MULTIPLE ANTENNA ARRAY SIGNAL COLLECTING AND PROCESSING

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Summary

This paper analyzes an optimal data collection and processing system for detecting a random signal field in a random noise field employing multiple antennas. The statistical model consists of a Gaussian random signal field and an independent Gaussian noise field. The time optimal data processing system to be analyzed is specified in terms of array deployment and signal/noise space time correlation structure. An exact expression for the characteristic function of the test statistic computed by the space-time signal processor is derived under the assumptions of stationarity and a long observation time. Analytical difficulties involved in evaluating and inverse transforming the characteristic function are considered through case studies. The test statistic is analytically shown to reduce to a Gaussian random variable in the threshold case for the array situation. The dependence of receiver performance on the space-time correlation structure of the signal and noise fields, the antenna deployment, and the optimal performance criteria (outage rate, pre-detection signal-to-noise ratio, etc.) is considered. Optimal antenna deployment is considered, for the threshold case, in terms of several physically realistic signal and noise field space-time correlation structure models.

Introduction

The problem of detecting a random signal field in the presence of a random noise field has received various treatments in the literature. Despite certain limited analytical treatment this is an important problem that occurs in many areas, such as fading on long range communication and radar links, sonar, radio astronomy, etc. The treatment addressed to this problem thus far consists almost exclusively of the derivation of the optimal receiver structure. The early work was done for the scalar (one antenna) case by Price* who only considered white noise and Middleton2/3 who considered colored noise. In these cases the derivations were performed using sampling point analysis rather than the more rigorous and useful Karhunen-Loeve expansion4 approach employed herein. Middleton2,3 also considered system performance with results presented in sampling point notation.

An approach for enhancing detectability in this problem area is the employment of an antenna array. The antenna array collects spatial samples of the signal and noise fields. Key issues are (1) optimal spatial coupling (antenna deployment), (2) optimal time processing (receiver structure), and (3) the performance improvement resulting from use of antenna array collection. The time optimal processing issue for the general case of detecting a random signal in random noise with an array receiver has been resolved by Middleton and Groginsky[5] and by VanTrees.6 However, efforts toward determining exact receiver performance and optimal coupling have been less successful for both array and single antenna coupling. This paper analytically develops the relationships essential for an exact performance evaluation for the antenna array case.

Problem Formulation

Statistical Model

The incident signal coupled by the n elements of the antenna array is represented by a column vectorx(t), and is assumed to contain a signal component and a noise component under the indicated hypothesis structure.

$$\begin{aligned} \underline{r}(t) &= \underline{s}(t) + \underline{N}(t) \quad ; \ H_1 \\ &= \underline{0} + \underline{N}(t) \quad ; \ H_0 \end{aligned} \tag{1}$$

The restriction to the "on-off" case is made to render the mathematics less cumbersome, and its removal is a straightforward extension of the results contained herein. The amplitude time structure of the signal s.(t) to be detected can be a narrow band, complex stochastic process y_(t) which has been modulated with some deterministic and possibly complex narrow band function f(t) 7 If the carrier frequency is w , then

$$\underline{s}(t) = R[\underline{y}(t)\, f(t)\, e^{j\omega_c t}] . \tag{2}$$

The orthogonal components of y.(t) are assumed to be Gaussian distributed and independent. This model can be used to represent situations such as multiplicative Rayleigh fading with a uniformly

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distributed additive phase.[8,9] $\underline{N}(t)$ is assumed independent of $\underline{y}(t)$ with independent, Gaussian distributed orthogonal components. Thus $\underline{r}(t)$ has means (specular components)

$$\underline{m}_1(t) = E[\underline{r}(t)|H_1] = R\{f(t)e^{j\omega_c t} E[\underline{y}(t)|H_1]\} + E[\underline{N}(t)|H_1] \tag{3}$$

$$\underline{m}_0(t) = E[\underline{r}(t)|H_0] = E[\underline{N}(t)|H_0]$$

and covariance matrices

$$\begin{aligned} \underline{K}_1(t,u) &= \underline{K}_s(t,u) + \underline{K}_N(t,u) \ ; H_1 \\ \underline{K}_0(t,u) &= \underline{K}_N(t,u) \ ; H_0 \end{aligned} \tag{4}$$

where

$$\underline{K}_s(t,u) = 2f(t)\, f^*(t)\, \underline{K}_y(t,u). \tag{5}$$

Time Optimal Continuous Data Processing

The time optimal continuous data processing for optimally detecting the above signal involves calculation of a test statistic ℓ which is compared with a threshold and can be represented in a number of forms corresponding to different implementations of the decision equation. Two such forms are:[6]

(1) The "phasing/weighting/combining-estimator-correlator" form which is specified by

$$\ell = \frac{1}{2}\int\int\int_{T_i}^{T_f} \underline{r}^T(t)\, \underline{G}(t,v) \underline{Q}_N(v,u) \underline{r}(u)\, du\, dt\, dv$$

$$= \frac{1}{2}\int_{T_i}^{T_f} \hat{\underline{s}}^T(v)\, \underline{Z}(v)\, dv \tag{6}$$

where $\underline{G}$ and $\underline{Q}_N$ are matrix functions defined by

$$\int_{T_i}^{T_f} \underline{K}_1(t,u)\, \underline{G}(u,v)\, du = \underline{K}_s(t,v) \tag{7}$$

and

$$\int_{T_i}^{T_f} \underline{K}_N(t,u)\, \underline{Q}_N(u,v) = \delta(t-v)\underline{I}. \tag{8}$$

Since (7) is a Wiener-Hopf integral equation,[6] then

$$\hat{\underline{s}}(v) = \int_{T_i}^{T_f} \underline{G}^T(t,v)\, \underline{r}(t)\, dt \tag{9}$$

is the usual optimal (minimum mean square error) estimate of $\underline{s}(v)$ under hypothesis H_1. The test statistic is this optimal estimate correlated with a filtered version

$$\underline{Z}(v) = \int_{T_i}^{T_f} \underline{Q}_N(v,u)\, \underline{r}(u)\, du \tag{10}$$

of the received signal.

(2) The energy detector form specified by

$$\ell = \frac{1}{2}\int_{T_i}^{T_f} \underline{x}^T(t)\, \underline{x}(t)\, dt = \frac{1}{2}\int_{T_i}^{T_f} ||\underline{x}(t)||^2\, dt \tag{11}$$

$$= \frac{1}{2}\int_{T_i}^{T_f} \sum_{i=1}^{n} |x_i(t)|^2\, dt,$$

where

$$\underline{x}(t) \triangleq \int_{T_i}^{T_f} \underline{k}^T(t,v)\, \underline{r}(v)\, dv \tag{12}$$

is a filtered version of $\underline{r}(t)$, with the matrix function filter $\underline{k}$ being defined by

$$\int_{T_i}^{T_f} \underline{k}(v,t)\, \underline{k}^T(v,u)\, dv = \int_{T_i}^{T_f} \underline{G}(u,v)\, \underline{Q}_N(v,t)\, dv. \tag{13}$$

The above structures can also be expressed in simplified linear matrix operator notation used by Middleton and Groginsky[5] as

$$\ell = \frac{1}{2}\underline{r}^T\, \underline{G}\, \underline{Q}_N\, \underline{r} = \frac{1}{2}\, \hat{\underline{s}}^T\, \underline{z}$$

and (14)

$$\ell = \frac{1}{2}\|\underline{x}\|^2 = \frac{1}{2}\, \|\underline{k}^T\, \underline{r}\|^2$$

The block diagram illustrations of these two equivalent forms are indicated in Figure 1. For complex signals, the "T" superscript indicates complex conjugate as well as transpose so that from the filter-squarer form, it is evident that ℓ is always positive. The above structures are preceded by a time delay (or phasing) network $\underline{\tau}$ (or φ) in Figure 1 to allow for the signal phasing (beam steering) requirement. Figure 2 illustrates the details of the structures in Figure 1 for $n = 2$ antenna coupling elements.

Characteristic Function Derivation

In order to evaluate system performance and specify how spatial sampling (array/distributed aperture coupling) might enhance performance, it is necessary to find the probability density functions of ℓ under all hypotheses in the problem structure (two in this case). This can be done by first calculating the characteristic functions of ℓ. The above decision equations (receiver structures) were derived by expanding $\underline{r}(t)$ in a truncated vector Karhunen-Loeve series expansion, writing the log likelihood ratio in terms of this expansion, and then allowing the number of terms to become unbounded.[4,6] The mean of $\underline{r}(t)$ is assumed to be the same under both hypotheses in order to further general non-threshold analytical calculation. This results in the test statistic

$$\ell = \lim_{K\to\infty} \ell_K = \lim_{K\to\infty} [(\underline{r} - \underline{m})^T \underline{L}(\underline{r} - \underline{m})] \tag{15}$$

where $\underline{L}$ is a diagonal matrix with diagonal elements

$$L_i = \frac{1}{2}\left[\frac{1}{\lambda_{io}} - \frac{1}{\lambda_{i1}}\right] \tag{16}$$

and where $\{\lambda_{ij}\}$ are the eigenvalues corresponding to the i^{th} eigenfunctions in the Karhunen-Loeve expansion under hypothsis H_j, $j = 0,1$. The characteristic function is then calculated as

$$M_{\ell j}(\omega) = \lim_{K\to\infty} M_{\ell kj}(\omega) = \lim_{K\to\infty} E[e^{j\omega\ell_K}|H_j]. \tag{17}$$

Since $\underline{r}$ is complex, write

$$\underline{r} - \underline{m} = (\underline{x} - \underline{m}_x) + j(\underline{y} - \underline{m}_y) \tag{18}$$

so that

$$M_{\ell Kj}(\omega) = \prod_{k=1}^{K} \{\iint_{-\infty}^{\infty} \frac{1}{2\pi\lambda_{kj}} \exp\Big[j\omega[(x_k - m_{xk})^2 + (y_k - m_{yk})^2] L_k - \frac{1}{2}[(x_k - m_{xk})^2 + (y_k - m_{yk})^2]/\lambda_{kj}\Big] dx_k\, dy_k\}$$

$$= \prod_{k=1}^{K} (1 - 2j\omega\lambda_{kj}L_k)^{-1} \tag{19}$$

Now let

$$D(z) = \lim_{K\to\infty} \prod_{k=1}^{K} (1 + c_{kj}z) \tag{20}$$

where

$$c_{kj} = 2\lambda_{kj}L_k = \lambda_{kj}\left(\frac{1}{\lambda_{ko}} - \frac{1}{\lambda_{k1}}\right). \tag{21}$$

Equation (20) is similar to the Fredholm determinant[7] and can be simplified in a similar manner. Then

$$\ln D(z) = \lim_{K\to\infty} \sum_{k=1}^{K} \ln(1 + c_k z) = \lim_{K\to\infty} \sum_{k=1}^{K} \int_0^z \frac{c_k du}{1 + c_k u} \tag{22}$$

Letting

$$\underline{H}(r,t;u) \triangleq \lim_{K\to\infty} \sum_{k=1}^{K} \frac{c_k}{1 + c_k u} \underline{f}_k(r)\underline{f}_k^T(t) \tag{23}$$

and

$$h(r,t;u) \triangleq Tr[\underline{H}(r,t;u)] \tag{24}$$

where the set of vector functions $\{\underline{f}_k(t)\}$ are complete and orthonormal. Note that h is similar to the resolvent kernel discussed in Courant-Hilbert, Chapter III.[10] Then

$$M_{\ell j}(\omega) = \lim_{K\to\infty} M_{\ell K_j}(\omega) = \exp\{-\int_0^{-j\omega} \int_{T_i}^{T_f} h(t,t;u)dt\,du\} \tag{25}$$

The next step is to interpret $\underline{H}(r,t;u)$ and hence $h(r,t;u)$ in terms of the space-time structure of the signal and noise fields. First it is assumed that the c_k's represent a set of eigenvalues (not necessarily all positive). Then if

$$\underline{\Phi}(r,t) \triangleq \lim_{K\to\infty} \sum_{k=1}^{K} c_k \underline{f}_k(r)\underline{f}_k^T(t) \tag{26}$$

converges for some complete, orthonormal set of vector functions $\{\underline{f}_k(t)\}$, $\underline{\Phi}(r,t)$ will be a kernel having the eigenvalues c_k and the eigenfunctions $\{\underline{f}_k(t)\}$, viz.

$$c_k \underline{f}_k(t) = \int_{T_i}^{T_f} \underline{\Phi}(t,r)\underline{f}_k(r)\,dr. \tag{27}$$

Then $\underline{H}(r,t;u)$ is a solution of the coupled system of integral equations

$$\underline{H}(r,t;u) + u\int_{T_i}^{T_f} \underline{\Phi}(r,s)\,\underline{H}(s,t;u)\,ds = \underline{\Phi}(r,t), \tag{28}$$

which can be verified by substituting the series expansions for $\underline{\Phi}$ and $\underline{H}$ in (28).

The problem is now one of relating $\underline{\Phi}$ to the signal and noise statistics by obtaining the $\{\underline{f}_k(t)\}$. The $\underline{K}_r$ covariance functions have eigenvalue and eigenfunction sets defined by

$$\lambda_{kj}\underline{\varphi}_{kj}(t) = \int_{T_i}^{T_f} \underline{K}_{rj}(t,v)\,\underline{\varphi}_{kj}(v)\,dv;\ j = 0,1. \tag{29}$$

(Note that the λ's and $\underline{\varphi}$'s are different under the two hypotheses H_0 and H_1.) It can be shown that

$$\frac{1}{\lambda_{kj}} \varphi_{kj}(t) = \int_{T_i}^{T_f} Q_{Tj}(t,v) \varphi_{kj}(v)\, dv \qquad (30)$$

by integrating the product of (29) and (30) over time. Then _if_ the eigenfunctions were the same under both hypotheses ($\varphi_{k1} = \varphi_{ko}$ for all k),

$$\left(\frac{1}{\lambda_{ko}} - \frac{1}{\lambda_{k1}}\right) \lambda_{kj} \varphi_k(t)$$

$$= \int_{T_i}^{T_f} [\underline{Q}_o(t,r) - \underline{Q}_1(t,r)] \int_{T_i}^{T_f} \underline{K}_j(r,v)\, \varphi_k(v)\, dv\, dr$$

$$= \int_{T_i}^{T_f} \{ \int_{T_i}^{T_f} [\underline{Q}_o(t,r) - \underline{Q}_1(t,r)] \underline{K}_j(r,v)\, dr\} \varphi_k(v)\, dv \qquad (31)$$

so that

$$\underline{\Phi}_j(t,v) = \int_{T_i}^{T_f} [\underline{Q}_o(t,r) - \underline{Q}_1(t,r)] \underline{K}_j(r,v)\, dr. \qquad (32)$$

The next step is the solution of the integral equation (28) for $\underline{H}$. This is facilitated by introducing the assumptions that the time (T_i, T_f) is long and that the processes are satisfactory so that

$$\underline{\Phi}_j(\omega) = \mathcal{F}\{\underline{\Phi}_j(\tau)\} \cong \int_{T_i}^{T_f} \underline{\Phi}_j(\tau) e^{-j\omega\tau} d\tau, \quad \tau = t - v. \ (33)$$

It can be shown that[6]

$$\mathcal{F}\{\underline{Q}_0(t) - \underline{Q}_1(t)\} = \underline{S}_{Q0}(\omega) - \underline{S}_{Q1}(\omega)$$

$$= \underline{S}_0^{-1}(\omega)\, \underline{S}_s(\omega) \underline{S}_1^{-1}(\omega) = \underline{S}_1^{-1}(\omega) \underline{S}_s(\omega) \underline{S}_0^{-1}(\omega) \ (34)$$

so that

$$\underline{\Phi}_1(r-t) = \int_{T_i}^{T_f} \underline{Q}_0(r-v)\, \underline{K}_s(v-t)\, dv$$

$$\underline{\Phi}_0(r-t) = \int_{T_i}^{T_f} \underline{Q}_1(r-v)\, \underline{K}_s(v-t)\, dv \qquad (35)$$

Substituting (35) into (28), taking the Fourier transform, and solving for $\underline{H}(\omega;u)$[11] then yields

$$\underline{H}_j(\omega;u) = [\underline{I} + u\, \underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)]^{-1} \underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)$$

$$= \frac{1}{u} \{\underline{I} - [\underline{I} + u\, \underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)]^{-1}\} \qquad (36)$$

where j, g = 0,1 and $j \neq g$, and

$$\underline{H}_j(\omega;u) = \mathcal{F}\{\underline{H}_j(\tau;u)\} = \mathcal{F}\{ \int \underline{H}_j(t-\tau,\ t;u)\, dt\} \qquad (37)$$

Note that this is the complete solution for $\underline{H}_j$ because of the above assumption of a long observation period. Then

$$\int_{T_i}^{T_f} h_j(t,t;u)\, dt \triangleq \mathrm{Tr}\, \underline{H}_j(\tau=0;u) = \mathrm{Tr} \int_{-\infty}^{\infty} \underline{H}_j(\omega;u) \frac{d\omega}{2\pi} \qquad (38)$$

so that

$$M_{\ell j}(\omega) = \exp\{- \int_0^{-j\omega} \int_{-\infty}^{\infty} \mathrm{Tr}\ \underline{H}_j(x;u) \frac{dx}{2\pi}\ du\}$$

$$= \exp\{- \int_{\infty}^{-j\omega} \int_{-\infty}^{\infty} \mathrm{Tr} \frac{1}{u} \left[\underline{I} - [\underline{I} + u\, \underline{S}_g^{-1}(x)\, \underline{S}_s(x)]^{-1}\right] \frac{dx}{2\pi}\, du\} \qquad (39)$$

Optimal Receiver Performance

The optimal receiver performance is found by (a) evaluating equation (39) to obtain the characteristic function $M_\ell(\omega)$ of the test statistic ℓ, (b) inverse transforming $M_\ell(\omega)$ to get the p.d.f. $p(\ell)$ of ℓ, and (c) integrating $p(\ell)$ over the appropriate limits to get the error probabilities. Unfortunately this cannot be done in closed form for the general case. An exception to this is the classical and useful threshold case which will now be developed for array coupling.

Threshold Case

The threshold condition is broadly defined as corresponding to a low SNR and the analytical condition for this assumption is

$$\mathrm{Tr} \int_{-\infty}^{\infty} [\underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)\, \underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)] \frac{d\omega}{2\pi}$$

$$<< \mathrm{Tr} \int_{-\infty}^{\infty} [\underline{S}_g^{-1}(\omega)\, \underline{S}_s(\omega)] \frac{d\omega}{2\pi} \qquad (40)$$

for g = 0,1. If (40) is satisfied, then

$$[\underline{I}+u\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)]^{-1}$$

$$\cong \underline{I} - u\underline{S}_g^{-1}(\omega)\underline{S}_s(\omega) + u^2\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)\,\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega) \quad (41)$$

and

$$\int_{T_i}^{T_f} h_j(t,t;u)\,dt = m_j - u\sigma_j^2$$

$$= \mathrm{Tr}\int_{-\infty}^{\infty}[\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega) - u\,\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)\underline{S}_g^{-1}(\omega)\underline{S}_s(\omega)]\frac{d\omega}{2\pi} \quad (42)$$

where m_j and σ_j^2 are constants. Then

$$M_{\ell j}(\omega) = \exp[-\int_0^{-j\omega}\int_{T_i}^{T_f} h_j(t,t;u)\,dt\,du]$$

$$= \exp[j\omega m_j - \frac{1}{2}\omega^2\sigma_j^2] \quad (43)$$

which is seen to be the form of the characteristic function for a Gaussian random variable with mean m_j and variance σ_j^2. It can be shown in a straightforward manner that the error probabilities with a Gaussian distributed test statistic ℓ are

$$P_F = \mathrm{erfc}(\frac{\ell_o - m_o}{\sigma_o})$$

$$P_D = \mathrm{erfc}(\frac{\ell_o - m_1}{\sigma_1}) \quad (44)$$

where ℓ_o is the optimal threshold and

$$\mathrm{erfc}(X) = \int_X^{\infty}\frac{e^{-y^2/2}}{\sqrt{2\pi}}\,dy \quad (45)$$

Middleton[2] has demonstrated that the test statistic becomes Gaussian in the scalar (one antenna) case under threshold conditions using sampling point analysis. The above results hold for continuous waveform processing and multiple antennas.

<u>Optimal Threshold</u>

The system performance is then completely specified if a closed form expression for the optimum threshold ℓ_o can be found. The analytical mechanics involved closely parallel those used to develop the characteristic function and only a general outline of the steps will be given here. In the Karhunen-Loeve derivation of the optimum test statistic,[6] the series expansion for the optimum threshold was found to be

$$\ell_s = \ln\eta - 2\sum_{k=1}^{\infty}\ln(\frac{\lambda_{k0}}{\lambda_{k1}}) \quad (46)$$

where η is a bias constant determined by the optimality criteria employed (Bayes, Neyman-Pearson, etc.) and the λ's are as previously defined. In order to obtain a closed form expression for this series it is convenient to assume that the noise also contains a white component of spectral intensity $N_o/2$. This assumption is physically realistic because of receiver noise and as will be seen later, is also related to the determination of optimal antenna spacing. Then

$$\lambda_{ki} = N_o/2 + \lambda_{cik} \;;\; i = 0,1 \quad (47)$$

and

$$\ell_s = \ln\eta + \sum_{k=1}^{\infty}\ln(1+\frac{2}{N_o}\lambda_{c1k}) - \sum_{k=1}^{\infty}\ln(1+\frac{2}{N_o}\lambda_{c0k}) \quad (48)$$

Each summation is handled as before by defining a $D_1(z)$, $\underline{H}_1(r,t;u)$, and $h_1(r,t;u)$, and showing that $\underline{H}_1$ is the solution of

$$\underline{H}_i(r,t;u)+u\int_{T_i}^{T_f}\underline{K}_i(r,s)\,\underline{H}(s,t;u)\,ds = \underline{K}_i(r,t) \quad (49)$$

where $\underline{K}_i$ is the covariance matrix of the non-white portion of the received voltage under hypothesis i, i = 0,1. Assuming stationarity and a long observation time then yields

$$\ell_o = \ln\eta + \mathrm{Tr}\int_0^{2/N_o}\int_{-\infty}^{\infty}\{[\underline{I}+u\underline{S}_1(\omega)]^{-1}\underline{S}_1(\omega)$$

$$- [\underline{I}+u\underline{S}_o(\omega)]^{-1}\underline{S}_o(\omega)\}\frac{d\omega}{2\pi}\,du \quad (50)$$

where

$$\underline{S}_1(\omega) = \underline{S}_{N_c}(\omega) + \underline{S}_s(\omega)$$

$$\underline{S}_o(\omega) = \underline{S}_{N_c}(\omega) \quad (51)$$

are the spectra of the non-white portions of the coupled voltages under the two hypotheses.

Optimal Antenna Dimensions

In order to maximize system performance it is necessary to simultaneously make Pf as small and PD as large as possible. This is done by maximizing the difference between the arguments in (44), or equivalently, by maximizing the square of the difference which is

$$\gamma \triangleq (E[\ell/H_0] - E[\ell/H_1])^2/\sigma^2_{\ell/H_0}$$

$$= (m_1 - m_0)^2/\sigma_0^2 \tag{52}$$

since in the threshold case $\sigma_1 \cong \sigma_o$. For the threshold case,

$$\underline{S}_1^{-1}(\omega) = [\underline{S}_s(\omega) + \underline{S}_N(\omega)]^{-1}$$

$$\cong [\underline{I} - \underline{S}_N^{-1}(\omega)\,\underline{S}_s(\omega)]\,\underline{S}_N^{-1}(\omega) \tag{53}$$

and from the above characteristic function analysis,

$$m_j = \mathrm{Tr}\int_{-\infty}^{\infty}[\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)]\,\frac{d\omega}{2\pi}$$

$$= \mathrm{Tr}\int_{T_i}^{T_f}\int \underline{Q}_g(t,v)\,\underline{K}_s(v,t)\,dv\,dt \tag{54}$$

$$\sigma_j^2 = \mathrm{Tr}\int_{-\infty}^{\infty}[\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)\,\underline{S}_g^{-1}(\omega)\,\underline{S}_s(\omega)]\,\frac{d\omega}{2\pi}$$

$$= \mathrm{Tr}\int\int\int\int_{T_i}^{T_f}\underline{Q}_g(t,v)\,\underline{K}_s(v,x)\,\underline{Q}_g(x,y)\underline{K}_s(y,t)\;\cdot\; dv\,dx\,dy\,dt.$$

so that

$$\gamma = \sigma_j^2 = m_1 - m_0$$

$$= \mathrm{Tr}\int\int\int\int_{T_i}^{T_f}\underline{Q}_N(t,v)\,\underline{K}_s(v,x)\,\underline{Q}_N(x,y)\underline{K}_s(y,t)\;\cdot\; dv\,dx\,dy\,dt \tag{55}$$

$$= \mathrm{Tr}\int_{-}^{\infty}[\underline{S}_N^{-1}(\omega)\,\underline{S}_s(\omega)\,\underline{S}_N^{-1}(\omega)\,\underline{S}_s(\omega)]\,\frac{d\omega}{2\pi}$$

Singular Detection

At this point the phenomena of spatial singular detection can arise In the maximization of Y. This phenomenon arises If receiver noise Is negllcted, i.e., if all the noise is assumed to be received through the antennas. Singular detection occurs when the $s_N(W)$ matrix becomes singular[2] and two such cases need to be distinguished.

In the first case *y* is maximized for zero antenna spacings, i.e., when the antennas are (ideally) located at the same point.[12] Later in this report it is shown that the S_N matrix becomes singular for zero spacings so that *y* appears to become unbounded. The physical significance of this phenomenon is that for zero spacings the noises in the antennas are identical and the optimum receiver structure phases and sums the antenna voltages In such a way as to completely eliminate the noise. Unfortunately the desired signals In the antennas are also Identical and are also completely eliminated by the optimum receiver.

The second case involves plane wave noise yielding noises In the antennas as delayed versions of each other. Thus It Is theoretically possible to completely eliminate the external noise without eliminating the signal provided the signal Is not received from the same direction as the noise. In this case the performance Is theoretically the same for all non-zero antenna spacings. The plane wave noise Is therefore not considered further In regard to optimal antenna spacing.

The zero spacings solution is invalid as is also evident from the fact that for singular detection the threshold assumption no longer holds since the inequality In equation (40) is reversed. If the threshold assumption does not hold, then equation (55) for Y IS Invalid and it is no longer evident that the optimum procedure is to maximize Y.

In order to obtain physically meaningful results It is necessary to avoid the spatially singular detection situation. This is done by including an additive receiver noise whose statistical properties are independent of the antenna spacings. The receiver noise need not be white but may reasonably be assumed uncorreleted. This is the approach taken by Gaarder1[3,14] in maximizing the antenna gain function and challenges the conclusion of Martel and Mathews who Indicate that near zero spacings are optimal.

Problem Simplification

The Insertion of receiver noise Increases the complexity of the Integrand In (55). Even In the most simple cases the exact evaluation of Y becomes so cumbersome as to render a direct approach practically useless. It is therefore desLrable to simplify the analytical task by finding some simpler expression which Is maximal for the same antenna deployment that maximizes *y.* The cases under which this is possible will now be developed.

A considerable simplification results If maximizing the Integrand Is the same as maximizing the Integral. The development which maximizes the Integrand will be a function of *w* unless the Integrand can be factored Into the product of a function of only the deployment and a function of frequency alone. With physically realistic antenna cross correlation spectra such a factorization Is not possible. Thus the two maximizations coincide only If the optimal deployment for the Integral equals that for the Integrand evaluated at a particular frequency; ω_0, perhaps the carrier frequency. This In turn occurs only when the Integrand contains, as a factor, $\delta(\omega-\omega_0)$. If the signal covarlance matrix contains a common spectral term $[\underline{S}_s(\omega)=S_s(\omega)\,\underline{P}_s(\omega)]$ then

$$\gamma = \int_{-\infty}^{\infty} S_s^2(\omega)\, \mathrm{Tr}[\underline{S}_N^{-1}(\omega)\, \underline{P}_s(\omega)\, \underline{S}_N^{-1}(\omega)\, \underline{P}_s(\omega)]\, \frac{d\omega}{2\pi} \qquad (56)$$

and the two maximizations colncide only if $S_s^2(\omega)$ approximates a delta functlon. This will be the case lf the 3 db width of $S_s(\omega)$ is much less than that of $\mathrm{Tr}[\underline{S}_N^{-1}(\omega)\, \underline{P}_s(\omega)\, \underline{S}_N^{-1}(\omega)\, \underline{P}_s(\omega)]$.

In some cases lt is also possible to use a simpler expresslon than the lntegrand ln the maximization. If λ_i are the eigenvalues of $\underline{A} = \underline{S}_N^{-1}\underline{P}_s$, then

$$\mathrm{Tr}[\underline{S}_N^{-1}\underline{P}_s\underline{S}_N^{-1}\underline{P}_s] = \sum_{i=1}^{n} \lambda_i^2 \qquad (57)$$

It would be much simpler to maximize $\sum_{i=1} \lambda_i$ Instead, but the two maximizations do not In general coincide. There are two obvious cases In which the maximizations do coincide, these being (1) when all the X's are equal and (2) when all the X's except one are zero.

If all the X's are equal, then (assuming A Is normal) A Is similar to a scalar multiple of the Identity matrix and hence Is Itself a scalar multiple of the Identity matrix. This in turn would require SN to be a scalar function of Ps , which Is not possible here because of the assumed additive receiver noise. Thus the case In which the X's are all equal Is physically unrealistic.

If all but one of the X's are zero, then A must have rank 1 and hence Ps must have rank 1. This then is the plane wave signal case in which $\underline{P}_s = \underline{1}$ (a matrix, all of whose elements equal one) and γ reduces to

$$\gamma = \int_{-\infty}^{\infty} S_s^2(\omega)\, G_0^2(\omega)\, \frac{d\omega}{2\pi} \qquad (58)$$

Here
$$G_0(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} S_N^{ij}(\omega) \qquad (59)$$

Is sometimes called the array gain function since It represents the SNR gain effected by the antenna array. The maximization of the antenna gain function by Gaarder13,14 Is valid only for narrow band plane wave signals (requiring that $S^2(W)$ approximate a delta function).

The Integrand can also be simplified by further selection of the $\underline{S}_N$ and $\underline{P}_s$ matrices. That this Is the case might be suspected because these matrices have the often convenient properties of being Hermltlan and positive definite. The product SNTs is Hermltlan only If S!" and P_s commute. S" and P„ will commute If and only If they have the same eigenvectors, or equlvalently If SN Is a scalar function of $\underline{P}_s$, or vice versa. (Note that these statements are true only because $\underline{S}_N$ and Ps are Hermltlan.) For the physically realizable cross covarlance spectra considered In this report, the restriction that S_N be a scalar function of Ps requires that the additive receiver noise be uncorrelated and that the external noise and signal sources be of the same type.

Case Study

At this point It Is desirable to Investigate (1) specific functional forms for the signal and noise statistics and (2) the plane wave signal case without the benefit of the threshold assumption. These will be considered as the case studies.

Physically Relevant Space-Time Correlation and Power Spectra Models

If physically meaningful conclusions are to be obtained, physically realistic space-time covarlance functions must be employed. The system designer must first model the physical noise field environment and then use the model to obtains; (w) and hence $\underline{S}_N(w)$. The procedures Involved, and some representative results will now be considered.

A treatment of this aspect has been given by Chllders[15] In terms of a vector antenna height function h(a>;0,cp) of the antennas used, which Is useful when the coupling elements are dlpoles. The voltage coupled In by an antenna Is the dot product of its vector height function and the electric field vector of the incident radiation field. The cross power spectral density function between the signals coupled in by two antennas (Figure 3) is given by the quadratic form

$$S_{N12}(\omega) = \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-j\frac{\omega z}{c}\cos\varphi}\, \underline{h}_1^T(\omega;\theta,\varphi) \cdot$$

$$\underline{S}_{N\Delta}(\omega;r,\theta,\varphi)\, \underline{h}_2^*(\omega;\theta,\varphi)\, \sin\varphi\, d\varphi\, d\theta \qquad (60)$$

where

$$\underline{S}_{N\Delta} = \begin{bmatrix} S_{N\theta\theta} & S_{N\theta\varphi} \\ & \\ S_{N\varphi\theta} & S_{N\varphi\varphi} \end{bmatrix} \qquad (61)$$

Is a power spectrum matrix of the 9 and cp components of the electric vector of the Incident field and where z is the separation between the antenna phase centers. Note that the auto power spectrum can be obtained from (60) by setting z=0 and hi. =Jh2. Note also that if h^Jlo/ then sN has rank 1 and hence is singular for z = 0, illustrating the singular detection phenomenon.

It Is readily apparent that various possible functional forms for Sn12 exist. Unfortunately except for a few special cases the resulting analytical forms of SN.12((D) In terms of field, environment, and coupling element variables, are usually very complex and lead to unwieldy mathematical complications. However, a number of special cases have been developed In the literature[16,17,18,19,20,21] which result In reasonably simple analytical forms for Sn12w). Four such physical cases are now considered. The assumption that the noise is homogeneous (SNA is Independent of r) Is common to all these cases and will not be separately stated for each case.

The first case Is one considered by Chllders and Reed[16] In which dlpole antennas and Isotropic (SN Is independent of 0 and cp), unpolarized ssNe0 =sNcpcp=sN and sN0 = SN0 = °) noise are assumed. The resulting expression for SKnoN12(w)) is then

SN12(w) = SNA[sIn0i sin02 cos(" 02>

$$[-\frac{\sin\omega z/c}{(\omega z/c)^3} + \frac{\cos\omega z/c}{(\omega z/c)^2} + \frac{\sin\omega z/c}{(\omega z/c)}]$$

$$+ 2\cos\theta_1\cos\theta_2\,[\frac{\sin\omega z/c}{(\omega z/c)^3} - \frac{\cos\omega z/c}{(\omega z/c)^2}]\} \qquad (62)$$

where the dipole orientations are defined by (θ_1,φ_1) and (θ_2,φ_2). In the case of white noise $(S_{N\Delta}(\omega) = N_o/4\pi)$ this inverse transforms into a reasonably simple cross correlation function between antennas.

$$R_{12}(\tau) = \frac{N_o}{z/c}\{\tfrac{1}{2}[1 + (\frac{\tau}{z/c})^2]\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)$$

$$+[1-(\frac{\tau}{z/c})^2]\cos\theta_1\cos\theta_2\};\ |\tau| < z/c. \qquad (63)$$

The remaining three cases involve isotropic antennas so that the quadratic form in (60) may be a scalar $S_{N\Delta}(\omega;\theta,\varphi)$:

$$S_{N12}(\omega) = \int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-j\frac{\omega z}{c}\cos\varphi} S_{N\Delta}(\omega;\theta,\varphi)\sin\varphi\, d\varphi\, d\theta. \qquad (64)$$

The second case assumes that the noise is isotropic ($S_{N\Delta}$ is independent of θ and φ) and results in the particularly simple expression

$$S_{N12}(\omega) = S_{N\Delta}(\omega)\,\mathrm{sinc}(\omega z/c). \qquad (65)$$

This Is the case most frequently found In the literature.[17,18,19,20] The isotropic assumption Is equivalent to the assumption that the random incident field is the result of a large number of independent random sources uniformly disturbed In a sphere centered around the antennas (See Figure 4). Such a situation Is encountered whenever the antennas are enclosed In a large volume of random scattering elements (any elements whose size approaches operating wavelengths) and where there are no nearby (In terms of wavelengths) boundary surfaces.[19,20]

The third case Is also frequently found In the literature[19,20,21] and Involves non-isotropic noise such as obtained when the noise Is generated by a large number of Independent noise sources uniformly distributed on a large circular area of any plane passed through the array volume (See Figure 4). Such a situation Is encountered when the random field is due to scattering from a "distant" plane surface.[19,20] The cross antenna power spectrum then has the form

$$S_{N12}(\omega) = S_{N\Delta}(\omega)\, J_o(\omega z/c\cos\gamma) \qquad (66)$$

where γ is the angle between a line connecting the antennas and the plane of the random sources.

The final case Involves a directional random field of the form

$$S_{N\Delta}(\omega;\theta,\varphi) = S_{N\Delta}(\omega)\, f(\theta)\, e^{-\beta|\cos(\varphi-\varphi_o)|} \quad (67)$$

so that the double integral In (64) factors Into the product of two Integrals. The function of f(0) can be arbitrary except that It should be normalized to make Its Integral equal to some convenient constant. The noise field will then have the shape f(0) In the 0 dimension and e-B |cos(cp-cpo) | In the cp dimension. This latter form Is that of a lobe centered at = -2 + cp_Q. This structure can be employed as a model for the situation Involving a distant random source (jamming, etc.) and a multlpath environment so that the noise appears to be coming from an angular region of arbitrary dimensions (See Fig 4). Narrow Lobe widths of the noise field correspond to large p values and for 3sln(|>$_0$ greater than about three, the rational cross antenna power spectrum

$$S_{N12}(\omega) = \frac{2\beta \sin\varphi_o S_{N\Delta}(\omega)}{(\beta \sin\varphi_o)^2 + (\omega z/c)^2}\, e^{j\frac{\omega d}{c}\cos\varphi_o} \quad (68)$$

Is obtained. The 3 db lobewldth of the noise field Is determined by B and Is

3 db lobewldth = 2 sin^{-1}(-Lin 2) . (69)

This spectrum agrees well with available experimental data for point sources in multlpath environment. (For example see the data given by Kurlchara.[22])

The limiting case of (68) In which B becomes unbounded corresponds to the plane wave noise field In which the noise comes from one particular angle. In this case the noises in the different antennas are Identical (spatial coherence) except possibly for a delay. Also, unless the signal field is of an identical space-time correlation nature, the noise can be completely eliminated by properly phasing, weighting and combining the signals, leaving only the system noise introduced in the receiver front end. However If the signal field Is exactly of this space-time form, no advantage can be obtained from space diversity since the spatial filtering of both signal and noise Is Identical.

Plane Wave (Point to Point Spatially Coherent) Signal Case

The trace operation In finding h (W,U) can be performed exactly without the threshold assumption In the special case In which the signal In the various antennas Is the same (spatially coherent) except possibly for a delay, which can be phased as shown In Figure 1. In this case all the elements of the $\underline{S}_s(\omega)$ matrix are the same; thus

$$\underline{S}_s(\omega) = S_s(\omega)\,\underline{1} \quad (70)$$

where $\underline{1}$ is a matrix, all of whose elements are 1. It can be shown that

$$\mathrm{Tr}[\underline{I} + a\underline{A}^{-1}\underline{1}]^{-1} = n - a\Sigma\Sigma A^{ij}/(1 + a\,\Sigma\Sigma A^{ij}) \quad (71)$$

where n is the rank and A^{ij} represents the ij element of $\underline{A}^{-1}$. Thus, with (71)

$$h_j(\omega;u) = \frac{1}{u}\mathrm{Tr}\{\underline{I} - [\underline{I} + u\underline{S}_g^{-1}(\omega)\, S_s(\omega)\,\underline{1}]^{-1}\}$$

$$= \frac{S_s(\omega) \sum_{i=1}^{n}\sum_{j=1}^{n} S_g^{ij}(\omega)}{1 + u S_s(\omega)\sum_{i=1}^{n}\sum_{j=1}^{n} S_g^{ij}(\omega)} \quad (72)$$

where again j = g. (Note that the summations are a result of the multiple antennas and have nothing to do with the Karhunen-Loeve expansion approach used earlier In the report.) The evaluation of (72) Is greatly simplified if the additive noise Is assumed to be white and uncorrelated from antenna to antenna, so that

$$\underline{S}_1^{-1}(\omega) = \frac{2}{N_o}[\frac{2}{N_o} S_s(\omega)\,\underline{1} + \underline{I}]^{-1}. \quad (73)$$

This yields

$$\sum_{i=1}^{n}\sum_{j=1}^{n} S_1^{ij}(\omega) = \frac{n(2/N_o)}{1 + n S_s(\omega)(2/N_o)} \quad (74)$$

so that

$$h_0(\omega;u) = \frac{n(2/N_o)\, S_s(\omega)}{1 + (1+u)\, n(2/N_o)\, S_s(\omega)}$$

and (75)

$$h_1(\omega;u) = \frac{n(2/N_o)\, S_s(\omega)}{1 + u\, n(2/N_o) S_s(\omega)}$$

For the assumption that $S_s(\omega)$ is rational, so that

$$S_s(\omega) = N(\omega^2)/D(\omega^2) \quad (76)$$

where the order of D exceeds that of N, then

$$h_o(\omega;u) = \frac{n(2/N_o)\, N(\omega^2)}{D(\omega^2) + (1+u)\, n(2/N_o)\, N(\omega^2)}$$

$$h_l(\omega;u) = \frac{n(2/N_o)\,N(\omega^2)}{D(\omega^2) + u\,n(2/N_o)\,N(\omega^2)} \tag{77}$$

and

$$\int_{-\infty}^{\infty} h(\omega;u)\,\frac{d\omega}{2\pi} \tag{78}$$

can be evaluated by residue theory. As an example, for S(t) plane wave and Markovian,

$$S_s(\omega) = 2\,\alpha K_o/(\omega^2 + \alpha^2). \tag{79}$$

The associated characteristic functions are

$$M_{\ell_0}(\omega) = \exp[\alpha(\sqrt{1+C} - \sqrt{1 + C - j\omega C})]$$
$$M_{\ell_1}(\omega) = \exp[\alpha(1 - \sqrt{1 - j\omega C})] \tag{80}$$

where

$$C = \frac{2nK_o}{\alpha(N_o/2)} = \frac{2nR}{\alpha} \tag{81}$$

$R = 2K_o/N_o$ being a measure of the signai-to-nolse ratio (s N R)°In this case. Neither of these characteristic functions can be Inverse transformed In closed form. It Is observed that the effect of the multiple antennas, In this case, is to Increase the SNR by a factor of n. This is a direct consequence of the assumptions of uncorrelated noise from antenna to antenna and plane wave (spatially coherent) signal.

Other assumptions which allow an exact evaluation of the trace in (39) are possible but usually lead to characteristic functions which cannot be inverse transformed in closed form. Numerical inversion can be used in these cases but the results are necessarily valid only for the specific case postulated and cannot be conveniently extrapolated to other cases or used to form broad, quantatlve conclusions. The exact performance analysis approach was therefore discontinued by the authors in favor of the more productive threshold case assumption, which allows simpler interpretations of steps to take in enhancing detectablllty by specific antenna deployment.

Critique

The contribution of this paper consists of (1) an analytical development of the exact characteristic functions for the test statistic computed by a time optimum data processing system employing multiple antennas for a random signal and noise field in terms of the space-time structure of the signal and noise fields and the antenna array configuration, (2) a simple, concise analytical demonstration that, in the threshold case the test statistic distributions for the array case become asymptotically Gaussian, and (3) analytical treatment in terms of physically realistic space-time correlation models of signal and noise field structures. The assumptions include Gaussian noise, statlonarlty, and "long" observation times.

Unfortunately the characteristic function form derived herein cannot be inverse transformed in closed form in the general case. Numerical solutions may be of value in specific instances but cannot be expected to provide broad quantitative Information. Thus the only route leading to quantitative results appears to be the threshold case for which closed form inversion is possible.

In the threshold case the test statistic is Gaussian distributed and hence its p.d.f. is uniquely determined by its first two moments. Even in non-threshold situations the accuracy to which the signal and noise fields can be statistically characterized may preclude a more accurate representation of the test statistic p.d.f. Thus the threshold assumption will probably yield results which are as meaningful as can be obtained even in non-threshold situations unless an unusually accurate statistical formulation is possible.

It is of interest to note the general effects of the array upon reception capability for specific classes of random noise fields and fading signal fields in terms of their space-time correlation structure and array deployment. These aspects are currently being pursued through a detailed investigation of a number of special situations by the authors and will be presented in a later report. A thorough discussion of anticipated results is at this time premature but a number of speculations can be projected:

(1) Performance capacity can Increase with the number of antennas, n, but the extent of any Improvement, as well as incremental gain, is greatly dependent upon the space-time correlation of the signal and noise fields and the antenna geometry employed.

(2) Deploying antennas so as to increase the noise cross-correlation between antennas increases the performance capability, assuming that the deployment does not render a similar effect upon signal cross-correlation between antennas. For example, for fully correlated (spatially coherent) noise between antennas, the Interesting case of singular detection is obtained in which theoretically the noise can be completely

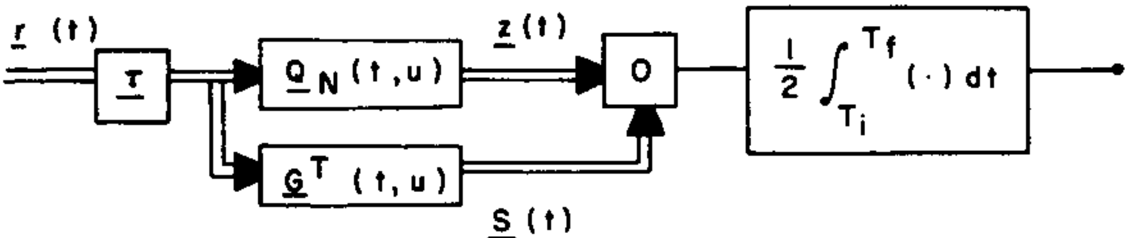
eliminated if that noise is the only source of interference and the antenna's front end can be ignored.

(3) The effect of signal cross-correlation between antennas (correlated fading) is such that increasing this correlation, by appropriate antenna deployment, will Increase the combined signal to noise level, but will also increase the "outage rate" due to spatially correlated fading. Thus deploying antennas to increase signal cross-correlation under these conditions increases performance capacity if SNR gain is the criteria and decreases performance capacity if signal loss or outage rate is the criteria.

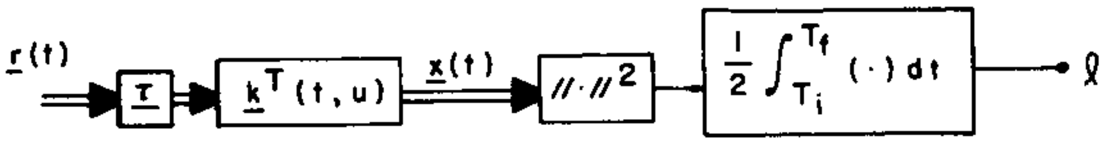
It should be emphasized that physically realistic space time correlation models are used in the preceding analysis. The set of cross-power spectra cited in .his paper should realistically model a number of situations of engineering Interest.

References

1. R. Price, "Optimum Detection of Random Signals In Noise, With Application to Scatter-Multipath Communication, I," IRE Trans, on Information Theory, Vol. IT-2, Dec. , 1956, pp. 125-135.
2. D. MIddleton, "On the Detection of Stochastic Signals in Additive Normal Noise - Parti, " IRE Trans, on Information Theory, Vol. IT-3, June, 1957, pp. 86-121.
3. D. MIddleton, Introduction to Statistical Communication Theory, McGraw-Hill, New York, I960, Sec. 20.4-7.
4. U. Grenander, "Stochastic Processes and Statistical Interference," Arkiv Mat. , Vol. 1, 1950, p. 295.
5. D. MIddleton and H. L. GrogInsky, "Detection of Random Acoustic Signals by Receivers with Distributed Elements," JASA, Vol. 38, No. 5, Nov., 1965, pp. 727-737.
6. H. L. Van Trees, Detection, Estimation, and Modulation Theory, Part I, John Wiley, New York, 1968.
7. C. W. Helstrom, Statistical Theory of Signal Detection, Pergamon Press, New York, 1960.
8. M. Schwartz, W. R. Bennett, and S. Stein, Communication Systems and Techniques, McGraw-Hill, New York, 1966.
9. S. Stein and J. J. Jones, Modern Communication Principles, McGraw-Hill, New York , 1967.
10. R. Courant and D. HIlbert, Methoden der MathematIschen PhvsIk. Springer, Berlin, 1931. English translation: Intersclence, New York, 1953.
11. P. M. Morse and H. Feshbach, Methods of Theoretical Physics, McGraw-Hill, New York, 1953.
12. H. C. Martel and M. V. Mathews, "Further Results on the DetectabIllty of Known Signals In Gaussian Noise," BSTJ, Vol. 40, March, 1961, pp. 423-451.
13. N. T. Gaarder, "The Design of Point Detector Arrays, I," IEEE Trans, on Information Theory, Vol. IT-13 , Jan., 1967, pp. 42-50.
14. N. T. Gaarder,"The Design of Point Detector Arrays, II, " IEEE Trans, on Information Theory, Vol. IT-12, April, 1966, pp. 112-120.
15. D. G. ChIlders, "Generalized SpatIotemporal Correlation Functions for Antennas," IEEE Trans, on Information Theory, Vol. IT-13, Jan. , 1967, pp. 121-122.
16. D. G. ChIlders and I. S. Reed, "Space-Time Cross-Correlation Functions for Antenna Array Elements In a Noise Field," IEEE Trans, on Information Theory, Vol. IT-11, April, 1965, pp. 182-190.
17. Finn Bryn, "Optimum Signal Processing of Three-DImensional Arrays Operating on Gaussian Signals and Noise," Jour. Acous. Soc. Am. , Vol. 34, March, 1962, pp. 289-297.
18. J. J. Freeman, "A Systematic Error In Underwater Acoustic Direction-Finding , " Jour. Acous. Soc. Am., Vol. 32, August, 1960, pp. 1025-1027.
19. B. F. Cron and C. H. Sherman, "Spatial-Correlation Functions for Various Noise Models," Jour. Acous. Am., Vol. 34, Nov., 1962, pp. 1732-1736.
20. M.J. Jacobson, "Space-Time Correlation In Spherical and Circular Noise Fields," Jour. Acous. Soc. Am., Vol. 34, July, 1962, pp. 971-978.
21. N. T. Gaarder, "The Design of Point Detector Arrays, I," IEEE Trans, on Information Theory, Vol. IT-13, Jan., 1967, pp. 42-50.
22. Y. KurIhara, "Trans-Horizon Microwave Propagation over Hilly Terrain," Proc. IRE, Vol. 43, Oct., 1955, pp. 1362-1368.

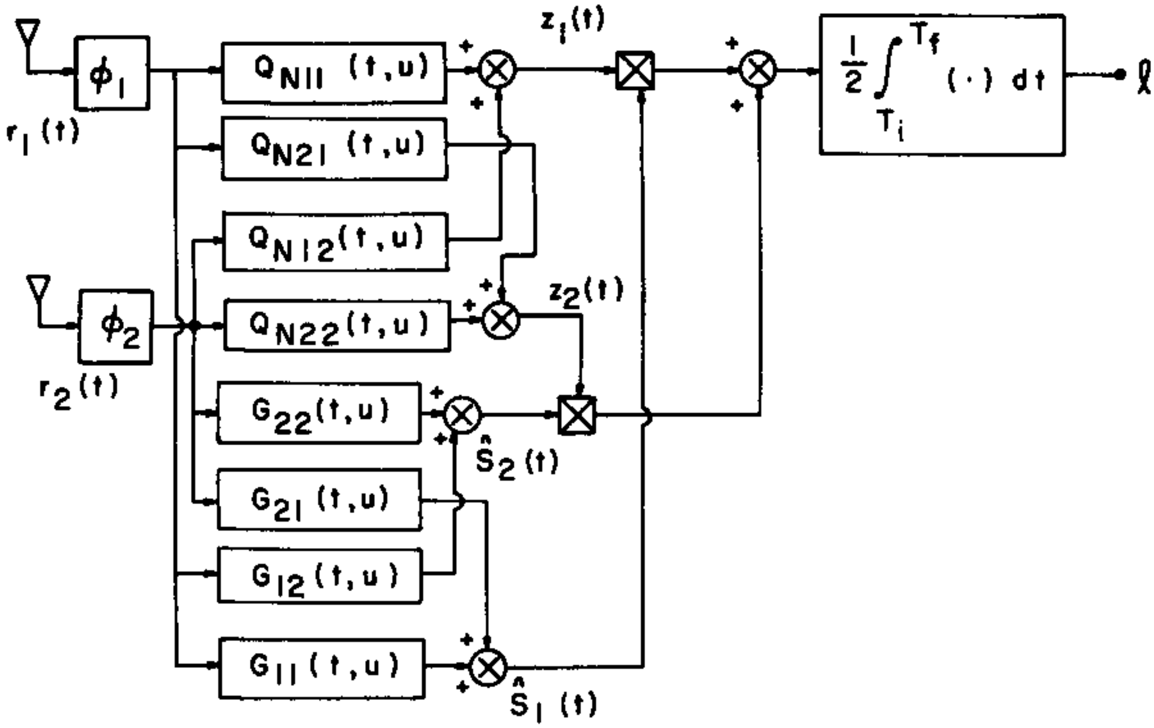


(a) Phaser/Weighter/Combiner-Estimator-Correlator Structure

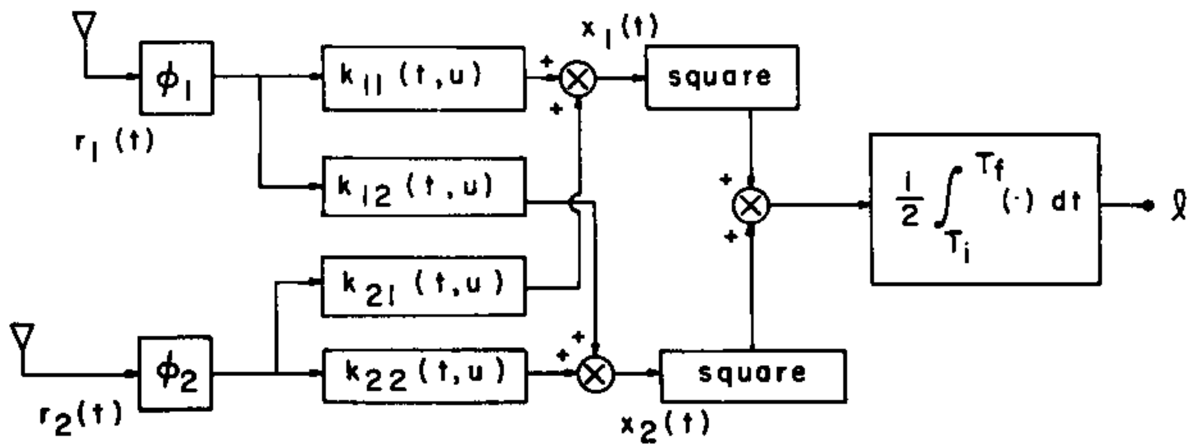


(b) Array Energy Detector Structure

FIGURE 1. OPTIMAL SIGNAL PROCESSOR STRUCTURES



(a) Phaser / Weighter / Combiner - Estimator - Correlator Struture



(b) Array Energy Detector Structure

FIGURE 2. OPTIMAL SIGNAL PROCESSOR STRUCTURES FOR n=2 ANTENNA ELEMENTS

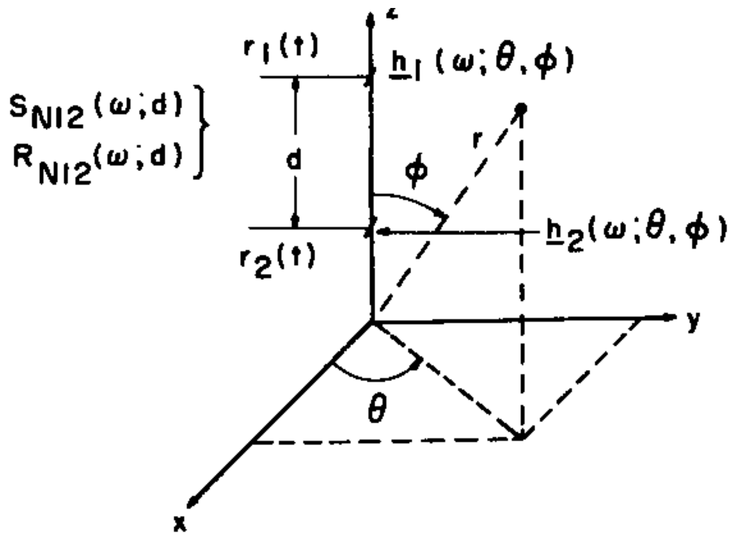


FIGURE 3. SPACE-TIME CORRELATION GEOMETRY.

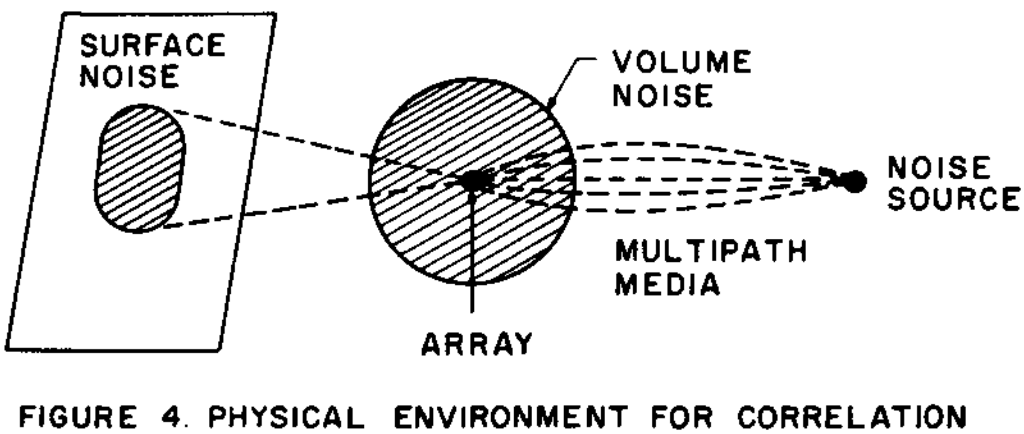


FIGURE 4. PHYSICAL ENVIRONMENT FOR CORRELATION MODELS.