the case that $\mathcal{F}(\mathcal{U}) = \emptyset$ for every $\alpha \in \mathcal{E}_-$ and $\mathcal{F}(\mathcal{U}) = \emptyset$ for every $\alpha \in \mathcal{E}_+$.

Unfortunately, it may be the case that there is no validatable repair plan addressing all defects. This may happen, for instance, if the user knows some answer is wrong but cannot pinpoint which assertion is at fault, as we illustrate next.

**Example 7.** Consider the QRP given by:

\[
\mathcal{K} = (\mathcal{T}_{\text{ex}}, \{\mathcal{F}(a), \mathcal{D}(a, b), \mathcal{G}(c)\})
\]

\[
\mathcal{U} = \{\exists x \mathcal{F}(a) \land \mathcal{D}(a, b) \land \mathcal{G}(c)\}
\]

Suppose that $\mathcal{F}(\mathcal{U}) = \emptyset$, $\mathcal{F}(\mathcal{U}) = \emptyset$, $\mathcal{F}(\mathcal{U}) = \emptyset$, $\mathcal{F}(\mathcal{U}) = \emptyset$, and $\mathcal{F}(\mathcal{U}) = \emptyset$. It is not possible to satisfy the wanted and unwanted answers at the same time, since adding the true assertion $\mathcal{D}(a, b)$ creates a cause for the unwanted answer that does not contain any assertion $\alpha$ with $\mathcal{F}(\mathcal{U}) = \emptyset$: the user does not know which of $\mathcal{D}(a, b)$ and $\mathcal{G}(c)$ is erroneous.

As validatable repair plans addressing all defects are not guaranteed to exist, our aim will be to find repair plans that are optimal in the sense that they address as many of the defects as possible, subject to the constraint that the changes must be validated by the user.

### 4 Optimal Repair Plans

To compare repair plans, we consider the answers from $\mathcal{U}$ and $\mathcal{W}$ that are satisfied by the resulting KBs, where:

- $q \in \mathcal{U}$ is satisfied by $\mathcal{K}$ if $\mathcal{K} \models_{\text{iar}} q$
- $q \in \mathcal{W}$ is satisfied by $\mathcal{K}$ if there exists $\alpha \in \mathcal{A}$ such that $\text{conf}(\alpha, \mathcal{K}) = \emptyset$ and there is no $\alpha \in \mathcal{A}$ with $\mathcal{F}(\alpha) = \emptyset$.

\[1\]In what follows, we often omit ‘w.r.t. user’ and leave it implicit.
Remark 1. Observe that for $q \in W$ to be satisfied by $\mathcal{K}$, we require not only that $\mathcal{K} \models_{\mathcal{L}} q$, but also that there exists a cause for $q$ that does not contain any assertions known to be false, i.e. $\mathcal{K} \models_{\mathcal{L}} q$ should hold ‘for a good reason’. We impose this additional requirement to avoid counterintuitive situations, e.g. preferring repair plans that remove fewer false assertions in order to retain a conflict-free (but erroneous) cause for a wanted answer.

We say that a repair plan $\mathcal{R} = (\mathcal{E}_-, \mathcal{E}_+)$ satisfies $q \in \mathcal{U} \cup W$ if the KB $\mathcal{K}_{\mathcal{R}} = (\mathcal{T}, (A, \mathcal{E}_-) \cup \mathcal{E}_+) \models q$, and we use $S(\mathcal{R})$ (resp. $S_U(\mathcal{R})$, $S_W(\mathcal{R})$) to denote the sets of answers (resp. unwanted answers, wanted answers) satisfied by $\mathcal{R}$.

Two repair plans $\mathcal{R}$ and $\mathcal{R}'$ can be compared w.r.t. the sets of unwanted and wanted answers that they satisfy: for $A \in \{\mathcal{U}, W\}$, we define the preorder $\preceq_A$ by setting $\mathcal{R} \preceq_A \mathcal{R}'$ iff $S_A(\mathcal{R}) \subseteq S_A(\mathcal{R}')$, and obtain the corresponding strict order ($\prec_A$) and equivalence relations ($\equiv_A$) in the usual way. If the two criteria are equally important, we can combine them using the Pareto principle: $\mathcal{R} \succeq_{(\mathcal{U}, W)} \mathcal{R}'$ iff $\mathcal{R} \preceq \mathcal{R}'$ and $\mathcal{R} \not\sim_W \mathcal{R}'$. Alternatively, we can use the lexicographic method to give priority either to the wanted answers ($\succeq_W \mathcal{R}$) or unwanted answers ($\succeq_U \mathcal{R}$) for the two criteria. We can then define the corresponding notions of $\succeq$-optimal repair plan.

Definition 3. A repair plan $(\mathcal{E}_-, \mathcal{E}_+)$ is globally (resp. locally) $\succeq$-optimal w.r.t. user if it is globally (resp. locally) $\succeq$-optimal w.r.t. user and there is no other repair plan $(\mathcal{E}_-, \mathcal{E}_+)$ such that $(\mathcal{E}_-, \mathcal{E}_+) \succeq (\mathcal{E}_', \mathcal{E}_+)$ (resp. such that $\mathcal{E}_- \subseteq \mathcal{E}_-$, $\mathcal{E}_+ \subseteq \mathcal{E}_+$ and $(\mathcal{E}_-, \mathcal{E}_+) \prec (\mathcal{E}_', \mathcal{E}_+)$).

Globally $\succeq$-optimal repair plans are those that are maximal with respect to the preference relation $\succeq$, whereas locally $\succeq$-optimal repair plans are those that cannot be improved in the $\succeq$ order by adding further assertions.

Remark 2. If a repair plan is validatable and addresses all defects of a QRP, then it is globally $\succeq_W$-optimal. If it additionally satisfies every $q \in W$ (ensuring that there is a ‘good’ cause for every $q \in W$), then it is globally $\succeq_W$-optimal for every $\succeq \in \{\succeq_W, \succeq_{(\mathcal{U}, W)}, \succeq_{\mathcal{U} \cup W}\}$.

The following example illustrates the difference between local and global optimality.

Example 8. Consider the QRP $((\mathbb{T}_{\mathbb{Q}}, A), \mathcal{W}, \mathcal{U})$ where

\[ A = \{\text{Teach}(a, e), \text{Adv}(a, b), \text{TakeC}(b, e), \text{TakeC}(b, e), \text{GrC}(e)\} \]

\[ \mathcal{W} = \{\exists x \text{Teach}(a, x), \exists x \text{Teach}(b, x) \wedge \text{GrC}(x)\} \]

\[ \mathcal{U} = \{\exists x y \text{Teach}(a, x) \wedge \text{Adv}(a, y) \wedge \text{TakeC}(x, y) \wedge \text{GrC}(x)\} \]

Suppose that user $(\text{Teach}(a, e)) = \text{user(GrC(e))} = \text{false}$, user $(\alpha)$ = unknown for the other $\alpha \in A$, and the user knows that Teach$(a, c), \text{Teach}(a, d)$ and GrC$(c)$ are true.

It can be verified that the repair plan $\mathcal{R}_1 = \{(\text{Teach}(a, e), \text{GrC}(e)), (\text{Teach}(a, c))\}$ satisfies the first answer in $\mathcal{W}$ and the (only) answer in $\mathcal{U}$. It is locally $\succeq_{(\mathcal{U}, W)}$-optimal since the only way to satisfy the second wanted answer would be to add GrC$(c)$, which would create a cause for the unwanted answer, which could not be repaired by removing additional assertions as the user does not know which of Adv$(a, b)$ and takeC$(b, c)$ is false. However, $\mathcal{R}_1$ is not globally $\succeq_{(\mathcal{U}, W)}$-optimal because $\mathcal{R}_2 = \{\{\text{Teach}(a, e), \text{GrC}(e)), \{\text{Teach}(a, d), \text{GrC}(c)\}\}$ satisfies all answers in $\mathcal{W} \cup \mathcal{U}$.

In order to gain a better understanding of the computational properties of the different ways of ranking repair plans, we study the complexity of deciding if a given repair plan is optimal w.r.t. the different criteria. Since validatability of a repair plan depends on user, in this section, we measure the complexity w.r.t. $|A|$, $|\mathcal{U}|$, $|\mathcal{W}|$, as well as the size of the set $\text{True}^\text{user}_{\text{rel}} = \{\alpha \in \text{True}^\text{user} \mid \text{there exists } q \in \mathcal{W} \text{ such that } \alpha \in C \text{ for some } C \in \text{causes}(q, A \cup \text{True}^\text{user})\}$ where Trueuser = $\{\alpha \mid \text{user}(\alpha) = \text{true}\}$. We make the reasonable assumption that Trueuser (hence True^reluser) is finite.

Theorem 1. Decision if a repair plan is globally $\succeq$-optimal is coNP-complete for $\succeq \in \{\succeq_{(\mathcal{U}, W)}, \succeq_{\mathcal{U} \cup W}, \succeq_{W}\}$, and in P for $\succeq \in \{\succeq_{\mathcal{U}}, \succeq_{W}\}$. Decision if a repair plan is locally $\succeq$-optimal is in P for $\succeq \in \{\succeq_{\mathcal{U}}, \succeq_{W}\}$.

For the coNP upper bounds, we note that to show that $\mathcal{R}$ is not $\succeq$-optimal (for $\succeq \in \{\succeq_{(\mathcal{U}, W)}, \succeq_{\mathcal{U} \cup W}\}$), we can guess another repair plan $\mathcal{R}'$ and verify in P that both plans are validatable and that $\mathcal{R}'$ satisfies more answers than $\mathcal{R}$. The lower bounds are by reduction from (variants of) UNSAT.

To establish the tractability results from Theorem 1, we provide characterizations of optimal plans in terms of the notion of satisfiability of answers, defined next.

Definition 4. An answer $q \in \mathcal{U} \cup \mathcal{W}$ is satisfiable if there exists a validatable repair plan that satisfies $q$. We say that $q$ is satisfiable w.r.t. a validatable repair plan $\mathcal{R} = (\mathcal{E}_-, \mathcal{E}_+)$ if there exists a validatable repair plan $\mathcal{R}' = (\mathcal{E}_', \mathcal{E}_+)$ such that $\mathcal{E}_- \subseteq \mathcal{E}_-'$, $\mathcal{E}_+ \subseteq \mathcal{E}_+$ and $(\mathcal{E}_-, \mathcal{E}_+) \succeq (\mathcal{E}_', \mathcal{E}_+)$.

Proposition 1. Decision if an answer is satisfied, satisfiable, or satisfiable w.r.t. a repair plan is in P.

Combining Prop. 1 with the following characterizations yields polynomial-time procedures for optimality testing.

Proposition 2. A validatable repair plan $\mathcal{R}$ is:

- globally $\succeq_{\mathcal{U}}$- (resp. $\succeq_{W_\mathcal{U}}$-) optimal iff it satisfies every satisfiable $q \in \mathcal{U}$ (resp. $q \in \mathcal{W}$).
- locally $\succeq_{\mathcal{U}}$-optimal iff it is locally $\succeq_{\mathcal{U}}$-optimal if it satisfies every $q \in \mathcal{U} \cup \mathcal{W}$ that is satisfiable w.r.t. $\mathcal{R}$.
- locally $\succeq_{W_\mathcal{U}}$-optimal iff it satisfies every satisfiable $q \in \mathcal{W}$ and every $q \in \mathcal{U}$ that is satisfiable w.r.t. $\mathcal{R}$.

Our complexity analysis reveals that the notions of global optimality based upon the preference relations $\succeq_{(\mathcal{U}, W)}$, $\succeq_{\mathcal{U} \cup W}$, and $\succeq_{\mathcal{U} \cup W}$ have undesirable computational properties; even when provided with all relevant user knowledge, it is intractable to decide whether a given plan is optimal. Moreover, while plans globally $\succeq_{\mathcal{U}}$- (resp. $\succeq_{\mathcal{W}}$-) optimal can be interactively constructed in a monotonic fashion by removing further false assertions (resp. and adding further true assertions), building a globally optimal plan for a preference relation that involves both $\mathcal{U}$ and $\mathcal{W}$ may require backtracking over answers already satisfied (cf. the situation in Example 8).
Algorithm OptRP_{\mathcal{U}}
Input: QRP (K = (\mathcal{U}, \mathcal{A}), \mathcal{U}, \mathcal{W}) Output: repair plan
A. \mathcal{E}_- \leftarrow \emptyset, \mathcal{E}_+ \leftarrow \emptyset
B. Display the assertions of \bigcup_{q \in \mathcal{U} \cup \mathcal{W}} \text{causes}(q, K) and
   \bigcup_{q \in \mathcal{W}, C \text{ causes}(q, K)} \text{conf}(C, K)
1. Ask user to mark all false (F) and true (T) assertions to
   while preserving \text{causes for the wanted answers not satisfied after Step B,}
   and every \text{cause of which contains at least one false assertion. Hence}
   \text{long to a cause of some}
2. \mathcal{E}_- \leftarrow \mathcal{E}_- \cup F \cup \text{conf}(T, K)
C. While \mathcal{W}' = \mathcal{W} \setminus \mathcal{S}_\mathcal{W}(\mathcal{E}_-, \mathcal{E}_+) \neq \emptyset: q \leftarrow \text{first}(\mathcal{W}')
1. Ask the user for true assertions \mathcal{T}_q (not already provided)
   to complete (or create) a cause for q
2. If \mathcal{T}_q = \emptyset (nothing to add): \mathcal{W} \leftarrow \mathcal{W} \setminus \{q\}, go to C.
3. \mathcal{E}_+ \leftarrow \mathcal{E}_+ \cup \mathcal{T}_q, \mathcal{E}_- \leftarrow \mathcal{E}_- \cup \text{conf}(\mathcal{T}_q, (\mathcal{T}, \mathcal{A} \cup \mathcal{T}_q))
4. Show assertions of every cause C of q such that
   \mathcal{T}_q \cap C \neq \emptyset and its conflicts: user indicates false,
   true assertions F', T': \mathcal{E}_- \leftarrow \mathcal{E}_- \cup F' \cup \text{conf}(T', K)
5. Show causes of assertions of every q' \in \mathcal{U} in A\mathcal{E}_- \cup \mathcal{E}_+:
   user gives false assertions F'': \mathcal{E}_- \leftarrow \mathcal{E}_- \cup F''
6. If there is q'' \in \mathcal{U} such that (\mathcal{T}, A\mathcal{E}_- \cup \mathcal{E}_+) \models \text{brave q''}
   and (\mathcal{T}, A\mathcal{E}_+) \not\models \text{brave q''}: \mathcal{E}_- \leftarrow \mathcal{E}_- \cup \mathcal{T}_q (\text{revert} \mathcal{E}_+)
D. Return (\mathcal{E}_-, \mathcal{E}_+)

Figure 1: Algorithm for constructing a globally \text{\leq_\mathcal{U}} and
   locally \text{\leq_{(\mathcal{U}, \mathcal{W})}}-optimal repair plan

For the preceding reasons, we target validatable repair plans that are both globally \text{optimal for} \text{\leq_\mathcal{U}} or \text{\leq_\mathcal{W}} (depending
   which is preferred) and \text{locally optimal for} \text{\leq_{(\mathcal{U}, \mathcal{W})}}. In
   Fig. 1, we give an interactive algorithm OptRP_{\mathcal{U}} for building
   such a repair plan when \mathcal{U} is preferred; if \mathcal{W} is preferred, we
   use the algorithm OptRP_{\mathcal{W}} obtained by removing Step C.6
   from OptRP_{\mathcal{U}}. The algorithms terminate provided the user
   knows only a finite number of assertions that may be inserted.
In this case, the algorithms output optimal repair plans:

Theorem 2. \text{The output of} OptRP_{\mathcal{U}} (resp. OptRP_{\mathcal{W}}) is \text{globally}
   \text{\leq_\mathcal{U}} (resp. \text{\leq_\mathcal{W}}) and \text{locally \leq_{(\mathcal{U}, \mathcal{W})}-optimal.}

Proof. We give the proof for OptRP_{\mathcal{U}}. First observe that at
   every point during the execution of the algorithm, the current
   repair plan is validatable, since only true assertions are added to
   \mathcal{E}_+ and false assertions are added to \mathcal{E}_- (they are either
   marked as false by the user or are in conflict with assertions
   that have been marked as true).
Step B adds to \mathcal{E}_- all assertions known to be false that be-
   long to a cause of some q \in \mathcal{U} \cup \mathcal{W} or a conflict of some
   cause of q \in \mathcal{W}. Thus, at the end of this step, \mathcal{E}_- satisfies
   every satisfiable answer in \mathcal{U}, that is, every answer in \mathcal{U}
   every cause of which contains at least one false assertion. Hence
   \text{(\mathcal{E}_-, \mathcal{E}_+)} is \text{\leq_\mathcal{U}-optimal at the end of step B. Moreover,}
   every false assertion that occurs in a cause or conflict of
   a cause of a wanted answer has been removed, so if q \in \mathcal{W}
   is not satisfied at this point, then it has no cause without any
   conflict in \mathcal{A}\setminus\{\alpha \mid \text{user}(\alpha) = \text{false}\}.

The purpose of Step C is to add new true assertions to cre-
   ate causes for the wanted answers not satisfied after Step B,
   while preserving \mathcal{S}_\mathcal{U}(\mathcal{E}_-, \mathcal{E}_+). For every q \in \mathcal{W}, while q
   is not satisfied, the user is asked to input true assertions to
   complete a cause for q in Step C.1. If he is unable to do so,
   at Step C.2, we remove q from \mathcal{W} (since it cannot be satisfi-
   ed w.r.t. user); otherwise, we update \mathcal{E}_- and \mathcal{E}_+ using \mathcal{T}_q
   (C.3). Note that since \mathcal{T}_q contains only true assertions, we
   can remove its conflicts without affecting already satisfied
   wanted answers; this step is necessary because \mathcal{T}_q may con-
   flict with assertions of \mathcal{A} that are not involved in the causes
   and conflicts presented at Step B. In Step C.4, we remove
   false assertions appearing in a new cause for q or its conflicts
   (such assertions may not have been examined in Step B). Step
   C.5 removes false assertions of new causes of unwanted
   answers, and Step C.6 undoes the addition of \mathcal{T}_q if it affects
   \mathcal{S}_\mathcal{U}(\mathcal{E}_-, \mathcal{E}_+). Thus, at the end of Step C, for every wanted
   answer, either it is satisfied, or the user is unable to supply a
   cause that does not deteriorate \mathcal{S}_\mathcal{U}(\mathcal{E}_-, \mathcal{E}_+). It follows that
   \text{(\mathcal{E}_-, \mathcal{E}_+)} is locally \text{\leq_{(\mathcal{U}, \mathcal{W})}-optimal.}

5 Optimal Deletion-Only Repair Plans

In this section, we restrict our attention to constructing opti-
   mal deletion-only repair plans. In this simpler setting, all of
   the previously introduced notions of optimality collapse into
   the one characterized in the following proposition.

Proposition 3. A validatable deletion-only plan is optimal iff
   it satisfies every q \in \mathcal{U} such that every C \in \text{causes}(q, K)
   has \alpha \in C with user(\alpha) = \text{false}, and every q \in \mathcal{W} for which
   there exists C \in \text{causes}(q, K) such that user(\alpha) \neq \text{false for every}
   \alpha \in C and user(\beta) = \text{false for every} \beta \in \text{conf}(C, K).

Constructing such repair plans can be done with one of the
   preceding algorithms, omitting Step C that adds facts. How-
   ever, it is possible to further assist the user by taking advan-
   tage of the fact that subsets of the ABox whose removal ad-
   dresses all defects of the QRP can be automatically identified,
   and then interactively transformed into optimal repair plans.
We call such subsets potential solutions.

An assertion is said to be relevant if it appears in a cause of
   some q \in \mathcal{U} \cup \mathcal{W} or in the conflicts of a cause of some
   q \in \mathcal{W}. If an assertion \alpha appears in every potential solution,
   either user(\alpha) = \text{false}, or there is no validatable potential
   solution. We call such assertions necessarily false. If \alpha
   appears in no potential solution, it is necessary to keep it in \mathcal{A}
   to retrieve some wanted answers under IAR semantics, so either
   user(\alpha) \neq \text{false, or it is not possible to satisfy all wanted
   answers. We call such assertions necessarily nonfalse.}

When a potential solution does not exist, a minimal correction
   subset of wanted answers (MCSW) is an inclusion-minimal subset
   \mathcal{W}' \subseteq \mathcal{W} such that removing \mathcal{W}' from \mathcal{W}
   yields a QRP with a potential solution. Because of the truth-
   fulness condition, we know that the absence of a potential so-
   lution means that some wanted answers are supported only by
   causes containing erroneous assertions (otherwise the wanted
   and unwanted answers would be contradictory, which would
   violate the truthfulness condition). Moreover, since removing
   all such answers from \mathcal{W} yields the existence of a potential
   solution, there exists a MCSW which contains only such an-
   swers, which we call an erroneous MCSW. This is why MC-
   SWs can help identify the wanted answers that cannot be satis-
   fied by a deletion-only repair plan.
Theorem 3. For complexity w.r.t. $|A|$, $|U|$ and $|W|$, deciding if a potential solution exists is NP-complete, deciding if an assertion is necessarily (non)false is coNP-complete, and deciding if $W' \subseteq W$ is a MCSW is BH$_2$-complete.

The lower bounds are proven by reduction from propositional (un)satisfiability and related problems. For the upper bounds, we construct in polynomial time a propositional CNF $\varphi = \varphi_U \land \varphi_W$ with:

$$
\varphi_U = \bigwedge_{q \in U} \bigwedge_{C \in \text{causes}(q,K)} \bigvee x_\alpha
$$

$$
\varphi_W = \bigwedge_{q \in W} \bigwedge_{C \in \text{causes}(q,K)} w_C
\land \bigwedge_{q \in W} \bigwedge_{C \in \text{causes}(q,K)} \bigwedge_{\alpha \in C} \neg w_C \lor \neg x_\alpha
\land \bigwedge_{q \in W} \bigwedge_{C \in \text{causes}(q,K)} \bigwedge_{\beta \in \text{conflicts}(C,K)} \neg w_C \lor x_\beta
$$

which has the following properties:

- there exists a potential solution iff $\varphi$ is satisfiable (satisfying assignments correspond to potential solutions);
- $\alpha$ is necessarily false iff $\varphi \land \neg x_\alpha$ is unsatisfiable;
- $\alpha$ is necessarily nonfalse iff $\varphi \land x_\alpha$ is unsatisfiable;
- there exist disjoint subsets $S, H$ of the clauses in $\varphi$ such that the MCSWs correspond to the minimal correction subsets (MCSs) of $S$ w.r.t. $H$, i.e. the subsets $M \subseteq S$ such that (i) $(S \setminus M) \cup H$ is satisfiable, and (ii) $(S \setminus M) \cup H$ is unsatisfiable for every $M' \subseteq M$.

We present in Fig. 2 an algorithm OptDRP for computing optimal deletion-only repair plans. Within the algorithm, we denote by $R(K, U, W, A')$ (resp. $N_i(K, U, W, A')$, $N_{-f}(K, U, W, A')$) the set of assertions from $A' \subseteq A$ that are relevant (resp. necessarily false, nonfalse) for the QRP $(K, U, W)$ when deletions are allowed only in $A'$ (the set $A'$ will be used to store assertions whose truth value is not yet determined). The general idea is that the algorithm incrementally builds a set of assertions that are false according to the user. It aids the user by suggesting assertions to remove, or wanted answers that might not be satisfiable when there is no potential solution, while taking into account the knowledge the user has already provided. If there exists a potential solution, the algorithm computes the necessarily (non)false assertions and asks the user either to validate them or to input false and nonfalse assertions to justify why they cannot be validated, and then to input further true or false assertions if the current set of false assertions does not address all defects. When a potential solution is found, the user has to verify that each wanted answer has a cause that does not contain any false assertion. If there does not exist a potential solution at some point, either initially or after some user inputs, the algorithm looks for an erroneous MCSW by computing all MCSWs, then showing for each of them the assertions involved in the causes of each query of the MCSW. If there is a query which has a cause without any false assertion, the MCSW under examination is not erroneous, nor are the other MCSWs that contain that query. Otherwise, the MCSW is erroneous and its queries are removed from $W$, and we return to the case where a potential solution exists.

Theorem 4. The algorithm OptDRP always terminates, and it outputs an optimal deletion-only repair plan.

Proof idea. Termination follows from the fact that every time we return to Step B, something has been added to $E_-$ or deleted from $W$, and nothing is ever removed from $E_-$ or added to $W$. Since we only add false assertions to $E_-$, the output plan is validatable. If the algorithm ends at Step B.2.b.ii, then $E_-$ satisfies every answer characterized in Prop. 3. Indeed, since $E_-$ is a potential solution, it satisfies every unwanted answer. Moreover, the answers removed from $W$ at Step C.2.c do not fulfill the conditions of Prop. 3 since all their causes contain some false assertion, and for every remaining $q \in W$, we ensure that there is a conflict-free cause of $q$ that contains no false assertions. If the algorithm ends at
Step B.2.c.i, the user was asked to indicate false or true assertions at Step B.2.c and was not able to input anything, so the user has deleted all false assertions he knows among the relevant assertions, and thus it is not possible to improve the current repair plan further. A similar argument applies if the algorithm ends at Step C.3.

To avoid overwhelming the user with relevant assertions at Step B.2.c, it is desirable to reduce the number of assertions presented at a time. This leads us to propose two improvements to the basic algorithm. First, we can divide QRPs into independent subproblems. Two answers are considered dependent if their causes (and conflicts in the case of wanted answers) share some assertion. Independent sets of answers do not interact, so they can be handled separately. Second, at Step B.2.c, the assertions can be presented in small batches. Borrowing ideas from work on reducing user effort in interactive revision, we can use a notion of impact to determine the order of presentation of assertions. Indeed, deleting or keeping an assertion may force us to delete or keep other assertions to get a potential solution. Relevant assertions can be sorted using two scores that express the impact of being declared false or true. For the impact of an assertion \( \alpha \) being false, we use the number of assertions that becomes necessarily (non)false if \( \alpha \) is deleted. The impact of \( \alpha \) being true also takes into account the fact that the conflicts of \( \alpha \) can be marked as false: we consider the number of assertions that are in conflict with \( \alpha \) or become necessarily (non)false when we disallow \( \alpha \)’s removal. We can rank assertions by the minimum of the two scores, using their sum to break ties.

6 Preliminary Experiments

We report on experiments made on core components of the above OptDRP algorithm. We focused on measuring the time to decide whether a potential solution exists (Step B), to compute necessarily (non)false and relevant assertions (Step B.1), to rank the relevant assertions w.r.t. their impact (Step B.2.c), and to find the MCSWs (Step C).

The components were developed in Java using the CQAPri system (www.lri.fr/~bourgaux/CQAPri) to compute query answers under IAR and brave semantics, with their causes, and the KB’s conflicts. We used SAT4J (www.sat4j.org) to solve the (UN)SAT reductions in Section 5.

We borrowed from the CQAPri benchmark [Bienvenu et al., 2016a] available at the URL above its: (i) TBox which is the DL-Lite\(_R\) version of the Lehigh University Benchmark [Lutz et al., 2013] augmented with constraints allowing for conflicts, (ii) c5 and c29 ABoxes with \( \sim 10 \) million assertions and, respectively, a ratio of assertions involved in conflicts of 5\%, that we found realistic, and of 29\%, and (iii) queries \( q_1, q_2, q_3, q_4 \), shown below. We slightly modified the original queries by changing some constants or variables in order to obtain dependent answers (whose causes and conflicts of causes share some assertions). We built 13 QRPs per ABox, by adding more and more answers of these queries to \( U \) or \( V \); the size of \( U \cup V \) varies from 8 to 121.

\[
q_1 = \exists y \text{ Person}(x) \land \text{takesCourse}(x, y) \land \text{Person}(G131) \land \text{GraduateCourse}(y) \land \text{takesCourse}(G131, y)
\]

\[
q_2 = \exists x \text{ Employee}(x) \land \text{memberOf}(x, D2) \land \text{degreeFrom}(x, y)
\]

\[
q_3 = \exists y \text{ teacherOf}(x, y) \land \text{degreeFrom}(x, U532)
\]

\[
q_4 = \exists x \text{ Employee}(x) \land \text{degreeFrom}(x, U532) \land \text{memberOf}(x, z) \land \text{Employee}(y) \land \\
\text{degreeFrom}(y, U532) \land \text{memberOf}(y, z)
\]

In all of our experiments, deciding if a potential solution exists, as well as computing the set of relevant assertions, takes just a few milliseconds. The difficulty of computing the necessarily (non)false assertions correlates with the number of relevant assertions induced by the QRP. For the c5 QRPs involving 85 to 745 relevant assertions, this computation took between 30ms to 544ms, while it took 24ms to 1333ms for the c29 QRPs involving 143 to 1404 relevant assertions. While these times seem reasonable in practice, ranking the remaining relevant assertions based on their impact is time consuming as it requires a number of calls to the SAT solver quadratic in the number of assertions: it took less than 10s up to \( \sim 150 \) assertions, less than 5mn up to \( \sim 480 \) assertions, and up to 25mn for 825 assertions. For all of the QRPs we built, computing the MCSWs takes a few milliseconds; somewhat surprisingly, we always found at most one MCSW.

7 Discussion

The problem of modifying DL KBs to ensure (non)entailments of assertions and/or axioms has been investigated in many works, see e.g. [De Giacomo et al., 2009; Calvanese et al., 2010; Gutierrez et al., 2011].

Our framework is inspired by that of [Jiménez-Ruiz et al., 2011], in which a user specifies two sets of axioms that should be entailed or not by a KB. Repair plans are introduced as pairs of sets of axioms to remove and add to obtain an ontology satisfying these requirements. Deletion-only repair plans are studied in [Jiménez-Ruiz et al., 2009] where heuristics based on the confidence and the size of the plan are used to help the user to choose a plan among all minimal plans.

When axiom (in)validation can be partially automatized, ranking axioms by their potential impact reduces the effort of manual revision [Meiilicke et al., 2008; Nikitina et al., 2012]. In our setting, we believe that validating sets of necessarily (non)false assertions requires less effort than hunting for false assertions among all relevant assertions, leading us to propose a similar notion of impact to rank assertions to be examined.

Compared to prior work, distinguishing features of our framework are the specification of changes at the level of CQ answers, the use of inconsistency-tolerant semantics, and the introduction of optimality measures to handle situations in which not all objectives can be achieved.

In future work, two aspects of our approach deserve further attention. First, when insertions are needed, it would be helpful to provide users with suggestions of assertions to add. The framework of query abduction [Calvanese et al., 2013], which was recently extended to inconsistent KBs [Du et al., 2015], could provide a useful starting point. Second, our experiments revealed the difficulty of ranking relevant assertions, so we plan to develop optimized algorithms for computing impact and explore alternative definitions of impact.
Acknowledgements
This work was supported by contract ANR-12-JS02-007-01.

References