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### ABSTRACT

Many learning-by-example problems can be viewed as inferring structured non recursive procedures from sets of their traces knowing the basic functions and predicates, used to write them. The on-line system PAPE does this as follows, an example (data  $\diamond$  trace) is given. PAPE first checks whether the already learned procedure gives the same trace with the given input data. If not, the trace is transformed into a procedure body (in  $O(t^p)$  time, where  $t$  is the length of the trace, and,  $p$  an integer depending upon the procedure and the basic functions and predicates), and then merges this body with the actual one to get the new procedure body.

### 0/ INTRODUCTION

Many problems of learning from examples (e.g.: drawing objects on a screen [v 2], complex actions of an industrial robot, commands of a computer system, queries of a database...) can be viewed as inferring non-recursive procedures from sets of their traces knowing the functions and the predicates used to write them.

These applications suggest that we can restrict ourselves to non-recursive procedures written in PASCAL-like language in which :

- there is no composition of functions in an assignment, and, of a function and a predicate (of a loop, or, of a conditional).
- all constants are introduced by an assignment ( $x := \text{constant}$ ).
- the sets of identifiers of variables, and, of formal parameters of the procedures are disjoint.
- all procedures to synthesize are supposed structured.

### 1/ THE SYSTEM PAPE

We describe here a program named PAPE [V 1], which, like BIERMANN and KRISHNASWAMY'S program

AUTOPROGRAMMER [BiK], is an on-line system learning a non-recursive procedure (a structured one for PAPE) from a set of this traces, while SIKLOSSY and SYKES' program SYN [sis] infers a recursive procedure from a single trace. But, unlike AUTOPROGRAMMER which requires from its programmer symbolic traces (sequences of assignments) and, the predicates for loops and conditionals at the places they are used, PAPE, like SYN, works automatically once the set of traces and the basic functions and predicates have been given.

An example  $E$  for PAPE is a 4-tuples  $E = (N, D, R, T)$  where  $N$  is the name of the procedure to synthesize,  $D$  the input data (actual parameters),  $R$  the output data, and,  $T$  the trace (a set of pairs of a variable and its value) free of errors.

All examples are supposed correct.

PAPE is mainly composed of two functions  $A1$  and  $A2$ .

It works as follows :

An example  $E$  is proposed.

-If  $E$  is the first one, PAPE creates a procedure-head with identifier  $N$  and  $A1$  ( $E$ ) as a body.

-If not, PAPE starts with an execution of a call of  $N$  with  $D$  as actual parameter. If the trace obtained matches  $T$ , then PAPE stops (awaiting for some other example if any). Otherwise the partial body  $A1$  ( $E$ ) is compared and merged with the actual body of  $N$  by  $A2$ .

$A1$  has a worst-case time complexity in  $O(t^p)$  where  $t$  is the length of the trace  $T$ , and,  $p$  is an integer depending upon the procedure to synthesize, and, the sets of functions, and, of predicates.

In the following the behaviour of PAPE will be described on an example :

Suppose we wish to learn the procedure MULTIPLY which performs the multiplication of two positive integers using  $F$  - ( $\_ \diamond$  (addition),  $\text{pred}$  (predecessor function),  $/2$  (integral division by 2),  $* 2$  (multiplication by 2))  $J$  and  $P = \{= 0, \text{even } J\}$  respectively as sets of functions, and, of predicates.

We have in mind a procedure of the form :

Procedure MULTIPLY (M1, M2/N) ;

loc Z ;

X := M1; Y := M2; Z := 0 ;

while X = 0 do

if even X then X := X/2 ; Y := Y \* 2

else X := X-1 ; Z := Z + Y

fi

od

N:=Z

corp

## 2/ THE SYNTHESIS FUNCTION A1

A1 is composed of three functions. Their behaviours will be shown on an example :

$E_0 = (\text{MULTIPLY}, (18, 4), (N=72), ((X, 18)(Y, 4)(Z, 0)(X, 9)(Y, 8)(Z, 8)(X, 4)(Y, 16)(X, 2)(Y, 32)(X, 1)(Y, 64)(X, 0)(Z, 72)(N, 72)))$ .

- First A11 generates a set of symbolic traces together with assertions (conjunctions of true predicates) : each pair  $(X_j, a_j)$  of T is replaced by the set of all assignments  $X_j := F_m(X_1, \dots, X_n)$  when  $F_m(b_1, \dots, b_n) = a_j, b_i$  being the value of  $X_i$  before  $(X_j, a_j)$ .

If we call M1 and M2 the formal input parameters, we get :

$A1(E_0) = (X:=M1 \{ \text{even}(X) \} ; Y:=M2 \{ \text{even}(X) \wedge \text{even}(Y) \} ; Z:=0 \{ \text{even}(X) \wedge \text{even}(Y) \wedge Z=0 \} ; X:=X/2 \{ \text{even}(Y) \wedge Z=0 \} ; (Y:=Y+Y, Y:=Y*2, Y:=\text{pred } X) \{ \text{even}(Y) \wedge Z=0 \} ; (X:=X+Z, X:=\text{pred } X, X:=Y) \{ \text{even}(X) \wedge \text{even}(Y) \wedge Z=0 \} ; (Z:=Z+Y, Z:=Z+X, Z:=Y, Z:=X) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; (Y:=Y+Z, Y:=Y+Y, Y:=Z+Z, Y:=Y*2, Y:=Z*2) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; X:=X/2 \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; (Y:=Y+Y, Y:=Y*2) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; (X:=X/2, X:=\text{pred } X) \{ \text{even}(Y) \wedge \text{even}(Z) \} ; (Y:=Y+Y, Y:=Y*2) \{ \text{even}(X) \wedge \text{even}(Z) \} ; (X:=X/2, X:=\text{pred } X) \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; Z:=Z+Y \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; N:=Z \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} )$

This is a purely combinational process.

- Then A12 yields a single symbolic trace. This done by using :

a/ The "nice sequence" algorithm [V 1].

(A sequence is said "nice" if it has a partition in which each part has a common initial subpart and the remaining is either empty, or a nice sequence, or a sequence of patterns having at least one occurrence in some other part. This notion captures the syntactical properties of the symbolic traces of an iteration). The assertions being not modified we get :

$(X:=M1; Y:=M2; Z:=0; X:=X/2; (Y:=Y+Y, Y:=Y*2); X:=\text{pred } X; Z:=Z+Y; X:=X/2; (Y:=Y+Y, Y:=Y*2); X:=X/2; (Y:=Y+Y, Y:=Y*2); (X:=\text{pred } X, X:=X/2); (Y:=Y+Y, Y:=Y*2); X:=\text{pred } X; Z:=Z+Y; N:=Z)$

This algorithm does not decide when sequences are equally "nice". Heuristics are used to get a single trace.

b/ The "regularity" heuristic [V 1] (patterns are chosen to give either the longest, or, the more repeating subsequences) :  $X:=X/2$  is preferred to  $X:=\text{pred } X$  since  $X:=X/2$  is always followed by  $(Y:=Y+Y, Y:=Y*2)$ , and,  $X:=\text{pred } X$  by  $Z:=Z+Y$ .

c/ The "insane" heuristic [Sik, V 1] (the most general schema is chosen, e.g. : the one having the more occurrences of variables) :  $Y:=Y+Y$  is preferred to  $Y:=Y*2$ .

- Finally A13 introduces loops by first cutting A12(A1(E)) into the initial, the "nice" and the final sequences, and, retaining only assertion subparts :

$((X:=M1; Y:=M2; Z:=0) \{ \text{even}(X) \wedge \text{even}(Y) \wedge Z=0 \} ; ((X:=X/2; Y:=Y+Y) \{ \text{even}(Y) \wedge Z=0 \} ; (X:=\text{pred } X; Z:=Z+Y) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; ((X:=X/2; Y:=Y+Y) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; ((X:=X/2; Y:=Y+Y) \{ \text{even}(X) \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; ((X:=X/2; Y:=Y+Y) \{ \text{even}(Y) \wedge \text{even}(Z) \} ; (X:=\text{pred } X; Z:=Z+Y) \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; (N:=Z) \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} )$  then, loops are introduced, by using the "pumping lemma" heuristic [Mic, V 1] (an iteration  $S_0$  ; while do S od is inferred from occurrences of subsequences of the form  $S_0; S, \dots, S$ ) in two steps starting from the inside, the predicate being the largest common predicate included into all the assertion preceding the subparts and into the complement of the assertion following the last occurrence of this subpart :  $A1(E_0) = (X:=M1 ; y:=M2; Z:=0) \{ \text{even}(X) \wedge \text{even}(Y) \wedge Z=0 \} ; \text{while } \neg X=0 \text{ do } X:=X/2; Y:=Y+Y; \text{while } \neg \text{even}(X) \text{ do } X:=\text{pred } X; Z:=Z+Y \text{ od od } \{ X=0 \wedge \text{even}(Y) \wedge \text{even}(Z) \} ; N:=Z)$

## 3/ THE MERGING FUNCTION A2

A2 is composed of three functions, only A21 is fully worked out :

- First A21 looks to places where A1(E) differs from P, the actual body of N, and, makes a partition of both A1(E) and P between common and different parts, and, then infers a new body for N. There are three cases :

a/ They differ on a assertion, or, a predica-  
 te of a loop, or, of a conditional, A21 replaces it  
 P by their disjunction.

b/ They differ on a sequence of assertions and  
 instructions, starting by an assertion, A21 repla-  
 ces this part in P by a conditional with predicate  
 being the difference of the two assertions, and its  
 assertion being the largest common one. Suppose the  
 body P is :  $P=(X:=M1; Y:=M2; Z:=0; \{even(Y) \wedge Z=0\} ;$   
 $while \neg X=0 \text{ do } X:=pred X; Z:=Z+Y; while even(X) \text{ do }$   
 $X:=X/2; Y:=-Y \text{ od od } (X=0 \wedge even(Y) \wedge even(Z)) ;$   
 $N:=Z)$   
 This body P together with  $A1(E_0)$ , from above, yields:  
 $A21(P, A1(E_0))=(X:=M1; Y:=M2; Z:=0 \{even(Y) \wedge Z=0\} ;$   
 $if even(X) \text{ then } while \neg X=0 \text{ do } X:=X/2; Y:=-Y+Y;$   
 $while even(X) \text{ do } X:=pred x; Z:=Z+Y \text{ od od else while}$   
 $X=0 \text{ do } X:=pred X; Z:=Z+Y; while even(X) \text{ do } X:=X/2 ;$   
 $Y:=-Y \text{ od od fi } (X=0 \wedge even(Y) \wedge even(Z)) ; N:=Z)$

c/ They differ on a sequence of instructions  
 and assertions, not starting by an assertion : this  
 is a filtering error due to A12. This is taken care  
 by A22, a backtracking error recovery function (on  
 all examples looked after, we haven't met such an  
 error).

- Then A23 "normalizes" the body of N, by  
 doing a symbolic execution and another program infer-  
 ence to get a "better looking" body.

#### 4/ CONCLUSIONS

We can prove that the procedure M synthesized  
 by PAPE (with A 23) from a set of examples  $E_0, E_1, \dots$   
 has the following properties :

- For any example  $E_i$  of the set, the execution  
 of M with the input data  $D_i$  produces the trace  
 $T_i$  of  $E_i$ .

- Let N be the actual procedure to synthesize,  
 consider any infinite enumeration of the examples  
 of N,  $E_0, E_1, \dots$ , then there is a finite k such  
 that if M is synthesized from the set  $E_0, E_1, \dots, E_k$ .  
 Then for any  $i > k$ , M executed with  $D_i$  produces  $T_i$   
 of  $E_i$ . (But it is impossible to determine k).

So far only parts of PAPE have been implemen-  
 ted : A 11, A 12, A 13 and A 21. Many procedures  
 in arithmetics have being partially synthesized.  
 On the example given here, to get  $A 1(E_0)$  took 5.7.  
 seconds on an IBM 370/168 in LISP 1.5. The full  
 system described here is being implemented in  
 V-LISP on a DEC VAX 11/780.

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#### APPENDIX

The "nice sequence" algorithm, on  $A 11(E_0)$   
 above, behaves as follows :

- it first scans the sequence, ignoring the  
 assertions, looking for the first set of assignment,  
 i.e :  $(Y:=Y+Y, Y:=Y*2, Y:=pred X)$ .

then checks whether a subset of it has repea-  
 ting occurrences in the sequence, here  $(Y:=Y+Y,$   
 $Y:=Y*2)$ . This subset is setted apart. If it has  
 more than one element, it checks whether there is  
 a single repeating subsequence formed with either  
 the preceding, or the succeeding assignment, or  
 both, and so on, until nothing repeats, or, comes  
 up with this single subsequence.

- and then, does the same for the next set.

At the end, only setted apart subsets are  
 retained, and, repeating subsequences are paren-  
 thesized.