A MECHANIZED PROOF PROCEDURE FOR FREE INTENSICNAL LOGICS*

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Included amongst the intensional logics are those dealing with obligation (deontic logic), necessity (modal logic), time (tense logic), and various other intensional concepts. These logics were developed to formalize the semantics of natural language discourses. The intensional logics studied here are extensions of some of the sentential logics known as the normal intensional logics. These logics include the modal logics K, M, B, S4, and S5 and their deontic and tense logic counterparts. The closure of the accessibility relation in these logics can be readily computed, a convenient property for mechanizing proofs.

In free logics, individual names need not have a referent, and the domain of discourse is not required to be non-empty. There is general agreement that a quantified intensional logic must have a free quantificational base if it is to capture the logic of natural language discourses. By having a free base, it is possible, e.g., for two worlds to be compatible where, in one world, it is declared that something exists and, in the other world, that nothing exists. Or a term can refer at one time but not at another. More technically, neither the Barcan formula (Vx 0A=> D\ixA in a modal logic) nor its converse (DVxA^ Vx DA) are provable in a free intensional logic.

If the free intensional logics are found to constitute the possible logical bases of semantic descriptions of natural languages, as many believe, then an adequate system for automatic natural language processing must handle these logics correctly. A mechanized proof procedure for these logics must be devised for such tasks as disambiguating discourses, question-answering, and guiding rule-governed behavior.

Failure of the Barcan formula and its converse rules out natural extensions of two theorem proving techniques in standard first-order logic to the free intensional logics. First, in the tradition of Gentzen, Beth, and Smullyan, the device of using a single tree in a derivation is difficult to develop, as the same term on the one and only tree might have to refer in one place but not in another. Second, the resolution method - presupposes a prenex normal form theorem, but this fails in the free intensional logics. If a computational method could be developed for the free intensional logics using either the device of a single tree or resolution, it is likely that it would be somewhat cx>unter-intuitive.

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Instead, a mechanized proof procedure is presented in which there is a tree for each "possible world" and a scheme of branch numbering to identify corresponding branches of different trees. This tree method provides a new view of what constitutes a proof, and an appropriate semantics is developed. It applies to an ordered set of sets of sentences (known as an <u>evolving theory</u>) and not just to a single set of sentences as other deductive systems do. Interleaved derivations are carried out in a finite number of worlds (trees) which interact. These interacting proofs could also be executed in parallel.

References

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