

Learning to Rank for Synthesizing Planning Heuristics

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Abstract

We investigate learning heuristics for domain-specific planning. Prior work framed learning a heuristic as an ordinary regression problem. However, in a greedy best-first search, the *ordering* of states induced by a heuristic is more indicative of the resulting planner’s performance than mean squared error. Thus, we instead frame learning a heuristic as a learning to rank problem which we solve using a RankSVM formulation. Additionally, we introduce new methods for computing features that capture temporal interactions in an approximate plan. Our experiments on recent International Planning Competition problems show that the RankSVM learned heuristics outperform both the original heuristics and heuristics learned through ordinary regression.

1 Introduction

Forward state-space greedy heuristic search is a powerful technique that can solve large planning problems. However, its success is strongly dependent on the quality of its heuristic. Many domain-independent heuristics estimate the distance to the goal by quickly solving easier, approximated planning problems [Hoffmann and Nebel, 2001; Helmert, 2006; Helmert and Geffner, 2008]. While domain-independent heuristics have enabled planners to solve a much larger class of problems, there is a large amount of room to improve their estimates. In particular, the effectiveness of many domain-independent heuristics varies across domains, with poor performance occurring when the approximations in the heuristic discard a large amount of information about the problem.

Previous work has attempted to overcome the limitations of these approximations by learning a domain-specific heuristic correction [Yoon *et al.*, 2006; 2008]. Yoon *et al.* formulated learning a correction for the FastForward (FF) heuristic [Hoffmann and Nebel, 2001] as a regression problem and solved it using ordinary least-squares regression. While the resulting planner is no longer domain-independent, the learning process is domain independent, and the learned heuristic is more effective than the standard FF heuristic.

In this paper, we improve on these results by framing the learning problem as a learning to rank problem instead of an

ordinary regression problem. This is motivated by the insight that, in a greedy search, the ranking induced by a heuristic, rather than its numerical values, governs the success of the planning. By optimizing for the ranking directly, our RankSVM learner is able to produce a heuristic that outperforms heuristics learned through least-squares regression.

Additionally, we introduce new methods for constructing features for heuristic learners. Like Yoon *et al.*, we derive our features from an existing domain-independent heuristic [Yoon *et al.*, 2006; 2008]. However, our features focus on the ordering and interaction between actions in approximate plans. Thus, they can be based on any existing heuristic that implicitly constructs an approximate plan, such as the context-enhanced additive (CEA) heuristic [Helmert and Geffner, 2008]. These features can be easily constructed and still encode a substantial amount of information for heuristic learners.

In our experiments, we evaluate the performance of the different configurations of our learners on several of the International Planning Competition learning track problems [Vallati *et al.*, 2015]. We find that the learned heuristics using the RankSVM approach allow more problems to be solved successfully than using the popular FF and CEA heuristics alone. Additionally, they significantly surpass the performance of heuristics learned through ordinary regression.

2 Related Work

Prior work in learning for planning spans many types of domain-specific planning knowledge [Jiménez *et al.*, 2012]; our focus in this paper is on learning heuristic functions.

Yoon *et al.* were the first to improve on a heuristic function using machine learning [Yoon *et al.*, 2006; 2008]. They centered their learning on improving the FF Heuristic [Hoffmann and Nebel, 2001], using ordinary least-squares regression to learn the difference between the actual distance-to-go and the estimate given by the FF heuristic. Their key contribution was deriving features using the relaxed plan that FF produces when computing its estimate. Specifically, they used taxonomic syntax to identify unordered sets of actions and predicates on the relaxed plan that shared common object arguments. Because there are an exponential number of possible subsets of actions and predicates, they iteratively introduced a taxonomic expression that identifies a subset greedily based on which subset will give the largest decrease in

mean squared error. This process resulted in an average of about 20 features per domain [Xu *et al.*, 2009]. In contrast, our features encode ordering information about the plan and can be successfully applied without any taxonomic syntax or iterative feature selection.

Xu *et al.* built on the work of Yoon *et al.* and incorporated ideas from structural prediction [Xu *et al.*, 2007; 2009]. They adapted the learning-as-search optimization framework to the context of beam search. They learn a discriminative model to rank the top b successors per state to include in the beam search. In subsequent work, they used RankBoost to more reliably rank successors by bootstrapping the predictions of action-selection rules [Xu *et al.*, 2010]. Although we also use a ranking approach, we use ranking as a loss function to train a heuristic from the position of states along a trajectory, resulting in a global heuristic that can be directly applied to greedy best-first search.

Arfae *et al.* learned heuristics by iteratively improving on prior heuristics for solving combinatorial search problems [Arfae *et al.*, 2011]. They used neural networks and user defined features. Finally, Virseda *et al.* learned combinations of existing heuristics values that would most accurately predict the cost-to-go [Virseda *et al.*, 2013]. However, this strategy does not use features derived from the structure of the heuristics themselves.

Wilt *et al.* investigated greedy heuristic search performance in several combinatorial search domains [Wilt and Ruml, 2012]. Their results suggest that heuristics that exhibit strong correlation with the distance-to-go are less likely to produce large local minima. And large local minima are thought to often dominate the runtime of greedy planners [Hoffmann, 2005; 2011]. They later use the Kendall rank correlation coefficient (τ) to select a pattern database for some of these domains [Wilt and Ruml, 2015]. Their use of τ as a heuristic quality metric differs from our own use because they score τ using sampled states near the goal while we score τ by ranking the states on a plan.

3 Planning domains and training data

Our goal is to learn a heuristic that will improve the coverage, or the number of problems solved, for greedy forward-search planning on very large satisficing planning problems. Secondary goals are to decrease the resulting plan length and time to solve these problems. The search control of our planners is greedy best first search (GBFS) with alternating, dual open lists [Richter and Helmert, 2009]. The preferred operators in the second open list are computed by the base heuristic which, as we will later see, is used to generate our learning features [Hoffmann and Nebel, 2001]. We use the lazy variant of greedy best first search which defers heuristic evaluation of successors. We consider STRIPS planning problems [Fikes and Nilsson, 1971] with unit costs, and without axioms or conditional effects, but our techniques can be straightforwardly generalized to handle them.

Definition 1 (Planning Domain). A planning domain $\mathcal{D} = \langle \mathcal{P}, \mathcal{A} \rangle$ consists of a set of predicate schemas \mathcal{P} and a set of action schemas \mathcal{A} . Each action schemas contains a set of precondition predicates and effect predicates. A predicate

schema or action schema can be instantiated by assigning objects to its arguments.

Definition 2 (Planning Problem). A planning problem $\Pi = \langle \mathcal{D}, O, s_0, g, \rangle$ is given by a domain \mathcal{D} , a set of objects O , an initial state s_0 , and a goal partial-state g . The initial state s_0 is fully specified by a set of predicates. The goal partial-state g is only partially specified by its set of predicates.

The overall approach will be, for each *planning domain*, to train a learning algorithm on several *planning problem* instances, and then to use the learned heuristic to improve planning performance on additional planning problems from that same domain. Note that the new problem instances use the same predicate and action schemas, but may have different universes of objects, initial states, and goal states.

In order to learn a heuristic for a particular domain, we must first gather training examples from a set of existing training problems within the domain [Jiménez *et al.*, 2012]. Suppose that we have a distribution over problems for a domain \mathcal{D} , which will be used to generate testing problems. We will sample a set of training problems $\{\Pi^1, \dots, \Pi^n\}$ from \mathcal{D} . From each problem Π^i , we generate a set of training examples in which the j th training example is the pair $\langle x_j^i, y_j^i \rangle$ where $x_j^i = \langle s_j^i, \Pi^i \rangle$ is the input composed of a state s_j^i and the problem Π^i . Let y_j^i be the length of a plan from s_j^i to g^i . Ideally, y_j^i would be the length of the shortest plan, but because obtaining optimal plans is intractable for the problems we consider, we construct approximately optimal plans and use their lengths as the y values in the training data.

We use the set of states on a single high-quality plan from the initial state to the goal state as training examples. Unfortunately, we have observed that using low-quality plans, which are more easily found, can be dangerous, as it introduces large amounts of noise into the training data. This noise can produce conflicting observations of y_j^i for similar x_j^i , which can prevent the learner from identifying any meaningful predictive structure. Reducing at least this kind of local noise is important for the learning process even if the global plan is still suboptimal. Thus, we post-process each candidate plan using two local search methods: action elimination and plan neighborhood graph search [Nakhost, 2010].

In separate experiments, we attempted learning a heuristic by instead using a sampled set of successors on these plans as training examples. However, we found that the inclusion of these states slightly worsened the resulting performance of the learners. Our hypothesis is that the inclusion of successor states improves local accuracy at the expense of global accuracy. Because the runtime of greedy search methods is often dominated by the time to escape the largest local minima [Hoffmann, 2005; 2011; Wilt and Ruml, 2012; 2015], it is a worthwhile tradeoff to reduce the size of large local minima at the cost of increasing the size of small local minima.

4 Feature Representation

The majority of machine learning methods assume that the inputs are represented as points in a vector space. In our case, the inputs x_j^i are a pair of a state and a planning problem, each

of which is a complex structured symbolic object. So, we need to define a feature-mapping function ϕ that maps an x value into a vector of numeric feature values. This can also be done implicitly by defining a kernel, we restrict our attention to finite-dimensional ϕ that are straightforwardly computable.

The objective in designing a feature mapping is to arrange for examples that are close in feature space to have similar output values. Thus, we want to reveal the structural aspects of an input value that encode important similarities to other input values. This can be particularly challenging in learning for planning: while problems within the same domain share the same schemas for predicates and actions, the set of objects can be arbitrarily different. For example, a feature representation with a feature for each predicate instance present in s_j^i or g^i will perform poorly on new problems, which may not share any predicate instances with the problems used to create the feature representation.

Yoon et al. used information from the FF heuristic to construct additional features from the resulting relaxed plan [Yoon *et al.*, 2006; 2008]. The relaxed plan compresses the large set of possible actions into a small plan of actions that are likely to be relevant to achieving the goal. Many modern heuristics either explicitly or implicitly generate approximate plans, similar to FF’s relaxed plan, that can be represented as directed acyclic graphs (DAG) where each action is a vertex, and directed edges indicate that the outgoing action is supported by the incoming action. We provide feature mappings that are applicable to any heuristic that gives rise to such a DAG, but in this paper, we focus on the FF [Hoffmann and Nebel, 2001] and CEA [Helmert and Geffner, 2008] heuristics. Our method can be extended to include additional features for example derived from landmark heuristics or domain-dependent heuristics, although we do not consider these extensions here.

We can now view our training inputs as $x_j^i = \langle s_j^i, g^i, \pi_h^{ij} \rangle$ where π_h^{ij} is the DAG generated by heuristic h for state s_j^i and goal g^i . The computation time of each feature affects the performance of the resulting planner in a complex way: the feature representation is computed for every state encountered in the search, but good features will make the heuristic more effective, causing fewer states to be encountered.

4.1 Single Actions

The first feature representation serves primarily as a baseline. Each feature is the number of instances of a particular action schema in the DAG π_h^{ij} . The number of features is the number of action schemas $|\mathcal{A}|$ in the domain and thus around five for many domains. This feature representation is simple and therefore limited in its expressiveness, but it can be easily computed in time $O(|\pi_h^{ij}|)$ and is unlikely to overfit. If we are learning a linear function of $\phi(x)$, then the weights can be seen as adjustments to the predictions made by the DAG of how many instances of each action are required. So, for instance, in a domain that requires a robot to do a “move” action every time it “picks” an object, but where the delete relaxation only includes one “move” action, this representation would allow learning a weight of two on pick actions, effectively predicting the necessity of extra action instances.

4.2 Pairwise Actions

The second feature representation creates features for pairs of actions, encoding both their intersecting preconditions and effects as well as their temporal ordering in the approximate plan. First, we solve the all-pairs shortest paths problem on π_h^{ij} by running a BFS from each action vertex. Then, consider each pair of actions $a_1 \rightarrow a_2$ where a_2 descends from a_1 , as indicated by having a finite, positive distance from a_1 to a_2 in the all-pairs shortest paths solution. This indicates a_2 must come after a_1 on all topological sorts of the DAG; i.e., π_h^{ij} contains the implicit partial ordering $a_1 \prec a_2$. Moreover, if there is an edge (a_1, a_2) in π_h^{ij} , then $a_1 \prec a_2$ is an explicit partial ordering because a_1 directly supports a_2 .

For every pair of action schemas (A_1, A_2) , we include two features, counting the number of times it happens that, for an instance a_1 of A_1 and instance a_2 of A_2 ,

1. $a_1 \prec a_2, \text{EFF}(a_1) \cap \text{PRE}(a_2) \neq \emptyset$
2. $a_2 \succ a_1, \text{EFF}(a_2) \cap \text{PRE}(a_1) \neq \emptyset$

The current state and goal partial-state are included as dummy actions with only effects or preconditions respectively.

This feature representation is able to capture information about the temporal spread of actions in the DAG: for example, whether the DAG is composed of many short parallel sequences of actions or a single long sequence. Additionally, the inclusion of the preconditions and effects that overlap encodes interactions that are not often directly captured in the base heuristic. For example, FF and CEA make predicate independence approximations, which can result in overestimating the distance-to-go. The learner can automatically correct for these estimations if it learns that a single sequence can be used to achieve multiple predicates simultaneously.

In contrast to the single-action feature representation, the computation of the pairwise representation takes $O(|\pi_h^{ij}|^2)$ in the worst case. However, the DAG frequently is composed of almost disjoint subplans, so in practice, the number of pairs considered is fewer than $\binom{|\pi_h^{ij}|}{2}$. Additionally, this tradeoff is still advantageous if the learner is able to produce a much better heuristic. Finally, for both the single and pairwise feature representations, we add three additional features corresponding to the original heuristic value, the number of layers present in the DAG, and the number of unsatisfied goals.

5 Models for heuristic learning

We consider two different framings of the problem of learning a heuristic function f . In the first, the goal is to ensure that the $f(x)$ values are an accurate estimate of the distance-to-go in the planning state and problem encoded by $x = (s, \Pi)$. In the second, the goal is to ensure that the $f(x)$ values accurately rank the distance-to-go for different states s within the same planning problem Π , but do not necessarily reflect that actual distance-to-go values.

These different framings of the problem lead to different loss functions to be optimized by the learner and to different optimization algorithms. Because our learning algorithms cannot optimize for search performance directly, the loss function serves as a proxy for the search performance. A good loss function will be highly correlated with perfor-

mance of learned heuristics. We restrict ourselves to linear models that learn a weight vector w , and make a prediction $f(x) = \phi(x)^T w$.

5.1 Heuristic value regression

Because learning a heuristic is, at face value, a regression problem, a natural loss function is the root mean squared error (RMSE). A model with a low RMSE produces predictions close to the actual distance-to-go. Because each training problem Π^i may produce a different number of examples m_i , we use the average RMSE over all problems. This ensures that we do not assign more weight to problems with more examples. If f is a prediction function mapping a vector to the reals, then:

$$\text{RMSE} = \frac{1}{n} \sum_{i=1}^n \sqrt{\frac{1}{m_i} \sum_{j=1}^{m_i} (f(x_j^i) - y_j^i)^2}.$$

The first learning technique we applied is ridge regression (RR) [Hoerl and Kennard, 1970]. This serves as a baseline to compare to the results of Yoon et al. [Yoon et al., 2008]. Ridge regression is a regularized version of Ordinary Least Squares Regression (OLS). The regularization trades off optimizing the squared error against preferring low magnitude w using a parameter λ . This results in the following optimization problem. Letting $\phi(X)$ be the design matrix of concatenated features $\phi(x_j^i)$ and Y be the vector concatenation of y_j^i for all i, j , we wish to find

$$\min_w \|\phi(X)w - Y\|^2 + \lambda \|w\|^2.$$

This technique is advantageous because it can be quickly solved in closed form for reasonably sized $\phi(X)$, yielding the weight vector

$$w = (\phi(X)^T \phi(X) + \lambda I)^{-1} Y.$$

Optimizing RMSE directly, with no penalty λ , will yield a weight vector that performs well on the training data but might not generalize well to previously unseen problems. Increasing λ forces the magnitude of w to be smaller, which prevents the resulting f from "overfitting" the training data and therefore not generalizing well to new examples. This is especially important in our application as we are trying learn a heuristic that generalizes across the full state-space from only a few representative plans.

We select an appropriate value of λ by performing domain-wise leave-one-out cross validation (LOOCV): For different possible values of λ , and in a domain with n training problem, we train on data from $n - 1$ training problems and evaluate the resulting heuristic on the remaining problem according to the RMSE loss function, and average the scores from holding out each problem instance. We select the λ value for which the LOOCV RMSE is minimized over a logarithmic scale.

5.2 Learning to Rank

The RMSE, however, is not the most appropriate metric for our learning application. We are learning a heuristic for greedy search, which uses the heuristic solely to determine open list priority. The value of the heuristic per se does not

govern the search performance which depends most directly on the ordering on states induced by the heuristic. In this context, any monotonically increasing function of a heuristic results in the same ranking and performance. A heuristic may have arbitrarily bad RMSE despite performing well.

For these reasons, we consider the Kendall rank correlation coefficient (τ), a nonparametric ranking statistic, as a loss function. It represents the normalized difference between the number of correct rankings and incorrect rankings for each of the ranking pairs. As with the RMSE, we compute the average τ across each problem. The separation of problems is even more important here. Our τ only scores rankings between examples from the same problem as examples from separate problems are never encountered together in the same search. This provides a major source of leverage over an ordinary regression framework. Heuristics are not penalized for producing inconsistent distances-to-go values across multiple problems, allowing them to provide more effort to improve the per-problem rankings.

Let $s(i; j, k)$ score the concordance or discordance of the ranking function f for examples $\langle x_j^i, y_j^i \rangle$ and $\langle x_k^i, y_k^i \rangle$ from the same problem Π^i :

$$s(i; j, k) = \begin{cases} +1 & \text{if } \text{sgn}(f(x_k^i) - f(x_j^i)) = \text{sgn}(y_k^i - y_j^i) \\ -1 & \text{if } \text{sgn}(f(x_k^i) - f(x_j^i)) = -\text{sgn}(y_k^i - y_j^i) \\ 0 & \text{if } f(x_k^i) - f(x_j^i) = 0 \end{cases}.$$

Then the Kendall rank correlation coefficient is specified by

$$\tau = \frac{1}{n} \sum_{i=1}^n \frac{2}{m_i(m_i - 1)} \sum_{j=1}^{m_i} \sum_{k=j+1}^{m_i} s(i; j, k).$$

Note that each y_j^i is unique per problem Π^i because our examples come from a single trajectory. Observe that $\tau \in [-1, 1]$; values close to one indicate the ranking induced by the heuristic f has strong positive correlation to the true ranking of states as given by the actual labels. Conversely, values close to negative one indicate strong negative correlation.

If our loss function is τ , it is more effective to optimize τ directly in the learning process. To this end, we use Rank Support Vector Machines (RankSVM) [Joachims, 2002]. RankSVMs are variants of SVMs which penalize the number of incorrectly ranked training examples. Like SVMs, RankSVMs also have a parameter C used to provide regularization. Additionally, their formulation uses the hinge loss function to make the learning problem convex. Thus, a RankSVM finds the w vector that optimizes a convex relaxation of τ . Our formulation of the RankSVM additionally takes into account the fact that we only wish to rank training examples from the same problem. Our formulation is the following:

$$\begin{aligned} \min_w \quad & \|w\|^2 + C \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{k=j+1}^{m_i} \xi_{ijk} \\ \text{s.t.} \quad & \phi(x_j^i)^T w \geq \phi(x_k^i)^T w + 1 - \xi_{ijk}, \forall y_j^i \geq y_k^i, \forall i \\ & \xi_{ijk} \geq 0, \forall i, j, k. \end{aligned}$$

The first constraint can also be rewritten to look similar to the original SVM formulation. In this form, the RankSVM can

be viewed as classifying if x_j^i, x_k^i are properly ranked.

$$(\phi(x_j^i) - \phi(x_k^i))^T w \geq 1 - \xi_{ijk}, \forall y_j^i \geq y_k^i, \forall i$$

Notice that number of constraints and slack variables, corresponding to the number of rankings, grows quadratically in the size of each problem. This makes training the RankSVM more computationally expensive than RRs or SVMs. However, there are efficient methods for training these, and other SVMs, when considering just the linear, primal form of the problem [Joachims, 2006; Franc and Sonnenburg, 2009]. It is important to note that we generate a number of constraints that is quadratic only in the length of any given training plan, and do not attempt to rank all the actions of all the training plans jointly; this allows us to increase the number of training example plans without dramatically increasing the size of the optimization problem.

An additional advantage of RankSVM is that it supports the inclusion of the non-negativity constraint $w \geq 0$ which provide additional regularization. Because each feature represent a count of actions or action pairs, the values are always non-negative, as are the target values. We generally expect that DAGs with a large number of actions indicate that the state is far from the goal. The non-negativity constraint allows us to incorporate this prior knowledge in the model, which can sometimes improve the generalization of the learned heuristic. As in RR, we select C using a line search over a logarithmic scale, to maximize a cross-validated estimate of τ . As a practical note, we start with an over-regularized model where $C \approx 0$ and increase C until reaching a local minimum because SVMs are trained much more efficiently for small C .

6 Results

We implemented our planners using the FastDownward framework [Helmert, 2006].¹ Each planning problem is compiled to a representation similar to SAS+ [Bäckström and Nebel, 1995] using the FastDownward preprocessor. However, the predicates that represent each SAS+ (variable, value) pair are still stored so, actions and states can be mapped back to their prior form. We used the `dlib` C++ machine learning library to implement the learning algorithms [King, 2009].

We experimented on four domains from the 2014 IPC learning track [Vallati *et al.*, 2015]: *elevators*, *transport*, *parking*, and *no-mystery*. For each domain, we constructed a set of unique examples with the competition problem generators by sampling parameters that cover competition parameter space. We use a variant of the 2014 FastDownward Stone Soup portfolio [Helmert *et al.*, 2011] planner, with a large timeout and memory limit, to generate training example plans. We trained on at most 10 examples randomly selected from the set of problems our training portfolio planner was able to solve, and then tested on the remaining problems.

¹Because heuristic values are required to be integers in this framework, we scale up and then round predicted heuristic values, in order to capture more of the precision in the values. Recall that scaling will not alter the planner’s performance because arbitrary non-negative, affine transformations to $f(x)$ will not affect the resulting ranking in greedy search.

For each experiment, we report the following values:² **Cov**: coverage, or total number of problems solved; **Len**: mean plan length; **Run T**: mean planning time in seconds; **Exp**: mean number of expansions; **RMSE**: RMSE of learned heuristic; τ : Kendall rank correlation coefficient of learned heuristic; λ/C : regularization parameter value (λ for RR and C for RankSVM); **Feat**: number of nonzero weights learned relative to the total number of features; and **Train T**: runtime to train the heuristic learner in seconds.

Each planner was run on a single 2.5 GHz processor for 30 minutes with 5 GB of memory. We only include the results of the original CEA heuristic on *elevators*, as the default heuristic was able to solve each problem and the heuristics learned using CEA all performed similarly.

The heuristics learned by RankSVM are able to solve more problems than those learned using ridge regression. Within a domain, τ seems to be positively correlated with the number of problems solved while the RMSE does not. The pairwise-action features outperform the single-action features in RankSVM, making it worthwhile to incur a larger heuristic evaluation time for improved heuristic strength. The CEA learned heuristics performed slightly better than the FF learned heuristics.

On *transport* and *parking*, the training portfolio planner was only able to solve the smallest problems within the parameter space. Thus, our RankSVM learners demonstrate the ability to learn from smaller problems and perform well on larger problems. In separate experiments, we observed that both artificially over-regularized and under-regularized learners performed poorly indicating that selection of the regularization parameter is important to the learning process.

The learned heuristics perform slightly worse than the standard heuristics on *no-mystery* despite having almost perfect τ values. In separate experiments using eager best-first search, the learned heuristics perform slightly better on *no-mystery*, but the improvement is not significant. This domain is known to be challenging for heuristics because it contains a large number of dead-ends. We observed that τ does not seem sufficient for understanding heuristic performance on domains with harmful dead-ends. Our hypothesis is that failing to recognize a dead-end is often more harmful than incorrectly ranking nearby states and should be handled separately from learning a heuristic. A topic for future work is to combine our learned heuristics with learned dead-end detectors.

Inclusion of the non-negativity constraint (NN) on *transport* significantly improved the coverage of the FF learned heuristic over the normal RankSVM formulation. We believe that this constraint can sometimes improve generalization in domains with a large variance in size or specification. For example, the *transport* generator samples problems involving either two or three cities leading to a bimodal distribution of problems.

Finally, we tested two learned heuristics on the five evaluation problems per domain chosen in the IPC 2014 learning

²We use arithmetic mean for plan length and geometric means for planning time and number of expansions, and report these statistics only for solved instances; RMSE and τ values are cross-validation estimates.

<i>elevators (35)</i>									
Method	Cov.	Len.	Run T.	Exp.	RMSE	τ	λ/C	Feat.	Train T.
FF Original	14	318	196	17833	34.370	0.9912	N/A	N/A	N/A
FF RR Single	22	546	504	34970	4.091	0.9948	100	9/9	3.133
FF RR Pair	15	561	308	20985	3.789	0.9971	1000	53/53	11.686
FF RSVM Single	34	375	403	23765	79.867	0.9967	0.1	9/9	55.681
FF RSVM Pair	34	631	123	7083	418.828	0.9996	1	53/53	140.786
FF NN RSVM Pair	35	655	61	10709	46.296	0.9992	1	51/53	125.702
CEA Original	35	397	163	4504	21.494	0.9973	N/A	N/A	N/A
<i>transport (35)</i>									
FF Original	5	588	470	18103	126.193	0.8460	N/A	N/A	N/A
FF RR Single	0	None	None	None	31.518	0.9303	100	6/6	3.569
FF RR Pair	4	529	560	27866	27.570	0.9392	10000	32/32	11.028
FF RSVM Single	21	1154	650	29452	149.003	0.9720	0.1	6/6	106.901
FF RSVM Pair	20	587	178	8896	162.141	0.9797	0.001	32/32	117.808
FF NN RSVM Pair	31	663	206	7803	141.273	0.9798	0.01	17/32	287.586
CEA Original	9	448	542	9064	57.819	0.9314	N/A	N/A	N/A
CEA RR Single	11	493	436	6921	33.032	0.9420	10000	6/6	4.536
CEA RR Pair	2	609	1602	40327	30.731	0.9318	100	45/45	15.716
CEA RSVM Single	18	722	588	11334	130.653	0.9748	0.1	6/6	158.523
CEA RSVM Pair	31	650	225	3526	159.139	0.9804	0.0001	45/45	244.164
CEA NN RSVM Pair	29	696	277	9006	191.064	0.9795	0.0001	29/45	528.665
<i>parking (10)</i>									
FF Original	0	None	None	None	6.101	0.9525	N/A	N/A	N/A
FF RR Single	0	None	None	None	4.571	0.9648	100	7/7	0.201
FF RR Pair	2	156	1419	33896	4.285	0.9757	100	40/40	0.570
FF RSVM Single	0	None	None	None	10.468	0.9745	0.01	7/7	8.423
FF RSVM Pair	8	185	208	2852	18.262	0.9918	0.1	40/40	7.030
FF NN RSVM Pair	6	183	358	5891	143.063	0.9941	10	26/40	7.119
CEA Original	0	None	None	None	15.885	0.9628	N/A	N/A	N/A
CEA RR Single	0	None	None	None	4.667	0.9669	0.01	7/7	0.277
CEA RR Pair	1	280	1230	48180	4.448	0.9660	10	47/47	0.738
CEA RSVM Single	0	None	None	None	7.950	0.9757	0.1	7/7	10.830
CEA RSVM Pair	10	272	81	2147	45.823	0.9918	1	47/47	10.237
CEA NN RSVM Pair	10	260	70	1690	140.297	0.9938	10	27/47	9.179
<i>no-mystery (10)</i>									
FF Original	4	31	583	5658745	3.462	0.9841	N/A	N/A	N/A
FF RR Single	4	30	1004	8385159	1.662	0.9861	100	6/6	0.085
FF RR Pair	2	31	700	3898861	1.622	0.9902	1000	21/21	0.193
FF RSVM Single	1	26	1411	16201215	21.069	0.9871	100	6/6	0.712
FF RSVM Pair	2	28	892	6894959	39.350	0.9968	1	21/21	0.914
FF NN RSVM Pair	1	29	1049	7973003	80.588	0.9972	10	17/21	1.024
CEA Original	3	30	73	107773	16.851	0.9579	N/A	N/A	N/A
CEA RR Single	2	28	9	26319	1.824	0.9890	100	6/6	0.069
CEA RR Pair	3	32	104	169434	1.717	0.9892	1000	32/32	0.342
CEA RSVM Single	2	28	12	33559	36.457	0.9916	1	6/6	1.283
CEA RSVM Pair	3	32	34	46501	6.358	0.9964	0.01	32/32	4.023
CEA NN RSVM Pair	3	31	190	264225	55.141	0.9970	1	16/32	62.608

Table 1: Results from the *elevators*, *transport*, *parking*, and *no-mystery* IPC Learning Track 2014 problems.

track. Both the FF RSVM Pair heuristic and the CEA RSVM Pair heuristic solved all 5/5 problems in *elevators*, *transport*, and *parking* but only 1/5 problems in *no-mystery*.

7 Conclusion

Our results indicate that, for greedy search, learning a heuristic is best viewed as a ranking problem. The Kendall rank correlation coefficient τ is a better indicator of a heuristic's quality than the RMSE, and it is effectively optimized using the RankSVM learning algorithm. Pairwise-action features outperformed simpler features. Further work involves com-

binning features from several heuristics, learning complementary search control using our features, and incorporating the learned heuristics in planning portfolios.

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