WALKING CONTROL AND DYNAMICS OP A SYSTEMS WITH TWO LEGS

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Abstract. The present report suggests and investigates a model of the two-legs walking. The walking apparatus is simulated by a heavy rigid body sypplied with a pair of multilink legs. The algorithm for solving the problem of the two-legs walking is composed. A number of the problems on construction of the two-legs walking are solved and analysed within the framework of the suggested algorithm.

Today one of the actual problem is to create the anthropomorphous locomotion systems. It appears the whole number of investigations concerning to the dynamics and the control of the two-legs walking [I-8] . This is a complex problem which has a great number degrees of freedom, a specific character of connections and controls. That is why the different dynamical models of the two-legs walking suggested earlier (even the simplest models) are very complex. It causes difficulties to analyse the problem.

Meanwhile the problem of the two-legs walking as well as any problem of dynamics and control needs in a model statement which is sufficiently reasonable to describe the principle effects and peculiarities of the processes investigated. This model statement should be also sufficiently simple to assume an effective analysis.

The represented report describes investigations carried out in this direction. In a number of cases suggested statements allow to obtain the process description in the analytical form. They also allow to give qualitative and parametrical analysis, and to construct the effective and simple algorithms for the numerical calculations.

I. Let us simulate the anthropomorphous system by means of a heavy rigid body (body-beam) which is supplied by a pair of imponderable non-inertial multilink legs suspended in one point of the body (fig.I). The contact of a leg with a surface is point and the interaction of the leg with the surface reduces to the resultant force applied to the point of the contact. In this case the algorithm for the construction of the motion and controls can be described by the following system of equations:

$$\vec r_0-\vec r_\lambda=\vec r_0-[(1-\lambda)\vec r_\pi+\lambda\vec r_\nu], \quad (1)$$

$$J\dot{\vec\omega}+\vec\omega\times J\vec\omega=[\vec\rho+(\vec r_0-\vec r_\lambda)]\times[\vec P-M\ddot{\vec r}_0]-M(\vec r_0-\vec r_\lambda)\times\{\dot{\vec\omega}\times\vec\rho+\vec\omega\times[\vec\omega\times\vec\rho]\}. \quad (2)$$

$$\vec R=-\vec P+M\{\ddot{\vec r}_0+\dot{\vec\omega}\times\vec\rho+\vec\omega\times[\vec\omega\times\vec\rho]\}, \quad (3)$$

$$\vec R_\pi=(1-\lambda)\vec R, \quad \vec R_\nu=\lambda\vec R \quad (4)$$

$$\vec U_j^i=(\vec r_j^i-\vec r_j)\times\vec R_j; \quad j=\pi,\nu; \quad i=0,1,2,\dots \quad (5)$$

$$|\vec r_j^{i+1}-\vec r_j^i|=l_i; \quad \vec r_\nu^0-\vec r_\pi^0=\vec r_0 \quad (6)$$

Here we designate: $\vec r_0(t)$ is a trajectory of the body's fixed point which coincides with a point of the leg suspension; $\vec r_\pi(t), \vec r_\nu(t)$ are the trajectories (discrete set) of the standpoints for the legs (a footstep trajectory). In this case ν refers to the leg going into the stage

of a support, $\mathcal{T}^2$ refers to the leg going out of the stage of a support. The process of the support transposition from one leg to another one is described by the arbitrary bunction $\lambda(t)$ which satisfies the conditions

$$\lambda(t_1)=0;\ \lambda(t_1+\tau)=1 \quad (7)$$

where t_1 is the beginning of a double-support phase of the motion and $t_1+\tau$ is the end of a double-support phase of the motion. J is a tensor of the body's inertia in the point of the leg suspension, $\vec{\omega}$ is a vector of the immediate angular velocity of the body, $\vec{\rho}$ is a vector fixed in the body from the point of the leg suspension into the center of mass of the body; $\mathcal{M}$ is a mass of the body; $\vec{P}$ is a vector of a gravity force, /f is a sum vector of the reactions applied in the points of the leg supporting ("the vector of reaction"), $\vec{R}_{\mathcal{T}}$ and $\vec{R}_v$ are components of this vector-force in each of the'supporting pointBi $\vec{u}_j^i$ is a vector of control in the i-th joint of the j-th leg, $\vec{\tau}_j^i$ is a trajectory of the i-th joint of the j-th leg; ℓ_i is a length of the i-th link of the leg. The corresponding matrix function can be introduced instead of the scalar function Aft) f and thereby it widens a class of the possible motions (equations (2) will have some different structure ffoj) • Alternativly if we assume $A=/$ we shall narrow a class of the motions up to the single-support walkings; in such walkings there are no a stage of support on two legs. The single-support walkings are a class of thei motions which is a boundary between the eigen-walking and running. Sequence of calculations: the functions of time, the required trajectory of the point of the leg suspension and the footstep trajectory and $\lambda(t)$, are given as explicit; equation (2) of the angular motion is integrated and among a set of the possible solutions we select ones which have the given properties (for example, the periodical solutions); the forces of reactions are calculated by formulae (3)-(4); the function of time of the trajectory of the leg joints are given as explicit and satisfying connections (6). According to (5) the controls are calculated forming the given motion.

Let us note that basic equations (2) and (3) do not depend on the motion of the leg joints (and depend only on the footstep trajectory and the trajectory of the suspension point). It represents the different possibilities for the eyethesis of the controls providing the given motion.

2. A number of problems on the construction of the two-legs walking are solved and analysed in the framework of the described algorithm.

We call the walking comfortable if the point of the leg suspension moves uniformly and rectilinearly and regular if each coordinate of the footstep trajectory is a step function obtaining the equal increments in equal intervals of time.

Let the walking be comfortable and regular. Assume also $|\vec{\rho}|=0$ (the point of the leg suspension coincides with the center of mass of the beam), and the beam is dynamically symmetrical. Then in virtue of (I)-(6) we come to the very simple model of the walking. This modes allows the complete analytical description: it is accurate in the plane case and approximate (but high-accurate) in the spatial case. The angular motions of the body are described by the three-parametric family of the periodical motions. It is shown in fig. 2. The angle 6 describes the normalized oscillations along the course counted from the vertical line, the angle φ describes these oscillations across the course. Factually, the trajectory of a "summit" of the walker (upper view) is shown here.

3. At any $|\vec{\rho}|\neq 0$ the body vibrations when the system moves can be small. Then equations (2) allow linearization. The

linear problems also allow the analytical solutions. So the problem on the comfortable motion by the steps and the inclined plane has been solved. The body vibrations in this case are described in the modified Bessel functions (fig.3). The shift of the average value of the body inclination at the expense of the inclination of the motion trajectory and the footstep path is demonstrative.

4. The considered statement allows effectively to solve the manoeuvring problems as well as the problem of motion on the rugged terrain, the problem of adaptation to the surface roughness and some other problems of control. The existance of the free parameters in the motion of the two-legs systems is very important. Such parameters are: the height of the point of the leg suspension, the supporting interval (the distance from the supporting point to the projection of the point of the leg suspension on the horizontal axis in the moment of change of the supporting leg). Fig. 4 illustrates an influence the parameter σ_o on the motion of the walker (σ_o is proportional to the supporting interval). If $\sigma_o > -1$ the apparatus walks with body inclination backwards on the average, if $\sigma_o = -1$ the apparatus vibrates on the average relative to the vertical line, if $\sigma_o < -1$ it inclines, on the average, forth. The corresponding possible configurations of the walker at the beginning and at the end of the half-period (i.e. at the moment of change of the supporting leg) are shown next to.

Thanks to the existence of the free parameters there is some possibility to optimize with respect to several of them. As an optimized value we can choose, for example, the work of the controlling moments during the corresponding angular movings for the halfperiod, i.e. during one step. The dependence of work on the value of the supporting segment for the plane model of the walker is constructed in fig.5- It is seen that at the fixed magnitudes of the remain parameters there exists some optimal value S minimizing the work.

5- The development of the described model is an analogous model having the ponderable legs. In this model the whole class of the plane motions is studied in detail. The algorithm is investigated which gives the legs' Kinematics. It calculates the motion of the point of the leg suspension. It construct the oscillational body motion, the algorithm calculates the reactions and the controls providing the fulfilment of the stated locomotional problem. On the whole the algorithm is reduced to the solution of the boundary value problem for one differential equation of the second order (describing the body vibration) and to the following calculations by the finite formulae. The boundary value problem can be solved by analytical way in the case of the small body vibration* then this solution is used as the first approximation to solve the nonlinear problem.

The parametrical analysis is obtained for the body vibration, for the control by the supporting and bearable legs, for the forces of reactions at the different kinds of motions. For the plane linear problem all motions and the acting forces are obtained in the closed analytical form.

Fig. 6 illustrates the solutions for the problem of walking (for three types of walking) so called the "full" walking (I) symmetrical man walking (II), non-symmetrical man walking (III). For each type of walking above we construct the trajectories of the represantabie points corresponding to the supporting and bearable legs on the plane of variables α, β (α, β are the angles which are formed vertically by a hip and a shin). In the case of "full" walking the represantabie point corresponding to the supporting leg passes the are of ellipse 1-5-3 &nd the represantabie point corresponding to the bearable leg passes the arc of ellipse I'-3'-5'. la the case of the symmetrical man walking the repre- aantable points corresponding to the supporting and bearable legs pass one and the same arc 1-3-3 but in the oppo- site directions. In the case of the non- symmetrical walking the represantable point oscillates along the limitted seg- ment of the ellipse arc. In all three walkings the bearable leg moves by "dragging" on the surface. It is not a matter of principle by the way. Below we illustrate the corresponding configura- tion of the walker for the different mo- ments of time. "Unnatural" motions of the walker's legs in the case of "full" walking are the analiticity fee of the obtained solutions. Dependence of the work on the period of motion is shown in fig. 7 for the indicated types of wal- kings (I, II, III). We can see that the non-symmetrical walkings are more econo- mical than symmetrical ones.

Pig. 8-12 illustrate some results of solutions of the walking problems for the plane nonlinear model; fig. 8-9 - for the symmetrical man walking, fig.10-12 - for the non-symmetrical comfortable wal- king. Pig. 8 showe the vertical component of reaction in the point of the support Ay depending on time (in the ha If pe- riod). It should be noticed that R_y is very close to the analogous component obtained while studying the man locomo- tion. Pig. 9 shows the dependence of the controlling moments in the joints on time during one step. The controls in the sup- porting leg is one an order more than in the bearable leg. In this case the cont- rols in a hip J-4 times more than the controls in a shin. Pig. 10 illustrates a strong influence of value of the sup- porting segment on the value of work of the inner forces. Depending on the value S the work can almost 4 times be chan- ged. Let us introduce the parameter ^ which is equal to the relation of the length of the supporting segment to the step length. At the fixed length of the step, for each set of parameters there exist $\varrho = \varrho_{min}$ where the work is minimal during one step. Dependence ϱ_{min} on the dimensionless value equal to the rela- tion of the leg mass of the walker to the whole mass of it is constructed in fig.II. At the fixed step there exists the opti- mal velocity of motion minimizing the energetic input for the walker.

REFERENCES

I. Vukobratovic M. A Contribution to the mathematical theory of the artifical biped motion. Труды У Международной конференции по нелинейным колебаниям, т.3, I970.

2. Вукобратович М., Франк А., Джурич Д., Об устойчивости передвижения на двух ногах. Сб.переводов "Механика" № I (I3I), I972.

3. Chow C.K., Jacobson D.H. Studies of Human Locomotion via Optimal Programming. Technical Report N 67, Division of Engineering and Applied Physics Harvard University, Cambridge, Mass.

4. Вукобратович М. Искусственные локомоционные системы: динамика, алгоритмы управления, устойчивость. Докторская диссертация. Москва, I972.

5. Белецкий В.В. Плоские линейные модели двуногого шагания. Препринт № 95, за I973 г., Ордена Ленина Институт Прикладной Математики Академии наук СССР, Москва.

6. Белецкий В.В. Динамика двуногой ходьбы. Препринт № 32, за I974 г., Ордена Ленина Институт Прикладной Математики Академии наук СССР, Москва.

7. Белецкий В.В. Управление ходьбой и динамика двуногих систем. УI Всесоюзное совещание по проблемам управления, рефераты докладов, часть Ш, изд-во "Наука", Москва, I974.

8. Белецкий В.В., Кирсанова Т.С., Чудинов П.С. Динамика двуногой ходьбы. "Проблемы нелинейных колебаний меха-

нических систем", тезисы докладов, изд-во "Наукова думка", Киев, 1974.

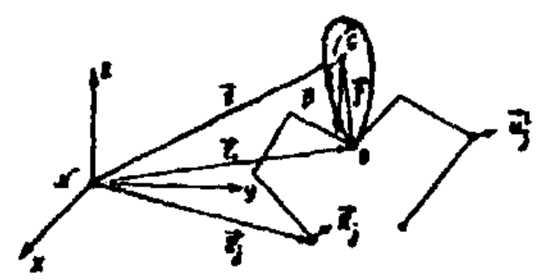
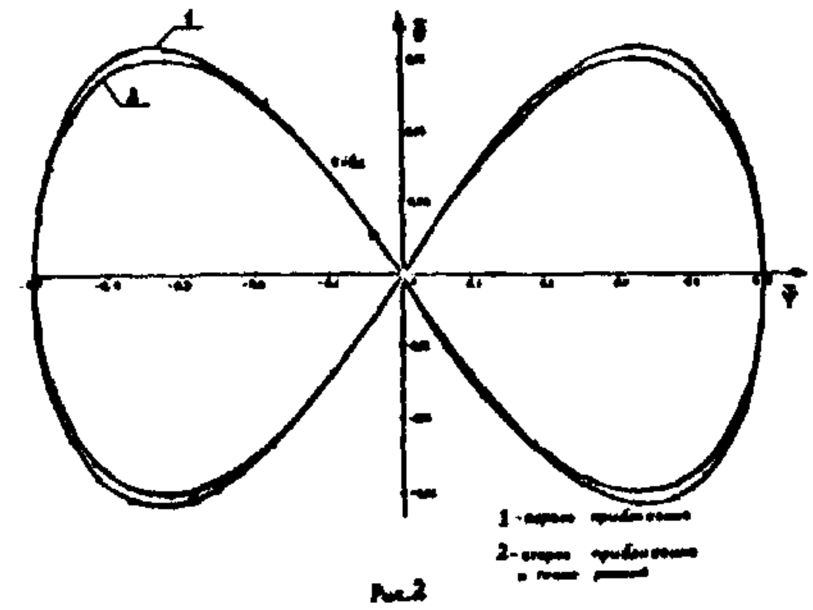


Рис.1



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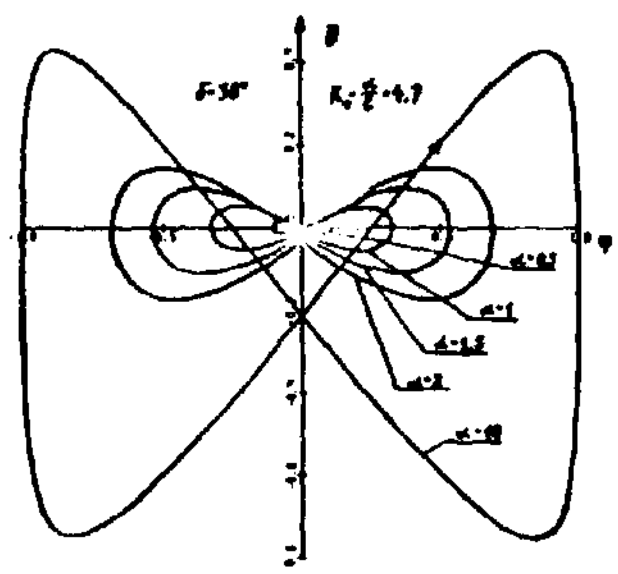

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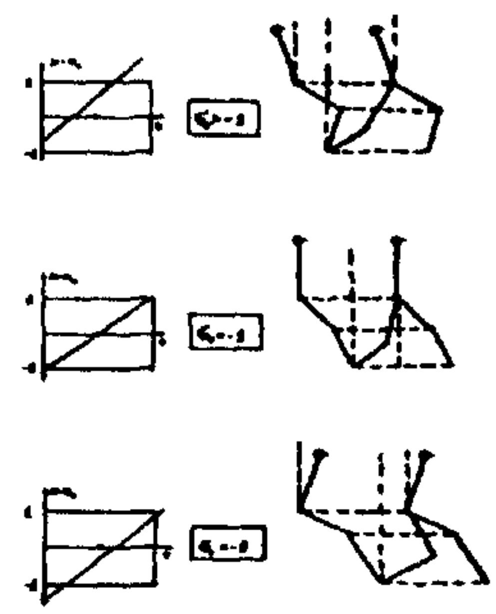

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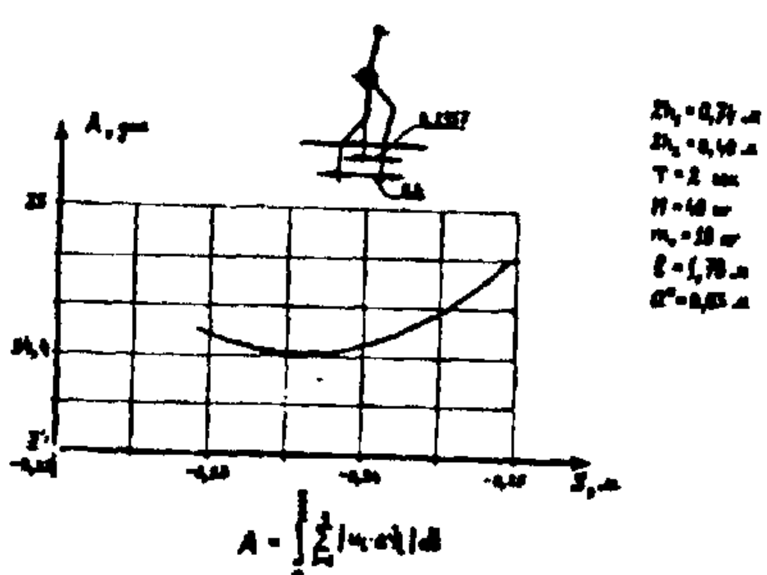

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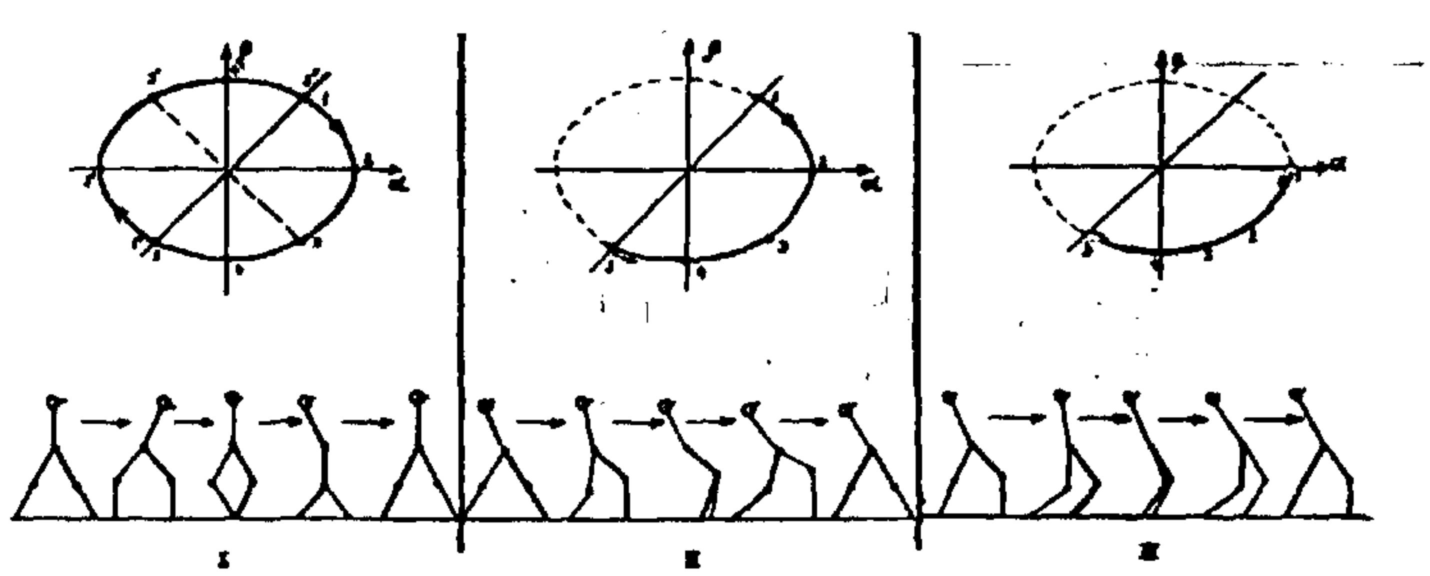

Рис.6

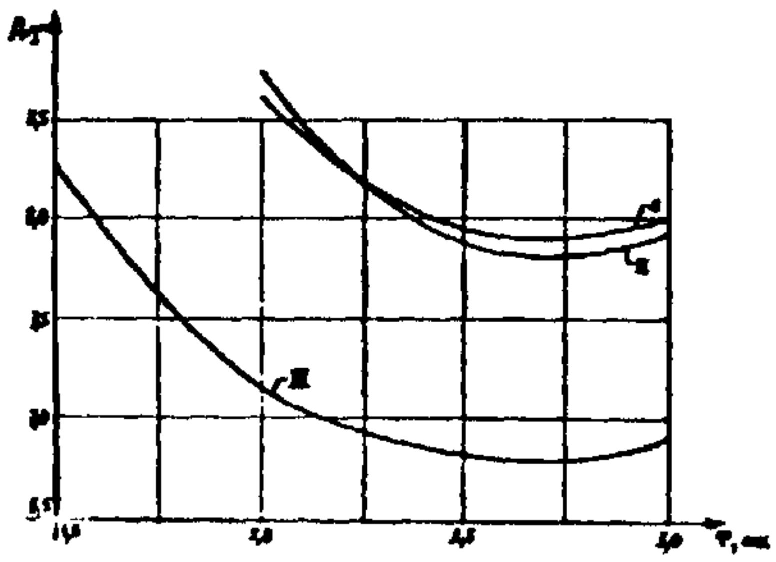

Рис. 7

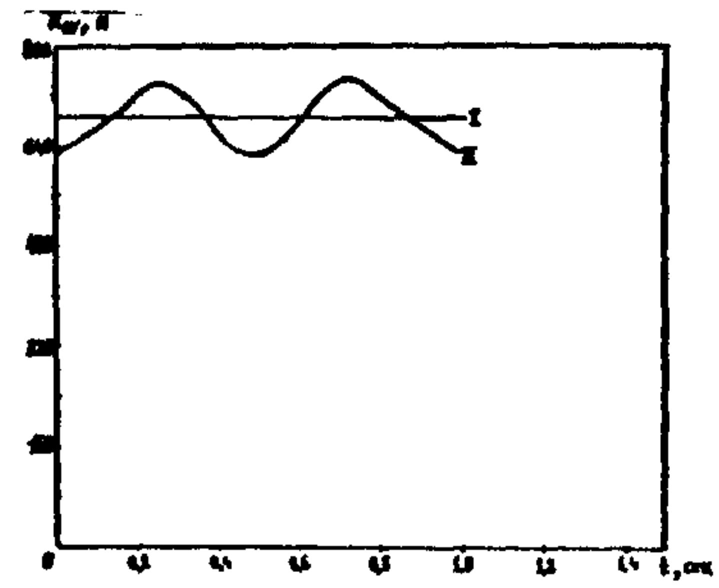

Рис. 8

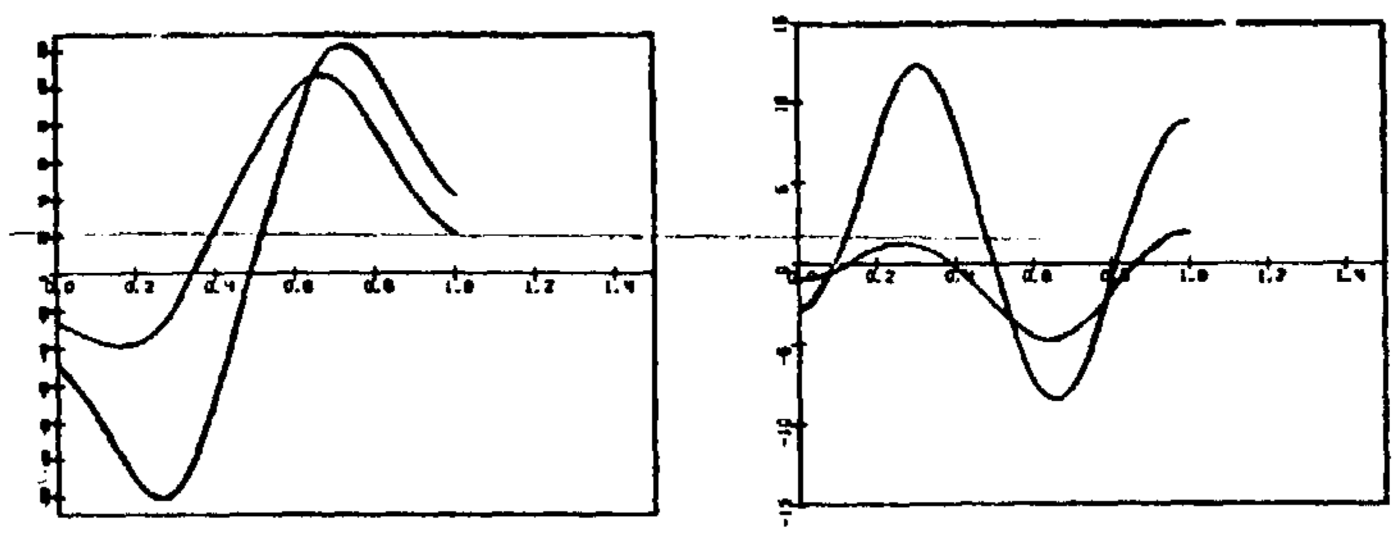

Рис. 9

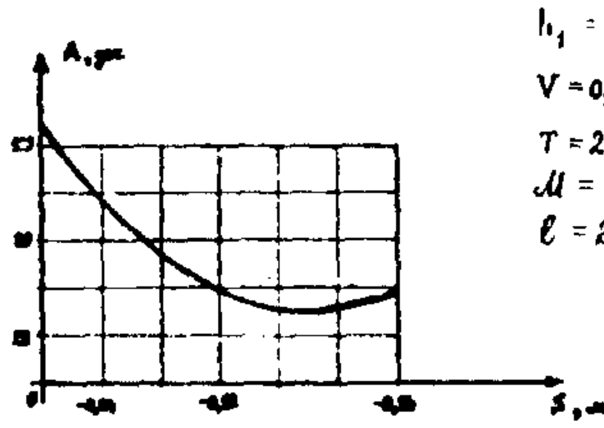

Рис. 10

$h_1 = 0,9$ м
$V = 0,3$ м/сек
$T = 2$ сек
$M = 70$ кг
$\ell = 2$ м

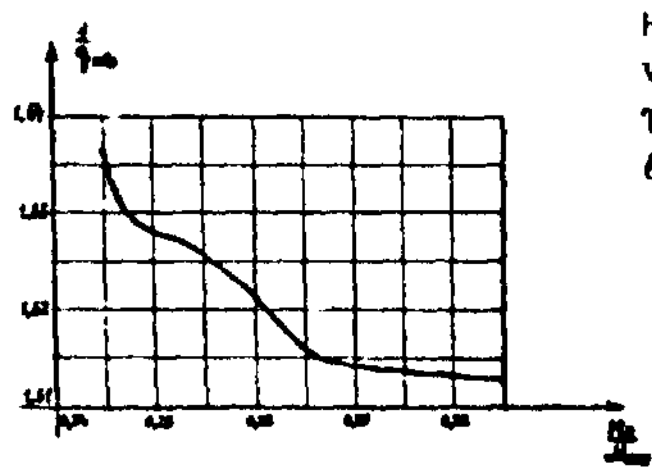

Рис. 11

$h_1 = 0,798$ м
$V = 0,4$ м/сек
$T = 2$ сек
$\ell = 1,7$ м

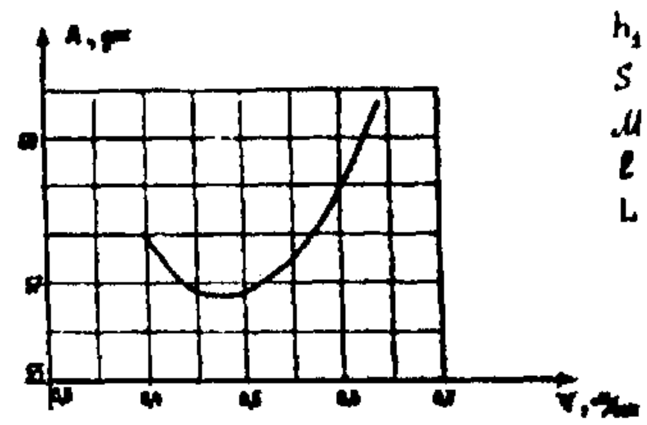

Рис. 12

$h_1 = 0,74$ м
$S = -0,2484$ м
$M = 70$ кг
$\ell = 1,7$ м
$L = 0,4$ м