COMPUTER SOLUTION OF CALCULUS WORD PROBLEMS*

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SUMMARY

A program was written to solve calculus word problems. The program, CARPS (CAlculus Rate Problem Solver), is restricted to rate problems. The overall plan of the program is similar to Bobrow's STUDENT, the primary difference being the introduction of "structures" as the internal model in CARPS. Structures are stored internally as trees, each structure holding the information gathered about one object.

It was found that the use of structures made CARPS more powerful than STUDENT in several respects.

1) CARPS is better able to recognize that two phrases describe the same object. (e.g., CONICAL PILE and PILE OF SAND)
2) Information about an object can be gathered in piecemeal fashion. In calculus word problems it is not uncommon to have two or three sentences providing information for one equation.
3) CARPS to a limited degree is able to use its knowledge to parse its input sentences. For example, in a problem about a filter, ALTITUDE was interpreted as ALTITUDE OF THE FILTER because CARPS knew that since the filter was a cone and cones have altitudes the filter had an altitude.

The program has solved 14 calculus problems, most taken (sometimes with slight modifications) from standard calculus texts.

I. INTRODUCTION

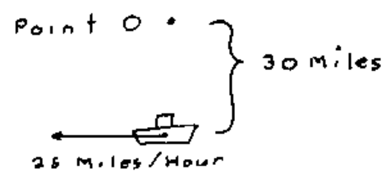
The research described in this paper had as its goal the creation of a program which solves freshman calculus word problems. The program, CARPS (CAlculus Rate Problem Solver), is restricted to rate problems. It is described in greater detail in MAC-TR-51 (Thesis)3 CARPS was primarily motivated by Bobrow's work on STUDENT[2] a program which solves high school algebra word problems. An understanding of STUDENT is sufficiently important to our work that we shall analyze Bobrow's program in the second section of the paper.

*This paper is based upon a thesis submitted in partial fulfillment for the degree of Master of Science at the Massachusetts Institute of Technology, Department of Electrical Engineering. The work was supported in part by Project MAC, an MIT research program sponsored by the Advanced Research Projects Agency, Department of Defense, under Office of Naval Research, contract Number Nonr-4102(01), and in part by the National Science Foundation.

CARPS is written in two languages. The bulk of the coding is in LISP. There are, however, large sections which require a great deal of pattern matching, something in which LISP is not particularly powerful. These sections were written in CONVERT[4]5 a language especially designed for pattern matching. Since CONVERT is embedded in LISP it was an especially convenient choice because we could easily switch back and forth between the two languages. Both of these languages were available on the Project MAC PDP-6 time sharing system which was used in this research. Also available were J. Moses' algebraic simplification and differentiation routines which are described in[7]. The PDP-6 system which has a quarter million words of core storage, gave us a decided advantage over Bobrow, whose program had to fit into a 32K 7094 LISP system, whereas ours wallows in the comparative luxury of 45K of memory.

Though CARPS is a fairly complex program, its basic organization is relatively straight forward. To demonstrate the underlying principles let us show how it would solve the following particularly simple problem. The parentheses and slashes are required by the PDP-6 LISP system.

```
(A SHIP IS 30.0 MILES SOUTH OF POINT 0
AND TRAVELING WEST AT 25.0 MILES PER
HOUR /. HOW FAST IS THE DISTANCE FROM
THE SHIP TO 0 INCREASING ?)
```



The diagram of this problem is for the benefit of the reader. CARPS does not use diagrams.

The program is divided into five sections. The primary goal of the first section is to tag words with their part of speech. At the same time the program accomplishes several other tasks such as checking for words indicating the type of problem it has been handed. Just before it turns the transformed problem over to the next section It will print out

```
(THE PR0BLH4 WITH TAGS ON IS)
(((A SHIP (IS VERB) 30.0 (MILE UNIT)
(SOUTH PN0UN)P0INT 0 AND (TRAVELING
VERB) (WEST PNOUN) (AT PREP) 25.0
(MILE UNIT) PER (HOUR UNIT)) (1.))
(((HOW QWORD) (FAST RWORD) (IS VERB) THE
DISTANCE (FROM PREP) THE SHIP TO 0
(INCREASING VERB)) (2.)))
```

(THE PROBLEM TYPE IS)
DISTANCE

At the moment the program is familiar with two types of problems, DISTANCE and VOLUME. The second section of the program takes the output of the first section and breaks the sentences into simple sentences. After it has done so it will print:

(THE SIMPLIFIED SENTENCES ARE)
(((A SHIP (IS VERB) 30.0 MILE UNIT)
(SOUTH PNOUN) POINT 0) (1.))
((A SHIP (TRAVELING VERB) (WEST PNOUN) (1.))
((A SHIP (TRAVELING VERB) (AT PREP) 25.0
(MILE UNIT) PER (HOUR UNIT)) (1.))
(((HOW QWORD) (FAST RWORD) (IS VERB) THE
DISTANCE (FROM PREP) THE SHIP TO 0
(INCREASING VERB)) (2.)))

Note that the first sentence has been broken into three, while the second has been left unchanged.

The third section is responsible for taking these simple sentences and transforming them into the model of the problem which the computer must have. The model used in the program is composed of equations and "structures". A "structure" is basically a tree which has as its head the name of some object, and at various levels beneath the head all the information the program was able to abstract from the problem. (The second level corresponds to the property list of the head atom. The third level corresponds to the property lists of the atoms on the second level.) In our problem there is only one structure. It is most easily visualized in the following form:

POSITION:G0007 VELOCITY:G0009

WRTO:T50008 VALU: (TIMES 30.0 MILE)/DIRECTION: (TIMES I.-J)

DIRECTION:(TIMES I.L) VALU:<QU0TIENT(TIMES 25.0 MILE)H0UR))

The expressions G0007, G0008, etc. are symbols generated by the LISP system. They are commonly called GENSYMS, and are used as fillers in the structures.

Looking at the right hand node of the structure we see that the velocity of the ship is 25 miles per hour, and this velocity is in the -J direction. (The program assumes a right-handed co-ordinate system so -J would be west as it should be.)

The fourth section will generate the equations which the final section will solve. In this problem it must change the expression (DISTANCE (SHIP)(GO008)) to an actual distance equation. After it has done so the program prints out

(THE EQUATION SET IS)
((EQUAL (GOO12) (EXPT (PLUS
(TIMES 899.99998 (EXPT MILE 2.))
(TIMES 624.99998 (EXPT TIM 2.) (EXPT MILE 2.)
(EXPT HOUR -2.))) 0.5)))

Noting that TIM is the symbol for time we see that this is just the equation written in more usual notation is

$$X = \sqrt{900\ MILES^2 + 625\ T^2\ MILES^2 / HOUR^2}$$

The final section of the program will differentiate, simplify, and finally print out

(THE ANSWER IS)
0.

This, of course, with either a little thought, or a little algebra can be shown to be the correct answer. As a point of interest CARPS took 41 seconds of machine time to solve this problem.

II. STUDENT

As we mentioned earlier Bobrow's program, STUDENT, solves algebra word problems. To really understand how STUDENT works we should go through a problem and see how the program solves it. A typical problem which STUDENT could solve is:

(THE GAS CONSUMPTION OF MY CAR IS 15
MILES PER GALLON. IF THE DISTANCE
BETWEEN BOSTON AND NEW YORK IS 250
MILES, WHAT IS THE NUMBER OF GALLONS
OF GAS USED ON A TRIP BETWEEN NEW YORK
AND BOSTON Q.)

In this problem we don't have any mandatory transformations, however many of the words will be tagged. (A typical mandatory transformation would be "twice" changed to "2 times".) After the words are tagged the problem would then look like:

(THE GAS CONSUMPTION (OF/OP)(MY/PRON)
CAR (IS/VERB) 15 MILES PER GALLON
(PERIOD/DIW) IF THE DISTANCE BETWEEN
BOSTON AND NEW YORK (IS/VERB) 250
MILES, (WHAT/QWORD) (IS/VERB) THE NUMBER
(OF/OP) GALLONS (OF/OP) GAS USED ON A
TRIP BETWEEN NEW YORK AND BOSTON (Q/DIM))

The next section of the program breaks the sentences into what Bobrow calls kernel sentences. In this problem the first sentence will not be changed, however the second will. It will become (for the sake of convenience we will drop the tags):

(THE DISTANCE BETWEEN BOSTON AND NEW
YORK IS 250 MILES. WHAT IS THE NUMBER
OF GALLONS OF GAS USED ON A TRIP BETWEEN
NEW YORK AND BOSTON Q.)

It accomplishes this by noting the "IF ... (-/QWORD) ... " construction and breaking the sentence into the two smaller ones above.

STUDENT next transforms the simple sentences into equations. The general rule used here is that the word "is" is changed to an equal sign, and words like "times", "divide" are changed to their algebraic equivalents. The equation formed by a question is an equality between a newly created atom and the quantity to which the question refers. Our problem will create the following equations:

(EQUAL X00001 (NUMBER OF GALLONS OF GAS USED ON TRIP BETWEEN NEW YORK AND BOSTON))

(EQUAL (DISTANCE BETWEEN BOSTON AND NEW YORK) (TIMES 250(MILES)))

(EQUAL (GAS CONSUMPTION OF MY CAR) (QUOTIENT(TIMES 15(MILES)) (TIMES I(GALLON))))

With the exception of X00001 (the newly created variable) our variables come directly from the words of the problem, ignoring any occurrences of "a" and "the".

STUDENT then tries to solve this set of equations and finds that it can not. It then prints out

(USING THE FOLLOWING KNOWN RE IAT IONS HIPS)
((EQUAL (DISTANCE) (TIMES (SPEED) (TIME)))
(EQUAL (DISTANCE((TIMES (GAS CONSUMPTION)
(NUMBER OF GALLONS OF GAS USED))))

(ASSUMING THAT)
((DISTANCE) IS EQUAL TO (DISTANCE BETWEEN BOSTON AND NEW YORK))

(ASSUMING THAT)
((NUMBER OF GALLONS OF GAS USED) IS EQUAL TO (NUMBER OF GALLONS OF GAS USED ON TRIP BETWEEN NEW YORK AND BOSTON))

(ASSUMING THAT)
((GAS CONSUMPTION) IS EQUAL TO (GAS CONSUMPTION OF MY CAR))

The equations are stored in a glossary under a key word. The first word of each variable is looked up in the glossary and the corresponding equations pulled out. STUDENT then matches up the variables in the new equations with the variables already in the problem. After having made these observations STUDENT again tries to solve the equations, succeeds, and prints out

(THE NUMBER OF GALLONS OF GAS USED ON A TRIP BETWEEN NEW YORK AND BOSTON IS 16.66 GALLONS)

A close examination of STUDENT shows that it has some properties which make it less general than one would like. The first of these is that once we have translated our sentences into equations, all algebra word problems are alike, that is, they are all sets of linear equations with as many equations as unknowns. Hence, once the problem has been reduced to equation form, STUDENT does not need any heuristics to solve the equations.

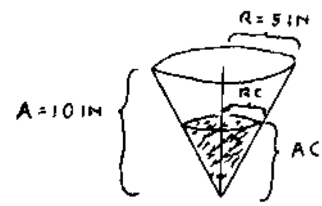
The second property is in my estimation even more serious. With one exception, each kernel sentence is translated into exactly one complete equation, and since STUDENT can count on its input always forming equations, there is no provision for any other form of information storage. However, in a typical calculus word problem we might have a sentence like "A ship is traveling east.", or 'tyater is flowing into a conical funnel." In each case there is information which should be stored, but clearly the equation form would not be the appropriate storage medium.

Both of these criticisms stem directly from the type of problem Bobrow sets out to solve. There are other difficulties with STUDENT which were probably left unsolved because they were not critical to the operation of the program and the size of the program had already reached the size of the available memory. These are 1) a very limited subset of English grammar. STUDENT cannot handle something as basic to English as a dependent clause. In general one must be very careful in one's choice of sentences when addressing STUDENT. 2) a lack of sophisticated heuristics to determine the equality of variables previously considered independent. 3) a small knowledge of the "real world" and a rather inflexible manner of using that knowledge.

III. <u>WORD TAGGING AND SENTENCE DECOMPOSITION</u>

If we are to really understand the workings of CARPS we should go through a problem in great detail. Consider the following problem:

(WATER IS FLOWING INTO A CONICAL FILTER AT THE RATE OF 15.0 CUBIC INCHES PER SECOND /. IF THE RADIUS OF THE BASE OF THE FILTER IS 5.0 INCHES AND THE ALTITUDE IS 10.0 INCHES /, FIND THE RATE AT WHICH THE WATER LEVEL IS RISING WHEN THE VOLUME IS 100.0 CUBIC INCHES /.)



In the first section of the program each word is checked to see if it has a property on its property list under the indicator GRAMMAR. If it does, the value of this property will be a function which will then be evaluated. The net result of the evaluation will be one of the following:

1) The current word (i.e., the word whose property list we Just checked) will be tagged with its part of speech. 2) The words of the question sentence will moreover be checked to see

if they give any clue as to the type of problem we are deailing with. In our problem the word LEVEL, is the clue that the problem is one which deals with volumes. 3) A current word may be noted as a key word (meaning it has equations associated with it in memory). The only key word occurring in our problem is CONICAL. 4) Common phrases will be changed to a standard form. For example the phrases AT THE RATE OF and AT THE RATE are changed to AT RATE, 5) Words ending in ING will be labeled as verbs.

After this section is through, our problem will look like this:

(((WATER (FLOWING VERB) {INTO PREP) A (CONICAL ADJ) FILTER (AT PREP) (RATE RWORD) 15.0 (IN3 UNIT) PER (SEC UNIT)) (1)) ((IF THE RADIUS OF THE BASE OF THE FILTER (IS VERB) 5.0 (IN UNIT) AND THE ALTITUDE (IS VERB) 10.0 (IN UNIT) , (FIND QWORD) (RATE RWORD) AT WHICH THE WATER LEVEL (RISING VERB) WHEN THE VOLUME (IS VERB) 100.0 (IN3 UNIT)) (2)))

The next section of the program breaks the sentences into simple sentences. Since we treat this as a problem of pattern matching, it is written in CONVERT. At the present moment we have sixteen rules in this part of the program. They cover a large portion of compound and complex sentences. They are listed with examples of their use in Appendix B.

Let us start with the second sentence, since it is somewhat easier to explain than the first. The pattern which will match this is

IF - ANYTHING - , - QUESTION WORD - ANYTHING

IF ⏟ **IF** | **THE RADIUS OF THE BASE OF THE FILTER IS 5.0 IN AND THE ALTITUDE IS 10.0 IN** ⏟ **ANYTHING** | **,** ⏟ **,** | **FIND** ⏟ **QUESTION WORD** | **RATE ATWHICH THE WATER LEVEL RISING WHEN THE VOLUME IS 100.0 IN3** ⏟ **ANYTHING**

The right side of the rule specifies that we begin the program over, but break the sentences in two. In our case the two sentences are THE RADIUS OF THE BASE OF THE FILTER IS 5 IN AND THE ALTITUDE IS 10.0 IN and FIND RATE ATWHICH THE WATER LEVEL RISING WHEN THE VOLUME IS 100 IN.

The second sentence of these two (FIND THE RATE ATWHICH...) will be broken by the rule

QUESTION WORD - ANYTHING - WHEN - ANYTHING

FIND ⏟ **QUESTION WORD** | **RATE ATWHICH THE WATER LEVEL RISING** ⏟ **ANYTHING**

WHEN ⏟ **WHEN** | **THE VOLUME IS 100.0 IN3** ⏟ **ANYTHING**

This rule is fairly straight forward in the sense that it splits off the dependent clause introduced by WHEN and makes it into a separate sentence. However, in the process the WHEN is removed from the sentence. The word WHEN in calculus problems is almost always used to indicate the boundary condition. That is, it precedes a value which should only be substituted into the equations after the differentiation is accomplished. In order to save this information the word WHEN is added to the tag of the second sentence created by this rule.

In the same way the rest of the problem is broken into simple sentences. Our problem now looks like (with the word tagged just to remind the reader that the tags are still there)

(((WATER (FLOWING VERB) (INTO PREP) A (CONICAL ADJ) FILTER) (1)) ((WATER (FLOWING VERB) (AT PREP) (RATE RWORD) 15.0 (IN3 UNIT) PER (SEC UNIT)) (1)) ((THE RADIUS OF THE BASE OF THE FILTER (IS VERB) 5.0 (IN UNIT)) (2)) ((THE ALTITUDE (IS VERB) 10.0 (IN UNIT)) (2)) (((FIND QWORD) (RATE RWORD) ATWHICH THE WATER LEVEL (RISING VERB)) (2)) ((THE VOLUME (IS VERB) 100.0 (IN3 UNIT)) (2 WHEN)))

IV. <u>TRANSFORMATION OF SIMPLE SENTENCES INTO THE INTERNAL MODEL</u>

Once the sentences are simplified the program goes to the third and probably most important stage. It is here that we translate our simple sentences into the internal representation of the problem (i.e. into structures and equations). This takes place in two phases. Let us call them A and B. The first phase identifies the basic form of a sentence. A list of these forms can be found in Appendix C. The second phase further analyzes the sentence components recognized by phase A, especially noun phrases. A list of the processes applied to noun phrases can be found in Appendix D. Phase B also relates the sentences to the entire problem of which it is a part. Phase B concludes by filling out the structures. The entire procedure Is applied to each sentence.

Upon entering phase A with the first sentence we check if the sentence is a question. In this case the sentence is routed to a special section which deals only with questions. The first sentence will not match any pattern and will be placed on the special list for unmatched sentences. The second sentence will match the pattern

```
WATER      FLOWING   AT     RATE       15.0     IN3
NO-VERB    VERB      PREP   ANYTHING   NUMBER   UNIT
PHRASE

PER   SEC
PER   TIMEUNIT
```

This format indicates that the object mentioned by the noun phrase is changing at a given rate. The rate is restricted to time simply because all our problems have only time rates.

We now enter phase B. It notices that IN3 is a unit of volume, so we get the structure:

```
              WATER
              /
      VOLUME:G0015
      /
VALU:(QUOTIENT (TIMES 15.0 (TIMES TIM
(EXPT IN 3)))SEC)
```

The program now realizes that this sentence indicates volume change. By the previously unused sentence it decides that WATER is the contents of the CONICAL FILTER. It then notes that CONICAL is an adjective describing SHAPE. Then this information, as well as the contents of the filter, is put into structure form and we get:

```
                    FILTER
                 /          \
CONTENTS:WATER               SHAPE:CONICAL
        /
VOLUME:G0015
      /
VALU:(QUOTIENT (TIMES 15.0 (TIMES TIM (EXPT
IN 3)))SEC)
```

The fact that atoms are unique in LISP comes in very handily here. When we put WATER on the property list of FILTER we are guaranteed that all the properties associated with WATER (VOLUME in this case) will tag along.

The third sentence (THE RADIUS OF THE BASE OF THE FILTER IS 5.0 IN) will match

```
THE RADIUS OF THE BASE OF THE FILTER   IS
            NOUN PHRASE                IS

5.0      IN
NUMBER   UNIT
```

This indicates a value assignment. THE RADIUS OF THE BASE OF THE FILTER is changed into structure form, and then the value, 5 IN, is tacked on. This gives us

```
            FILTER
            /
      BASE:G0016
         /
RADIUS:G0017
     /
VALU-.5 IN
```

To understand the fourth sentence (THE ALTITUDE IS 10.0 IN) is handled we must backtrack somewhat. When we encountered the phrase CONICAL FILTER we put conical on the property list of filter. The program knows that conical objects have many properties in common. For example they all have altitudes and bases, and bases have radii. So these facts are put on the structure of the filter. Hence much of the structure we have shown being created had already been placed in the structure as a common property of cones. Never the lees at each step the system actually did create the substructure finding, however, that the information was already on the property lists.

There is one other operation we have so far neglected. Every time a property X is put on the atom Y, a property MARKERON is placed on the atom X, and this has the value Y. This gives us backwards pointers so that given a property we can find out which atoms have that property. The point is that when the program checked to see if ALTITUDE was a marker on any atom, it discovered that it was on FILTER. The program then assumed that ALTITUDE referred to the altitude of the filter, and created the sub structure (A)

```
        FILTER               WATER
        /                        \
ALTITUDE:G9002          ALTITUDE:G0019
      /
VALU:(TIMES 10 .0 IN)

         A                      B
```

The next sentence is the question (FIND RATE ATWHICH THE WATER LEVEL RISING). The phrase WATER LEVEL becomes (B). Note that CARPS recognizes that in this context LEVEL means ALTITUDE. For a partial list of the knowledge CARPS has about words, see Appendix E. Once again WATER is only on the property list of FILTER, so we can trace back and find that what we are interested in is (G0019 WATER FILTER). This is the equation variable format. This variable represents, in English, the altitude of the contents of the filter. So we create the equation

(EQUAL G002 0 (DERIV(G0019 WATER FILTER)))

The interpretation of the last simple sentence (THE VOLUME IS 100.0 IN3) depends on the same sort of analysis. The word volume only appears as a marker on water, so the program assumes the sentence is referring to this volume. However since the sentence tag for this sentence has the word WHEN, the structure created is modified as seen below in the completed structure. The WHEN on the property list of WATER indicates that what follows beneath it is a boundary condition.

At this point all the necessary information has been extracted from the sentences, and this stage of the solution is over. The final structure is on the next page.

V. THE FORMATION OF THE EQUATIONS AND THEIR SOLUTION

The fourth section is concerned with establishing the equations which the fifth section will manipulate to solve the problem.

There will be two equations (other than the one already created by the question sentence). One relates the volume of the contents of a cone to the altitude and radius, and the other relates the height and radius of our cone to the height and radius of the contents of the cone. To see how the stored equations are manipulated consider this equation used in the problem.

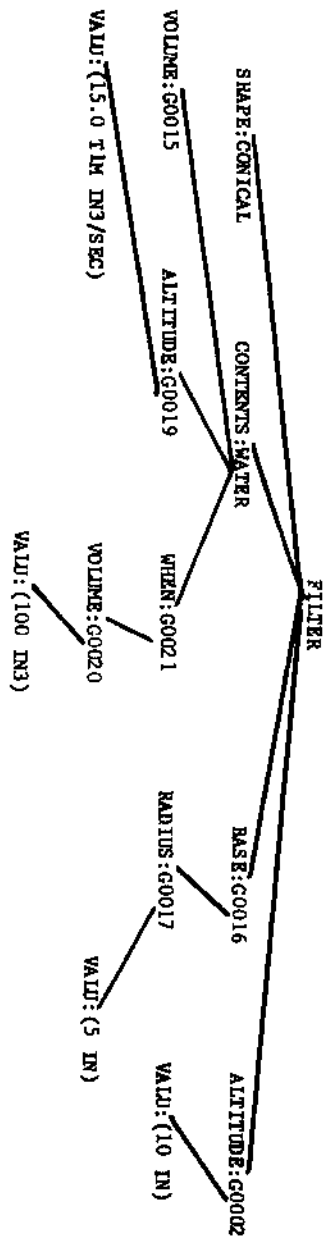
```
(EQUAL (TIMES (RADIUS BASE CONTENTS OBJ)
              (ALTITUDE OBJ))
       (TIMES (ALTITUDE CONTENTS OBJ)
              (RADIUS BASE OBJ)))
```

This is the relati R / RC = A / AC s seen in the diagram on page 3 .

Before the parameters are replaced an extra copy is set aside to be used later when we solve for the boundary condition. The program will then go through the list structure noting that EQUAL and TIMES are algebraic symbols. It will then come to (RADIUS. . . . .) and note that this is not an algebraic symbol, implying that the list of which RADIUS is the first element is a variable name. Processing the variable in reverse order, it first finds that the value of OBJ is FILTER. It then looks for the contents of FILTER, which is water, and then for the BASE of the WATER, which it does not find. Hence it leaves this variable with only the last two elements replaced (e.g., (RADIUS BASE WATER FILTER)). It will find the altitude of the filter however, and note that it has the property VALU whose value will then replace this entire expression in the equation. In this same manner the rest of the equation set will be filled, and the computer prints out.

```
(THE EQUATION SET IS)
1 ((EQUAL (G0005) (DERIV (G0004 WATER
FILTER)))
2 (EQUAL (QUOTIENT (TIMES 17.0 (TIMES
(EXPT IN 3) TIM)) SEC)
  (TIMES (G0004 WATER FILTER) 0.33333300 PI
  (EXPT (RADIUS BASE WATER FILTER)2)))
3 (EQUAL (TIMES (RADIUS BASE WATER FILTER)
(TIMES 12.0 IN))
  (TIMES (G0004 WATER FILTER)
(TIMES 5.0 IN))))
```

In usual notation this is



$$X = \frac{d}{dt} AC$$

$$17t \ IN^3/SEC = \frac{1}{3} \pi \ (RC)^2 \ AC$$

$$RC \cdot 12 \ IN = AC \cdot 5 \ IN$$

At this point section four of CARPS is completed, and we go on to five.

In section five we first get our desired variable (in our case the altitude of water) completely in terms of time, and then differentiate to find the rate as a function of time. Then the boundary condition is used to solve for time and this value is substituted into the rage equation to get our final answer.

(THE ANSWER IS)

(TIMES .53132943 IN (EXPT SEC -1.0)
(EXPT PI -0.33333332))

In conventional notation this would be

$$.53 \ (\pi)^{-1/3} \ IN/SEC.$$

VI. CONCLUSIONS

In section 2 we noted several aspects of STUDENT which we felt needed further work. Let us compare CARPS and STUDENT in these areas.

1) By storing information in terms of structures, CARPS is better able to recognize that two phrases describe the same object.
2) Again because of the use of structures CARPS can gather information about an object in piecemeal fashion. STUDENT was essentially required to generate one equation for each sentence in the problem description. In calculus word problems it is not uncommon to have two or three sentences providing information for one equation.
3) CARPS to a limited degree is able to use its knowledge to parse its input sentences. For example we saw how ALTITUDE was interpreted as ALTITUDE OF THE FILTER because CARPS knew that since the filter was a cone and cones have altitudes, the filter had an altitude.
4) Whereas STUDENT has only one solution method (i.e., solution of linear equations), CARPS has several and can decide which is appropriate for a given problem. CARPS machinery for solving its equations (e.g., differentiation, simplification) is also more complex than STUDENT'S.
5) CARPS utilizes a more sophisticated grammatical analysis of the sentences than STUDENT. This is used both in breaking up sentences and in generating the internal structures. Up to now 14 problems have been solved by CARPS. Most were taken from standard texts, sometimes with slight modifications. (Variations in the statements of problems would, of course, increase the number of problems solvable by CARPS.) The other problems solved by CARPS are in Appendix A.

Many of the improvements that we claim for CARPS were necessitated by the increased complexity of the problems that we expected it to solve. However many weaknesses of STUDENT are still present in some form in our design.

1) An important weakness in the program is due to its dependence on key words to signify the type of problem (i.e., distance or volume) and the method of solution to be used. What one would like to have in a calculus problem solver is a program which would use the information presented in the problem to figure out relationships among the elements (e.g., similar triangles) and actually propose the method of solution. The answers which could be provided by such a program are currently built into CARPS' data base for several cases, but this scheme severely limits the power of the program.
2) Another weakness of CARPS is its limited knowledge of English syntax. It would not be too difficult for CARPS to learn new syntactic rules by adding these rules to its CONVERT subroutines. Actually what would be more satisfying would be a different method of parsing the sentences into components of structures. Currently the CONVERT rules are attempted one at a time until one matches the sentence. A better approach would be an incremental left to right parse which, when finished with the sentence, would have.translated it into the internal model.
3) A very powerful calculus word problem solver will require a good deal of "common sense" knowledge. Consider this problem which we gave to CARPS

(A LADDER 20.0 FT LONG LEANS AGAINST A
HOUSE /. FIND THE RATE AT WHICH THE TOP
OF THE LADDER IS MOVING DOWNWARD IF ITS
FOOT IS 12.0 FT FROM THE HOUSE AND MOVING
AT THE RATE 2.0 FT PER SEC /.)

Much to our surprise CARPS was not able to solve it. A closer look at the problem shows why. The last phrase mentions that the ladder is moving at the rate 2 ft per second. CARPS has, as an internal check, the requirement that associated with each velocity must be the direction of the velocity. The point is that the problem never gave this direction. Most people, however, would assume that it was moving horizontally away from the house. The reason of course, is that a familiarity with ladders or the law of gravity tells us that this is the most likely way for it to be moving.

This is not an isolated incident. Consider the problem

(A BARGE WHOSE DECK IS 10 FT BELOW THE
LEVEL OF A DOCK IS BEING DRAWN IN BY
MEANS OF A CABLE ATTACHED TO THE DECK
AND PASSING THROUGH A RING ON THE DOCK.
WHEN THE BARGE IS 24 FT FROM AND
APPROACHING THE DOCK AT 3/4 FT / SEC,
HOW FAST IS THE CABLE BEING PULLED IN ?)

The ship is moving horizontally towards the dock. The problem does not mention this.

Nor is the need for a body of "real world" Information unique to calculus problems. Consider an example taken from a children's story. "Mike's costume had big ruffles. He refused to wear it." The question is "Why?". A program which will answer correctly must know that boys usually will not wear girl's clothes, and anything with ruffles on it must be for girls.

It is our belief that a program which is to understand second grade stories must have at its disposal a body of facts comparable to that of an average seven year old. More generally, computer understanding of any body of literature will require a data base similar to that of a human reader. The major problems facing computer understanding of natural language are, in our opinion, collecting, storing, and utilizing such large bodies of information.

APPENDIX A

1 (A MAN 6.0 FT TALL WALKS AT THE RATE 5.0 PER SECOND TOWARD A STREET LIGHT WHICH IS 16.0 FT ABOVE THE GROUND /. AT WHAT RATE IS THE TIP OF HIS SHADOW MOVING ?)

Taken from Thomas[8], page 100. The problem originally asked two questions, only the first is in our problem.

2 (SHIP P IS 15.0 MILES EAST OF 0 AND MOVING WEST AT 20.0 MILES PER HOUR ; SHIP B /, 60.0 MILES SOUTH OF 0 /, IS MOVING NORTH AT 15.0 MILES PER HOUR /. AT WHAT RATE ARE THEY APPROACHING OR SEPARATING AFTER 1.0 HOUR /.)

Problem taken from Ayres, page 59. Three changes were made. SHIP P was originally SHIP A, the question originally read "Are they approaching or separating after 1 hr and at what rate ?", and there were three questions.

3 (LADDER 20.0 FEET LONG LEANS AGAINST A HOUSE /. FIND THE RATE AT WHICH THE TOP OF THE LADDER IS MOVING IF ITS FOOT IS 12.0 FEET FROM THE HOUSE AND MOVING AWAY FROM THE HOUSE AT THE RATE 2.0 FEET PER SECOND /.)

Taken from Ayres page 59. The problem originally asked two questions.

4 (A TRAIN STARTING AT 11.0 AM /, TRAVELS EAST AT 45.0 MILES PER HOUR WHILE ANOTHER /, STARTING AT NOON FROM THE SAME POINT /, TRAVELS SOUTH AT 60.0 MILES PER HOUR /. HOW FAST ARE THEY SEPARATING AT 3.0 PM ?)

Taken from Ayres page 59, no changes.

5 (GAS IS ESCAPING FROM A SPHERICAL BALLOON AT THE RATE OF 2.0 CUBIC FEET PER MINUTE /. HOW FAST IS THE SURFACE AREA SHRINKING WHEN THE RADIUS IS 12.0 FEET ?)

Taken from Ayres page 57, no changes.

6 (A BALL IS RISING VERTICALLY OVER A POINT B AT THE RATE 15.0 FEET PER SECOND /. A POINT C IS LEVEL WITH B AND IS 30.0 FEET FROM B /. WHEN THE BALLOON IS 40.0 FEET FROM B /, AT WHAT RATE IS ITS DISTANCE FROM C CHANGING ?)

7 (UPON BEING HEATED /, A METAL DISC EXPANDS /. THE RADIUS OF THE DISK LENGTHENS AT THE RATE OF .01 PER SEC /. CALCUIATE THE RATE AT WHICH THE AREA OF THE DISK IS INCREASING WHEN THE RADIUS IS 3.0 IN /.)

Taken from Lightstone[5] page 145. The problem originally had two questions and the first two sentences above were connected by the phrase "in such a manner that".

8 (SAND FALLS INTO A CONICAL PILE AT THE RATE OF 10.0 FT3 PER MIN /. THE RADIUS OF THE BASE OF THE PILE IS ALWAYS EQUAL TO ONE HALF OF ITS ALTITUDE /. HOW FAST IS THE ALTITUDE OF THE PILE INCREASING WHEN IT IS 5 FT DEEP ?)

Taken from Thomas page 106, no changes.

9 (A RECTANGUIAR TROUGH IS 8.0 FT LONG /, 2.0 FT WIDE /, AND 4.0 FT DEEP /. IF WATER FLOWS INTO THE TROUGH AT THE RATE OF 2.0 FT3 PER MIN /, HOW FAST IS THE SURFACE OF THE WATER RISING WHEN THE WATER IS 1.0 FT DEEP ?)

Taken from Ayres page 59. Rather than WIDE the problem had "across the top", instead of INTO THE TROUGH the problem had "in", and instead of SURFACE OF THE WATER the problem had just "surface".

10 (A MAN WALKS AWAY FROM A STREET LIGHT AT 4.0 FT PER SEC /. HOW FAST IS THE POSITION OF THE TIP OF HIS SHADOW CHANGING IF HE IS 5.0 FEET TALL /, WHILE IT IS 20.0 FEET TALL /.)

11 (JOE RUNS FROM A STREET LIGHT WHOSE HEIGHT IS 30.0 FEET /. IF JOE IS 6.0 FEET TALL /, AND TRAVELING AT 10.0 MILES PER HOUR /, FIND THE RATE AT WHICH HIS SHADOW IS LENGTHENING ?)

12 (THE AREA OF A SPHERE IS INCREASING AT THE RATE OF 4.0 SQUARE INCHES PER SECOND /. AT WHAT RATE IS THE DIAMETER CHANGING AFTER 10.0 SECONDS 7)

APPENDIX B. Breaking sentences into simple sentences.

PATTERN	EXAMPLE	USE
IF - ANYTHING - , - QUESTION WORD-ANYTHING	If - the radius is 3 ft - , - what - is the altitude	To break up sentences which contain a subordinate clause introduced by IF
QUESTION WORD - ANYTHING - IF - ANYTHING	What - is the altitude - if - the radius is 3 ft	Same, except that clause occurs at the end of the sentence
QUESTION WORD - ANYTHING - WHEN ANYTHING	What - is the altitude - when - the radius is 3 ft	Separate off from the sentence a subordinate clause, introduced by WHEN
WHEN - ANYTHING - QUESTION WORD - ANYTHING	When-the radius is 3 ft-what-is the altitude	Same, except that clause occurs at the beginning of the sentence
ANYTHING - VERB - ANYTHING - AND/WHILE - VERB - ANYTHING	The radius - is - 3 ft - and - increasing - at 3 ft per sec	To break up a sentence which contains two predicates, connected by AND or WHILE
ANYTHING - VERB - ANYTHING - AND/WHILE - ANYTHING - VERB - ANYTHING	The radius - is - 3 ft - and - the altitude - is 5 in	To separate two complete sentences joined by AND or WHILE
ANYTHING - WHOSE - ANYTHING - VERB - ANYTHING - VERB - ANYTHING	The cone - whose - radius - is - 3 ft - has - an altitude of 5 ft	To separate a subordinate clause introduced by WHOSE
ANYTHING - VERB/PREP - RP - WHOSE - ANYTHING - VERB - ANYTHING	A man walks - toward - a light - whose - height - is 20 ft	Same, except that clause occurs at the end of the sentence
ANYTHING - VERB/PREP - RP - WHICH/THAT - VERB - NVP	A man walks toward - a light - which - is - 20 ft high	To separate a subordinate clause introduced by WHICH or THAT
RP - , - ANYTHING - , - VERB - ANYTHING	A man - , - starting at 5 PM - , - walks toward a light	To separate an appositive set off by commas
NVP - VERB - NVP - , - NVP - , - NVP	A box - is - 10 ft long - , - 3 ft wide - , - and 8 ft high	To separate a sentence with 3 objects or predicate adjectives
NVP - VERB - ADVERB - ANYTHING	A man - walks - directly - toward a light	To create a new sentence with the adverb and verb as its predicate
NVP - VERB - PREP - RP - PREP/ADVERB - ANYTHING	A man - walks - toward - a light - at - 2 miles per hour	To break up 2 predicate phrases, the first starting with a prep, the second with a prep or adverb
RP - NUMBER - UNIT - RP - VERB/PREP - ANYTHING	A man - 5 - ft - tall - walks - toward a light	To separate some appositives which are not indicated by commas

APPENDIX B: Continued

PATTERN	EXAMPLE	USE
ANYTHING - VERB/PREP - RP - NUMBER - UNIT - NVP - RP	A man walks-toward - a light - 20 ft - from - the ground	Same, except that the appositive occurs at the end of the sentence
NVP - IS - RP - AND - RP	A box - is - 12 ft long - and 8 ft high	To separate a sentence with two predicate adjectives

PATTERN	EXAMPLE	COMMENTS
VOLUME SECTION		
NVP - VERB - PREP - ANYTHING - NUMBER - UNIT - PER - TIME UNIT	A ship - traveling - at - - 20 - mile - per - hour	Indicates something changing at a constant rate.
NVP - IS - NUMBER - UNIT	A radius - is - 5 - in	The basic value assignment statement.
NVP - IS - NUMBER/NUMBER "TIMES"/ NUMBER "OF" - NO UNIT - NVP	The altitude - is - .5 times - the - radius of the base	This will create an equation.
NVP - IS - NUMBER - UNIT - SINGLE WORD	The cone - is - 8 - in - tall	
DISTANCE SECTION		
NVP - VERB - POSIT	A ship - traveling - north	Indicates direction of velocity.
NVP - VERB - AT - TIME INDICATOR	A ship - starting - at - 12 AM	Indicates zero time. The time indicator may also be "noon" or "midnight".
NVP - VERB - NUMBER - UNIT - POSIT - NVP	A ship - is - 30 - mile - north - point 0	Indicates value and direction of position with respect to a second object.
NVP - VERB - POSIT - NVP	A ship - is - north - point 0	Indicates direction of position with respect to a second object. Only used in distance problems.
QUESTION SECTION		
QUESTION WORD - RWORD - ANYTHING - VERB - RP/NULL - TIME INDICATOR - RP/NULL	Find - rate - atwhich they - separating - at - 3 PM - -	Boundary conditions explicitly involving time have different forms, this rule covers most.
QUESTION WORD - RWORD - ANYTHING - VERB - OR - VERB	What - rate - are they - approaching - or - separating	
QUESTION WORD - RWORD - "ATWHICH"/VERB - NVP - VERB	How - fast - is - the altitude - rising	This is the basic rate question format.

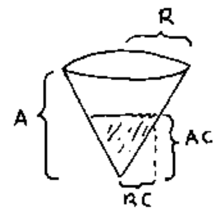
APPENDIX D: Noun phrase analysis

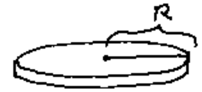
PATTERN	EXAMPLE		COMMENT
ANYTHING - OF - A/THE/PRONOUN - ANYTHING	the radius - of - the - base	BASE — RADIUS:G0001	
ANYTHING - OF - NO NUMBER	a pile - of - sand	PILE — CONTENTS:SAND	Only used in volume problems
ANYTHING - DISTANCE - FROM - ANYTHING	the train's - distance - from - the ship	TRAIN — POSITION:G0001 — DIRECTION:FROM, WRTO:SHIP	
ANYTHING - DISTANCE - FROM/BETWEEN - ANYTHING - TO/AND - ANYTHING	The - distance - between - the train - and - the ship	as above	
POSSESSIVE PRONOUN - ANYTHING	its - altitude	FILTER — ALTITUDE:G0001	Filter was the name of the previous non human structure mentioned in the problem
SINGULAR PRONOUN	he , she , it		Return the name of the last human or nonhuman structure mentioned
PLURAL PRONOUN	they		Return names of last two different structures mentioned
ADJECTIVE - ANYTHING	red balloon	BALLOON — COLOR:RED	

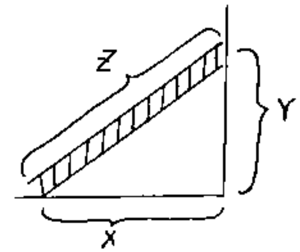
APPENDIX E: KNOWLEDGE ABOUT WORDS

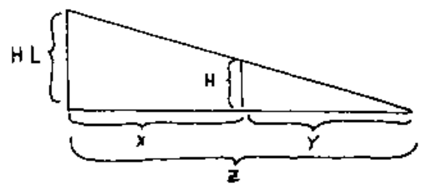
KNOWLEDGE IN THE FORM OF EQUATIONS

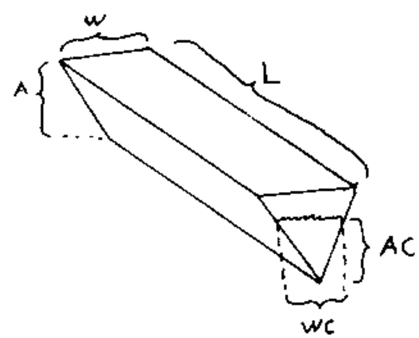
conical	$VOLUME\ OF\ CONTENTS = 1/3 * \pi * RC^2 * AC$
	$RC * A = AC * R$
disk	$AREA = 2 * \pi * R^2$
	$VOLUME = \pi * R^2 * THICKNESS$
ladder	$L^2 = X^2 + Y^2$
shadow	$Y * (HL - H) = H * X$
	$Z = X + Y$
spherical	$VOLUME = 4/3 * \pi * R^3$
	$AREA = 4 * \pi * R^2$
	$DIAMETER = 2 * R$
trough	$VOLUME\ OF\ CONTENTS = 1/2 * L * WC * AC$
	$WC * A = AC * W$











KNOWLEDGE ABOUT OBJECTS

cone	is a conical object
conical	has altitude, contents, base with radius
disk	has radius, width, volume, and area
sphere	is a spherical object

APPENDIX E: CONTINUED

spherical — has area, volume, diameter, and radius

trough — has altitude, width, and length

KEY WORDS FOR DETERMINING PROBLEM TYPE

DISTANCE KEYS

approaching, distance, separating; ladder, pole, rail; shadow

VOLUME KEYS

altitude, area, diameter, radius, surface, volume

ADJECTIVE TYPES

conical - shape   cylindrical - shape
green - color   rectangular - shape   red - color

spherical - shape

DIRECTION WORDS

above +K   east +fl   horizontally +1
level 4+1   north -J   over +K   south +J
vertically +K   west -I

OTHER KNOWLEDGE

altitude — implied by the words deep, depth, height, high, rising, and tall sometimes by surface and level

AM — implies the number preceding it in hours

balloon — spherical, it and its contents have the same volume, radius, etc.

heap — it and its contents have the same volume, etc.

Joe — is a person

length — is implied by the words long and lengthening

level — in a noun phrase implies altitude

Mary — is a person

midnight — zero hours

moving — implies position

noon — 12 hours

pile — it and its contents have the same volume, etc.

PM — adds 12 hours to the number preceding It

surface — if followed by AREA implies area, if followed by a verb which indicates altitude (such as rising) implies altitude, else implies area

vertical surface — is implied by fence, house, or wall

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