A HIERARCHICAL APPROACH TO THEOREM-PROVING *

Malcolm C. Harrison
Courant Institute of Mathematical Sciences, New York University, New York, N. Y. 10012

1. Introduction

A considerable amount of recent work in theorem-proving has been concerned with methods for increasing the power of inferences which can be made in special cases. This appears to be in response to the widespread recognition that general purpose theorem provers, particularly those using resolution [10] as their inference rule, have extreme difficulty with particular aspects of proofs for which there are fairly effective algorithms and heuristics. Oustanding examples are equality, commutativity, and associativity. It is generally recognized that such properties are more efficiently dealt with if they are somehow 'built in'; to the theorem prover, rather than proviced in the form of axioms. There is, however, an understandable reluctance to destroy generality or completeness by ad hoc mechanisms.

The usual approach has been to search for methods of replacing an axiom or a set of axioms with an inference rule. This has resulted in a number of suggested methods for treating equality [1,2,5, 6,7,8,11,12], associativity and commutativity [1,4,15], and other properties [14,16]. One of the problems with equality is that proving two terms equal is in general as difficult as proving an arbitrary theorem — equality is undecidable, and any theorem can be expressed using just the equality predicate. Associativity is not as difficult, but the fact that there are an infinite number of ways of unifying terms involving associative functions gives rise to significant problems.

Recently we have been looking at a method of incorporating axioms into inference rules which approaches this problem in a different way. The idea is to split an unsatisfiable set S of clauses into two disjoint subsets, say U and S-U, and to use a theorem prover working on clauses of U to generate inference rules which are applicable to clauses of S-U. If these inference rules are generated on demand, this method is not necessarily restricted to values of U for which an equivalent, finite set of inference rules are known. In this paper we discuss a simple inference rule called U-generalized-resolution which is complete for any subset U of S.

This approach grew out of some discussions with David Yarmush and Norman Rubin concerning the problem of equality. It was suggested that an appropriate generalization of resolution to deal with equality consisted of generalizing the unification procedure so that literals could be unified if they had the same predicate and a substitution could be found which would make the arguments equal, rather than identical. Since equality is in general undecidable, such a procedure would require the use of a theorem-prover within the unification procedure. We discuss this approach to equality, which we call U-equality-generalized resolution, in Section 2. The more general approach, which we call U-generalized resolution and which gives a framework for dealing with properties other than equality, is treated in Section 3.

2- U-Equality-generalized Resolution

Given two clauses (written as sets of literals)

$$A \cup \{L(x_1,\ldots,x_\ell)\} \qquad (1)$$

$$B \cup \{\neg L(y_1,\ldots,y_\ell)\} \qquad (2)$$

we can infer

$$[A\lambda - \{L(x_1,\ldots,x_\ell)\lambda\}] \cup [B\lambda - \{\neg L(y_1,\ldots,y_\ell)\lambda\}] \cup \bigcup_i C_i\lambda \qquad (3)$$

if we can prove

$$C_i \cup \{r_i = s_i\}, \quad i = 1,2,\ldots\ell \qquad (4)$$

and λ is the most general simultaneous unifier (mgsu) of the pairs of terms

$$(x_i,r_i), \quad (y_i,s_i), \quad i = 1,2,\ldots,\ell.$$

If (4) is deduced from a set U of clauses, we refer to (3) as a <u>U-equality-generalized</u> resolvent of (1) and (2).

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