

# DETERMINING VELOCITY MAP BY 3-D ITERATIVE ESTIMATION

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## I INTRODUCTION

Velocity information is not only important for determining velocities and trajectories of objects but also important as a cue for image segmentation. It may be used to link adjacent but visually dissimilar surfaces or to divide surfaces not easily separable by static criteria alone[1].

A straightforward approach to obtaining velocity information is to find and track prominent features by a matching method from frame to frame[2,3]. These methods can estimate velocities of prominent points accurately; however, a problem of these methods is that velocities of only a sparsely sampled set of points can be obtained, which are not dense enough to be useful for tasks such as image segmentation.

Another approach uses the fact that for images of moving objects the spatial and temporal intensity changes are not independent of each other and satisfy a relation

$$V_x \cdot G_x + V_y \cdot G_y = -Dt \quad (1)$$

Here,  $(V_x, V_y)$  are the two components of the velocity vector,  $(G_x, G_y)$  represent the two components of the spatial intensity gradient, and  $Dt$  represents the gray value difference between consecutive frames, at a point  $(x, y)$ .

Since the single constraint of Eq. (1) does not allow one to determine both components  $(V_x, V_y)$  of the velocity, additional assumptions are necessary. Cafforio and Rocca[4] and Fennema and Thompson[5] assumed that the velocity is constant within a region corresponding to the image of a moving object, and applied a clustering approach to determine the dominant velocities in the scene. By this global method, they obtained accurate velocities of objects; however, with that assumption, the method is applicable only to objects which translate parallel to the image plane. Horn and Schunck[6] studied a more general assumption; namely that the velocity varies smoothly. This assumption is interesting because it constrains the variability of velocities between

neighboring points and allows one to estimate the velocity of each point from local computation. However, the method seems to become more efficient and reliable if we have some initial estimates of velocities.

There has been no attempt to try to combine or relate the matching method by which accurate estimation of velocities (although of only a sparsely sampled set of points) are obtained and the nonmatching method by which velocities of a dense set of points can be estimated, but additional constraints are required. In this communication, we propose an approach which uses the velocity estimates of the prominent feature points as reliable initial estimates of velocities and propagates them to other points using the constraint relation between the neighboring points. The velocities estimated by the above method based on the local data may have errors caused by noise. To reduce or eliminate these estimation errors, we propose an iterative scheme which corrects and modifies the estimation from the velocity values of the spatial and temporal neighborhood of a point.

## II VELOCITY ESTIMATION BY PROPAGATION FROM PROMINENT POINTS

Local computation of the velocity is, in general, not possible since Eq. (1) does not allow one to determine both components of the velocity. This will be clear from Fig.1 where (a) shows a part of an object in the previous frame and (b) shows the same part in the next frame. It is not possible to determine the velocity of the local area enclosed by the rectangle since it can correspond to many areas in the next frame. However, as shown in Fig.2, it is not difficult to establish correspondences between the prominent feature points. Once these have been established, it is not difficult to establish correspondences of other points because the correspondences of the feature points constrain possible correspondences of other points, provided we can assume that the object is rigid and thus velocities vary smoothly.

Based on the above idea, we study, in this correspondence, the propagation of the velocity estimates from the prominent feature points using the constraint relation between the neighboring points. As we also assume that the velocities vary smoothly, we follow Horn and Schunck[6] and the constraint relation can be written as

$$E2 = \text{SQRT}[(\bar{V}_x - V_x)^2 + (\bar{V}_y - V_y)^2] \implies \text{Minimum} \quad (2)$$

where  $(V_x, V_y)$  represents the velocity of a point  $(x, y)$  and  $(\bar{V}_x, \bar{V}_y)$  represents the average velocity of its neighboring points. For each point  $(x, y)$ , we have the relation (1). Because there are estimation errors caused by noise, we rewrite it as

$$E1 = V_x * G_x + V_y * G_y + Dt \implies \text{Minimum} \quad (3)$$

By minimizing the total error  $E^2 = a^2 * E1^2 + E2^2$  ( $a$ : constant), we can derive the following two equations[6]:

$$V_x = \bar{V}_x - G_x * (G_x * \bar{V}_x + G_y * \bar{V}_y + Dt) / (a^2 + G_x^2 + G_y^2)$$

$$V_y = \bar{V}_y - G_y * (G_x * \bar{V}_x + G_y * \bar{V}_y + Dt) / (a^2 + G_x^2 + G_y^2) \quad (4)$$

Velocity estimation are propagated from the prominent points in the image using Eq. (4) as follows.

1. Estimate the velocities of the neighboring points of each prominent point in the image by Eq. 4, in this case, for  $(\bar{V}_x, \bar{V}_y)$  use  $(V_x, V_y)$  of the prominent point.
2. Estimate the velocities of the neighboring points of those whose velocities have been estimated at the previous propagation. In this case,  $(\bar{V}_x, \bar{V}_y)$  is calculated from a 5\*5 neighborhood of those points whose velocities have been estimated in the previous estimation.
3. Iterate the propagation until velocities of all the points in the image have been estimated.

#### 1.1 ESTIMATING THE VELOCITY IN FRAME SEQUENCES

Let us define the image sequence as  $I(x, y, t), t=1, \dots, N$ . Then we can obtain a sequence of velocity maps

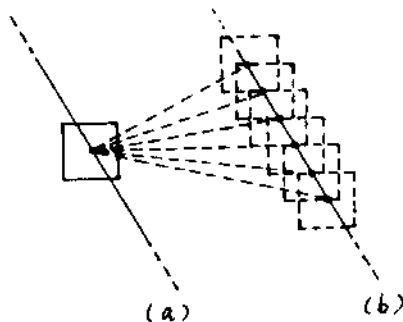


Fig.1 Local computation of the velocity is, in general, not possible.

$V(x, y, t), t=2, \dots, N$  by the method described in the previous section, such as  $V(x, y, t)$  from the two images  $I(x, y, t-1)$  and  $I(x, y, t)$ . However, when we have estimated the velocity  $V(x, y, 2)$  from the first two frames  $I(x, y, 1)$  and  $I(x, y, 2)$ , it is more efficient to predict velocity distribution in the next frame,  $V(x, y, 3)$ , from the velocity map  $V(x, y, 2)$ , and use the prediction map  $V(x, y, 3)$  in order to estimate the velocity of the next frame[7], if we can assume that velocities vary smoothly between consecutive frames[6]. This constraint between neighboring points in consecutive frames can be represented by EQ. (2) where, in this case  $(\bar{V}_x, \bar{V}_y)$  is the average of velocities of neighboring points in the prediction map and  $(V_x, V_y)$  is the velocity of the current frame to be estimated at the point  $(x, y)$ . Therefore we can estimate the velocity  $V(x, y, t)$  of the  $t$ -th frame from the velocity  $V(x, y, t-1)$  of the  $(t-1)$ -th frame using Eq. (4). This is iterated until the  $N$ -th frame has been estimated.

#### IV ITERATIVE CORRECTION OF VELOCITY ESTIMATION BY 3-D NEIGHBORS

We have estimated velocities  $V(x, y, t), t=2, \dots, N$ , of each point for the sequence of images. These velocities, however, have been estimated by local computation, and therefore may have estimation errors caused by noise. To eliminate or reduce these estimation errors, we propose an iterative method which corrects and modifies the estimation from the velocity values of spatial and temporal neighborhood of a point. We assume here that velocities vary smoothly between neighboring points and between consecutive frames. Then, Eq. (2) holds for the spatial and temporal neighbors. Then, we correct the estimation of a point  $(x, y)$  in the  $t$ -th frame by the average velocity  $(\bar{V}_x, \bar{V}_y)$  of neighboring points (5\*5 points) in the  $(t-1)$ th,  $t$ -th and  $(t+1)$ th frames (in total 5\*5\*3 points), using

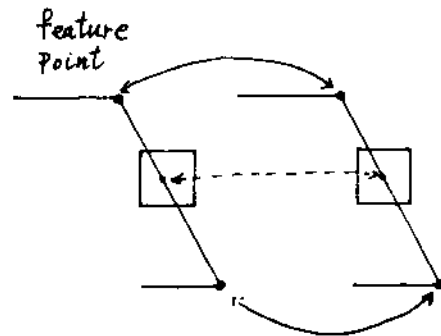


Fig.2 Correspondences of the feature points constrains possible correspondences of other points.

Eq. (4). We obtain the newly estimated velocities at every point for every frame,  $t^2, \dots, N$ . This newly estimated sequence of velocity maps,  $V(x, y, t), t^2, \dots, N$ , is again used as input for the next iteration. This process is iterated until major changes do not occur in the sequence of velocity maps.

Fig.3 shows some examples of the results.

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#### REFERENCES

- [1] J. W.D.Thompson, "Combining Motion and Contrast for Segmentation." IEEE-PAMI-2, No.6, pp.543-549, 1980.
- [2] H.P.Moravec, "Towards Automatic Visual Obstacle Avoidance." IJCAI-79, pp.598-600, 1979.
- [3] S.T.Barnard and W.B.Thompson, "Disparity Analysis of Images." IEEE-PAMI-2, pp.334-340, 1980.
- [4] C.Cafforio and F.Rocca, "Methods for Measuring Small Displacements of Television Images." IEEE-IT-22, pp.573-579, 1976.
- [5] C.L.Fennema and W.B.Thompson, "Velocity Determination in Scenes Containing Several Moving Objects." Computer Graphics and Image Processing, 9, pp.301-315, ~1979.
- [6] B.K.P.Horn and B.G.Schunck, "Determining Optical Flow." AI Memo-572, Artificial Intelligence Laboratory, MIT, Cambridge/MA, 1980.
- [7] M.Yachida, "M.Ikeda and S.Tsuji, Plan-Guided Analysis of Noisy Dynamic Images." IJCAI-79, pp.978-983, 1979.



(a)



(b)



(c)



(d)

Fig.3 Experimental results. (a) Car scene turning around a corner(4-th frame). (b) Close-up of initial estimates of the velocities by the propagation method(Second frame estimated from the first two frames). (c) Estimated velocity (6-th frame) from the velocity map of the 5-th frame. (d) Velocity map of the 6-th frame after 5-th iteration.