RESULTS ON TRANSLATING DEFAULTS TO CIRCUMSCRIPTION

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ABSTRACT

In this paper we define different concepts, of translating a set of defaults into circumscription The most important of these - modular translation requires that the additive changes of specific facts (not defaults) of default theory translate to the additive changes of the simple abnormality theory (corresponding to circumscription). We show that, surprisingly enough, the important class of defaults, normal defaults with prerequisites, do not have such a modular translation into circumscription (abnormality theory). We also establish the classes of defaults which are translatable into circumscription. For example we show that arbitrary combination (ordered combination) of seminormal defaults without prerequisites can be modularly translated into circumscription.

AREA: Automated reasoning. Common sense reasoning.

KEY WORDS: Nonmonotonic logic, default logic, circumscription, model theory, translation.

I Introduction

Three systems for nonmonotonic reasoning have been proposed so far (|1|, |2), (3|). Unfortunately little is known about their relative expressive power. It is clear that the logic of McDermott and Doyle is the most (may be over) powerful of the three. However, it is not clear at all what is the relationship between two most successful systems: default logic and circumscription. With the exception of the paper by Grosof |4| which dealt with translation of normal defaults into circumscription , very little has been done to explore this relationship..

Recently McCarthy |5| introduced the concept of generalised circumscription and simple abnormality theories. In this paper we would like to explore the problem of relative expressive power of default logic and generalised circumscription. We assume that the reader is familiar with the notion of generalised circumscription as presented in |6) and with the notion of simple abnormality theory as in (5|

Both systems have their advantages and disadvantages. The advantage of default logic lies in its conceptual simplicity (it is an easy specification language) and modular structure which is particularly useful in the presence of updates. The disadvantage is the lack of well developed reasoning methods. This is related to the fact that the language of default logic is not first order.

The situation is just the opposite as far as circumscription is concerned. Circumscription uses first order language (with possibly second order scheme) and hence inherits formal reasoning methods of first order logic. The representation of the problem in the form of circumscriptive scheme requires however more skill and is no longer so simple conceptually as the representation in default logic. The possibility of translation between two formalisms could help in combining their advantages; we could for example specify the system in the default logic making it conceptually simple and easy to update (modularity) and perform the reasoning using its circumscriptive translation.

A possible translation could also help in providing a semantical grounding for default theory. This is because circumscription, on the contrary to default logic, has a clear model theory, being a modification of the standard Tarskian semantics. Recently Lifschitz |6] showed that the new generalized circumscription (including the prioratized one) is in fact based still on the same semantic principle as the original circumscription - the concept of minimal models with respect to some relation of the partial order A possible translation from the default logic to circumscription could possibly lead to clear semantics for default logic based on the classical models instead of Kripke structures. Additionally, the semantics based on the notion of the partial order would be particularly attractive since it corresponds to our intuitions partial order captures the notion of <u>preference</u> implicitly present in the meaning usually associated with default formulas The semantics based on the partial order is also probably the simplest one which has the property of beeing global * on the contrary to the local semantics of the classical monotonic logic.

Finally, the description of the class of default theories which can be translated to circumscription would help to establish the limits of the expressive power of the semantics based on the partial order between the models. This is particularly important task since it could draw the line around generalised and prioritised circumscription.

In this paper we introduce the notion of translation between default theories and simple abnormality theories (circumscription). We prove a number of negative and positive results. We are particularly interested in the so called modular translations in which the introduction of the new specific fact does not require recomputation of the whole translation from the beginning. The addition of new defaults or so called general facts (laws) may certainly force us to do such a recomputation. Our definition of modularity requires "additivity" of changes <u>only</u> on the level of specific facts. In other words we are interested in translations of default theories with <u>fixed</u> set of general facts and defaults and varying set of specific facts.

We show that normal defaults with prerequisites do not enjoy the property of modular translatability into the circumscription. In fact we prove even a stronger result which is not restricted to "the current version" of circumscription. We prove that no mechanism based on the partial order relation (minimality vs this order) can capture normal defaults with prerequisites. This pessimistic fact is rather surprising, particularly in the context of the other, positive result saying that seminormal defaults without

•in a sense of McDertmott, the global semantics is the one in which the set of models of the set of formulas cannot be expressed as the intersection of the sett of models of the formulas of this set prerequisites can be translated into circumscription in the modular way. We also characterise the maximal class of default theories which have a modular translation into circumscription.

The paper is organized as follows In the second section we introduce the concept of order semantics and its relation to circumscription and default theories In the third second section the main results of the paper are presented.

II Basic notions

A. Order semantics

Without loss of generality we consider here only the propositional language. Let L denote such a language. The formulas of L are built from atomic propositions (usually denoted by letters X,Y) and usual propositional connectives. By M_L we denote the set of propositional truth assignments to the formulas of this language, i.e the set of functions from L to the set {T,F} of truth values. By m(A) where A is a sentence of L and m is the truth assignment we denote the truth value of A under m. We will call the truth assignment m the model of A iff m(A)-true. Let $\leq$ be the partial order relation (reflexive and transitive) defined over the set M_L. Let $W \cup \{\sigma\}$ be a subset of L. By $\models_{\leq}$ we will denote the semantic consequence relation defined as follows: $W \models_{\leq} \sigma$ iff is true in all models minimal with respect to < in the set of models of W

The order semantics has two important interpretations: one is related to circumscription, the other is more in a style of default logic It is not surprising that partial order relation provides a good semantical grounds for these two important nonmonotonic systems The notion of partial order between the models (truth assignments) corresponds closely to a concept of preference which is the important source of nonmonotonic behavior of the systems under consideration. The general semantics for nonmonotonic logic based on the notion of preference order will be elaborated in a forthcoming paper.

B. Generalised Circumscription and Order semantics

The generalised circumscription scheme |6), |5|, has the form:

$$A|X| \wedge \forall X'(A|X'| \supset \neg X' <|R| X)$$

where A is the formula, X and X' are the predicate (propositional) variables of the same type and $<|R|$ is the preorder defined between different predicate (propositional) variables of the same type. In this approach |6) all the predicates, functions and constants are treated as variables when predicate language is assumed. In our case, in the circumscription scheme, X and X' are propositional variables where X is identified with some atomic proposition of L and X' is additional propositional variable (metavariable really). In general, all the atomic propositions of L will be treated as propositional variables (not all of them, obviously will vary in the process of circumscription). Let us denote by Circum$(T|X|,X,R)$ the theory T together with the circumscription scheme. The above scheme with preorder relation was proposed to capture a new generalised version of circumscription proposed by McCarthy in [5]. Intuitively, the preorder relation R is intended to generalise the partial order relation (set theoretical inclusion of extensions of the predicates) corresponding to the original version of circumscription [1]. The scheme presented here shows that the new circumscription is essentially founded on the same principle - minimalisation with respect to the preorder R. The circumscription scheme expresses the fact that we are looking for the predicate X which satisfies A and is minimal with respect to the preorder relation R. It is easy to see, that having the order relation R defined between the predicate variables we can build the order relation $\leq$ between the models of the language under consideration in such a way that:

$$\text{Circum}(T|X|,X,R) \vdash_C \sigma \text{ iff } T \models_{\leq} \sigma$$

For example in the case of propositional language, if X is a propositional variable which is minimised in the circumscription, X' is the variable of the same type that X and Y is the set of propositional variables of the language L which are the parameters of the circumscription the corresponding order relation < can be defined as follows

$m1 \leq m2$ iff($m1(Y)=m2(Y)$ for every $Y \in Y$) and $(X|m1(X)) \leq |R| (X'|m2(X))$. Where (X|Y) denotes the result of the substitution of Y for X.

In case when X is the sequence of variables the substitution is defined componentwise.

We will further consider the so called simple abnormality theories |5). The language of these theories is extended by additional propositional variables (in his paper they are predicates, since he deals with the predicate language) called abnormality variables. These variables are the primary target of minimalisation in circumscription. In other words the corresponding order relation between the set of models is induced by the order between abnormality variables. Because of the lack of space here the reader is referred to |5| for a more detailed presentation. In the paper we will identify circumscription schemes with the partial order induced by them over the set of models . We will use more semantic oriented notation $(T,\leq)$ to denote the theory T and partial order relation induced by the circumscription scheme. In particular, when the theory is not circumscribed, the corresponding order relation will make all the models incomparable

C. Default logic and order semantics

We will use here a standard terminology introduced by Reiter [3) The defaults will therefore be denoted in a format of inference rules: $\frac{A:M(B)}{C}$ The formula A will be called the prerequisite of the default. The default will be called normal if B=C, seminormal if $B \supset C$ (|7]). By W we will denote the set of facts and by D the set of defaults of the default theory (W,D).

It turns out that the order relation between models has a direct and natural interpretations in terms of default statements. By E(m) let us denote the conjunction of all atomic propositions or their negations for which m is a model. This formula forms the finite representation of the set of all formulas which are true in the model m (we assume there is a finite number of propositional constants in the language).

On the beginning let us assume that the order relation $\leq$ has such a form that ml and m2 are the only models such that $m1<m2$ holds (strict inequality). All the other models are incomparable. We are going to "reconstruct" this order relation in terms of default and later generalize the same pattern of reasoning for the arbitrary order.

If ml<m2 we know that for any set W of formulas such that $m1 \in M(W)$ (M(W) is the set of models of W) we have:

$W \models_{\leq} \sigma$ iff σ is true in every model in $M(W)\setminus\{m2\}$.

The explanation of this is simple: since m2 cannot be minimal in the set of models of W (ml is there too), it can as well be eliminated. This can be equivalently expressed by the following default rule:

$$\frac{:M(E(m1))}{\neg E(m2)}$$

Indeed, this rule says that if E(ml) is consistent with W, which means that the model ml belongs to the set of models M(W), than -i E(m2) can be assumed The latter, in model theoretic interpretation means that the model m2 is eliminated from the set of models of W. This is, however precisely what the above, model theoretic interpretation of the partial order, says.

Therefore, more generally

$$W \models_{\leq} \sigma \text{ iff } (W, \frac{:M(E(m1))}{\neg E(m2)}) \vdash_D \sigma$$

where $\vdash_D$ denotes the syntactic consequence relation for default theories {i.e $(W;D)\vdash_D \sigma$ iff σ belongs to every extension of (W;D)) and $\leq$ is the specific partial order relation assumed above

Generally, with any partial order relation defined over the set of truth assignments of the propositional language we car associate the set of seminormal defaults without prerequisites in a sense that

$$(W;D(<))\vdash_D \sigma \text{ iff } W \models_{\leq} \sigma$$

Where $D(\leq)$ indicates that the set of defaults D depends on the order relation $<$. The proof is by a simple construction similar as above.

The default reconstruction of the partial order $\leq$ does not play an important role in this paper (at least in the first part) We presented it here to show that the notion of order can be used to construct semantics for defaults, at least for some classes of them. The construction above will also be helpful in understanding the proofs.

Later in the paper we will use the following simple lemma

Lemma 1: Let the order $\leq$ have the following property: For any model ml of the formula A there exist a model m2 such that $m2(\neg A)=true$ and $m2<m1$ than:

$\emptyset \models_{\leq} \neg A$, where $\emptyset$ denotes the empty set of formulas.

This follows from the above considerations about default reconstruction of partial order.

We will now formally define the notions of translation and representation.

III Main results

A. The concept of translation

Both default theories and simple abnormality theories have two components:

- specific facts
- general facts

General facts correspond roughly speaking to the time invariant properties of the real world which is modelled by a system under consideration and to some properties of the system itself. The general properties of the world have the form of general laws (like "all the ostriches are birds"), the defaults represent "general" attitude (preferences) of the reasoner Specific facts, on the other hand correspond to the particular instance of the world under consideration and may have the form "bird(joe)" or "ostrich(mary)" etc. The updates of specific facts and general facts have a different meaning - they occur on the different levels of abstraction McCarthy in his characterization of simple abnormality theories makes a sharp distinction between these two types of facts:

-the general facts involve usually abnormality variables

-the specific facts do not involve abnormality variables

We will make this distinction explicit and view a default theory as a triple $<W_f, W_r, D>$ where W_f is the set of facts, W_r is the set of general (monotonic) rules and finally D is the set of defaults. For the sake of conciseness of notation we will refer to both W and D by D (a set of defaults and general rules). Abnormality theories will be viewed simply as $<Ab_f, Ab_r, \leq>$ where the first two elements have the same meaning as in default theory and $\leq$ is the partial order corresponding to circumscription scheme.

One must be very careful when defining the notion of translation. First of all we have to decide - what are we going to translate?

By translation, we mean reformulating the <u>general facts</u> represented in default theory into <u>general facts</u> specified in abnormality theory. For example (see [5]) the translations of general facts to abnormality theory set may have the form $\neg ab(aspect1,x) \supset \neg flies(x)$ or $ostrich(x) \supset ab(aspect2,x)$ for the "bird" example. Intuitively, the abnormality predicates (propositions) reflect typical and atypical properties of objects under consideration. We insist that the specific facts remain unchanged since it is the case in simple abnormality theories (see [5]). Therefore we are interested in translating <u>specifications</u> not particular instances.

Formally,we say that the set of defaults D is <u>modularly translatable</u> or <u>representable</u> by the <u>order semantics</u> iff there exist a set of formulas Ab and the order relation $\leq$ such that for every set W of the of specific facts from the language L and for every formula *a* we have

$$(W,D)\vdash_D \sigma \text{ iff } W \cup Ab_r \models_{\leq} \sigma.$$

The set of formulas Ab_r is expressed in some extended language $L^* \supset L$, which results from L for example by simply adding propositional variable's ,like abnormality variables. The order $\leq$ is defined over the models of extended language L . In terms of circumscription this simply means that we can possibly minimise on abnormality predicates. In other words, the circumscription schema may involve additional variables (abnormality variables) which do not belong to L. Notice that we do not limit here in any way the number of additional variables introduced to the language L. In order to establish translatability it is sufficient to find any extension of L which would make a translation possible.

Modularity is the desirable properly of translation both from the conceptual and computational point of view This is the case simply because every time an update is made and a new specific fact is taken into account we do not have to "recompute" or change the existing content of our database (i.e incremental updates of specific facts in default theories are also incremental for abnormality ones) Besides, from the conceptual point of view it preserves the clear distinction between specific facts and general facts by translating "separately" (or modularly) specific and general facts Finally, the modular translation is the only case when we really can say that the translation of the set of defaults into circumscription exists, i.e there is a single reformulation of general facts ("time invariant" properties) expressed in the default theory in the abnormality theory.

We may certainly consider nonmodular translations (in fact as we will show in this paper, in many important cases we have to) in which the order $\leq$ is fixed (for the give D) but. the set Ab_r will depend on the set of specific facts W. In this case however each new specific fact added to the default theory will require a recomputation of the "old" translation. It is therefore hard to talk here about single translation of D, we rather get a family of translations A good example of such a nonmodular translation is provided in database theory. The translation of Closed World Assumption into the completed database [8] is nonmodular. Indeed, the addition of new atomic facts changes the completion axiom.

Generally speaking, it may be just as difficult to compute the nonmodular translation as just to compute the extensions of the default theory In such a situation translation does not make much sense. For instance, we could provide such an "expensive" translation, given the default theory $\frac{A:M(B)}{B}$ by constructing a translation to the ordinary propositional theory in the following way: if A is derivable from W and ~B is not than translate W to $W \cup \{B\}$ otherwise leave W unchanged. This is the kind of nonmodular translation which we certainly want to avoid.

Finally let us point out that if the set of defaults D is changed than obviously even the modular translation allows both changing the $\leq$ order relation and the set Ab_r. The same may happen if the new general rules are added. This is simply because we may then assume that this is not the instance of external world which has changed but rather specification of the system. We do not talk here about such changes restricting ourselves to the changes of the specific facts of the system.

B. Main Results - negative part

We are here interested in:

1. describing the class of default theories which can be modularly translated into abnormality theories.

2. Determining what is the advantage of abnormality propositions. In other words , we would like to establish whether there exists a class of defaults which are translatable when the abnormality variables are used, but are not translatable when we do not use these additional variables(predicates).

We present now our main negative result:

Theorem 2: The set of defaults $D \{ \frac{A:M(B)}{B} \}$ where B is independent of A (i e both $\{A, \neg B\}$ and {A,B} are consistent) is not modularly translatable into abnormality theory, no matter how many abnormality variables are introduced

Proof

Suppose there is such a translation Let Ab be the corresponding formula (Ab_r) and let $\leq$ be the resulting order relation Let us take W to be {A} or any set which implies A Without the loss of generality let us assume that both $\{A, \neg B\}$ and {A,B} are consistent. Than for any model ml such that $m1(A \wedge Ab \wedge \neg B) = true$ there must exist a model m2 such that $m2(A \wedge Ab \wedge B)$ is true and $m2 \leq m1$ Notice that $\{A, Ab, \neg B\}$ must be consistent. Indeed, otherwise the consistent theory {A,B} would be translated into the inconsistent one. Let now $W = \emptyset$. By lemma 1 and the definition of our order relation * we have $\emptyset \models_{\leq} Ab \supset (A \supset B)$. Hence, since Ab is true in every minimal model of < , we obtain simply by modus ponens, that

$$\emptyset \models_{\leq} A \supset B$$

This is however not the case in the original default theory: it is not true that $(\emptyset, D) \vdash_D (A \supset B)$. Since the order relation and the formula Ab were chosen arbitrarily, we have proved our theorem.

Notice here that if either {A,B} or {A, B} are inconsistent the situation is trivial, since the above default is totally redundant.

Let us now point out an important difference between the defaults $\frac{A:M(B)}{B}$ and $\frac{:M(A \supset B)}{A \supset B}$ The second default is easily translatable by circumscription, the first one as our theorem points out is not. In his paper Grosof [4] claimed that he has provided translation of the normal defaults with prerequisites (i.e. of the first type) into circumscription, while in fact his construction is correct only for the defaults of the second type. The important difference between these two types of defaults is best seen when we consider two default theories with defaults: $\frac{bird(x):M(fly(x))}{fly(x)}$ and $\frac{:M(bird(x) \supset fly(x))}{bird(x) \supset fly(x)}$ respectively and the same set of specific facts W={black(a)} . If we do not know whether "a" is a bird or is not we do not want to conclude that if "a" is a bird that it would fly, since"a" may turn out to be an atypical bird The first default would not give this answer $(i.e(Bird(a) \supset fly(a))$, while the second one would

The above result is definitely a pessimistic one, specially because normal defaults with prerequisites constitute the most important class of defaults. The result indicates that it is possible only to construct nonmodular translations. This in turn faces a danger that the overall cost of translation in terms of both the size of the abnormality theory and the time required to compute it may be higher than deduction using the original default theory. In fact the large size (the number of formulas) of abnormality theories corresponding even to simple defaults (see the papers |6], [5]) reinforces this apprehension.

Notice also that our result ran be formulated even in the stronger way as long as circumscription is based on the minimalization versus some arbitrary order, the modular translation of normal defaults with prerequisites is impossible. This is because, as we demonstrated, no partial order between the models can capture normal defaults with prerequisites.

C. Main Results - positive part

We will now characterise those defaults which are translatable into simple abnormality theories. We will also address the second of our previous questions describing the class of defaults whose translatability require the presence of additional abnormality variables, i.e. the defaults which otherwise are not translatable. Therefore, the answer for our second question about the usefulness of additional abnormality variables is positive.

Lemma 3: Seminormal and normal defaults without prerequisites are modularly translatable into circumscription even without abnormality variables.

Proof

Let $\frac{:M(B \wedge A)}{A}$ be a seminormal default without prerequisite We can show that if we define the order relation $\leq$ between the models as $m1 \leq m2$ iff $m1(B \wedge A)$=true and $m2(\neg A)$=true or if $m_1 = m_2$ than we have for every σ, W :

$$(W, \frac{:M(B \wedge A)}{A}) \vdash_D \sigma \text{ iff } W \models_{\leq} \sigma$$

where the partial order is defined on the truth assignments to the language L without introducing additional abnormality variables. Notice also that in this case $Ab_r = \emptyset$.

The translation obtained here is the direct consequence of the correspondence between defaults and partial order as described in the second section. We cannot apply this mechanism however for the arbitrary defaults which are not necessarilly seminormal. Indeed the default $\frac{:M(B)}{D}$ such that $B \wedge \neg D$ is not false would get us rather the order equivalent to the default $\frac{:M(B)}{B \vee D}$ This follows directly from the construction of the order relation as in section 2 or as in the lemma 2. It is interesting to observe that seminormal defaults are "special" from the point of view of the order semantics and that seminormaiity is in fact critical to translatability into the order semantics.

In general it can be showed that arbitrary combination (and also ordered combination) of seminormal defaults can be translated modularly into circumscription.

By the <u>multiple</u> default we mean the default of the form $\frac{:M(B1),...,M(Bn)}{C}$ where $n>1$. If n=1 we will talk about <u>singular</u> defaults. Now let us present the result related to the usefulness of additional abnormality variables.

Lemma 4: Multiple seminormal defaults are not modularly translatable into circumscription unless abnormality variables are introduced.

Proof:

Let us consider a seminormal default D of the form $\frac{:M(B1 \wedge A), M(B2 \wedge A)}{A}$ This default is not representable by any order relation $\leq$ if it is to be defined only on the truth assignments for the language L without additional variables ab_i. We will demonstrate this by showing that if this default is to be representable by the order $\leq$, this order relation would also represent the other defaults $\frac{:M(B1 \wedge A)}{A}$ and $\frac{:M(B2 \wedge A)}{A}$

Let $W=\{\neg B1 \vee \neg B2\}$. The order $\leq$ must be defined in such away that for any model m such that $m((\neg B1 \vee \neg B2) \wedge \neg A)$ is true there must exist a model m' such that $m' \leq m$ and $m'((\neg B1) \vee \neg B2) \wedge A)$ is true This must be the case since A must be implied. But then if we take $W=\{B1 \wedge \neg B2\}$ we will get $W \models_{\leq} A$ contrary to the definition of the default D. Therefore no order defined solely on truth assignments to L can help. In what sense, therefore, the extension of the language L by the new variables can help? To show this let us extend the the language L by the new propositional variable p. Let us now define the order relation on the structures of the extended language in the following way: For every model m1 such that $m1(\neg A \wedge p)$=true and every model m2 such that $m2(B1 \wedge A)$=true put $m2 \leq m1$ For every model m3 such that $m3(\neg A \wedge \neg p)$=true and every model m4 such that $m4(B2 \wedge A)$=true put $m4 \leq m3$ Make all the other models incomparable. It is easy to see that the order $\leq$ represents the default D.

Indeed for $W=\{\neg B1 \vee \neg B2\}$ the models of $B1 \wedge A$ and the models of $B2 \wedge A$ are the minimal elements of M(W).

Therefore:

$$W \models_{\leq} A$$

On the other hand if $W=\{\neg B1, B2\}$ then there are minimal models in M(W) in which $\neg A$ is true (the models of $\neg A \wedge p$). This blocks the inference of A. A similar situation arises when $W=\{B1 \wedge \neg B2\}$. Therefore the extension of the language L by a single propositional variable p allows to represent the default which otherwise would not be representable. This is achieved through "splitting" of the models of $\neg A$ into two classes: models of $\neg A \wedge p$ and models of $\neg A \wedge \neg p$. The variable p has the same character as abnormality variables - it is irrelevant by itself, it only "helps" to define the order relation properly.

One may argue that multiple defaults are not practically applicable. We do not discuss this point here. We have only established the technical result which shows the essential difference between multiple and singular defaults.

We will discuss now the problems of representation of the various combination of the set of defaults. We will introduce here the notion of <u>inference rule</u> as the generalisation of monotonic (modus ponens, generalisation, resolution etc) and nonmonotonic (defaults) inference rules.

<u>Definition</u> 1:

Let L be a propositional language. By the inference rule r over L we mean an arbitrary binary relation defined over $\mathcal{P}(L) \times L$ where $\mathcal{P}(L)$ is the power set of L. If $<W,\sigma> \in r$ then we will say that σ is derivable from W by a single application of the inference rule r. We will say that the rule r is <u>applicable</u> to the set W if there exist the tuple $<W,\sigma> \in r$ for some σ not from W. We will assume that $<W,\sigma> \in r$ for any $\sigma \in W$. By r(W) we will denote the following set $\{\varsigma: <W,\varsigma> \in r\}$. If r is inapplicable to W we will put $r(W)=W$. We will define the following operations on inference rules which derive new inference rules from the given ones:

1. Intersection. Denoted by $r_1 \cap r_2$. Intersection corresponds to a separate, independent application of the rules r_1 and r_2 and selecting only these formulas which could be obtained independently by the first and by the second inference rule.

2. Composition. $<W,\sigma> \in r_1.r_2$ iff $\sigma \in r_1.r_2(W)$. Composition of inference rules corresponds to priority (in this case of the second rule over the first one)

Our definition of inference rules is very general. We would like now to put some restrictions on them. First we assume that our rules are idempotent in a generalized sense that is $r_1.r_2.r_1(W=r_2.r_1(W)$ if $r_1(W) \neq W)$. In this way we will treat a standard first order syntactical consequence relation as single inference rule r (than of course we have $r.r(W)=r(W)$. To allow multiple extensions we do not assume in general that our inference rules have Church - Rosser property. We will now introduce the third operation of parallel composition, which under the above idempotence assumptions is definable in terms of the previously defined operations.

<u>Definition</u> 2.

By the parallel composition of the inference rules r_1 and r_2 we will mean the following operation:

$$r_1 | r_2 = r_2.r_1.r_2 \cap r_1.r_2.r_1$$

Parallel composition corresponds to the intersection of the set of formulas obtained by applying r_1 and r_2 in the different order. The reason for using "triple" composition is that the application of the rule r_1 may make the application of r_2 possible when it was not the case before. The second application of r_1 cannot however change anything according to our principle, therefore three compositions are enough.

Introduction of these formalisms will help in establishing the class of general inference rules which are representable by the order semantics. In a way therefore we will be able to determine the limits of this semantics. Besides that the formalism enables us to be more general and not to restrict ourselves to any particular semantics associated, for example, with combination of defaults (if they are our basic inference rules). For example for the same set of defaults a number of different definitions of the corresponding combination can be provided. Compare for example proposals of Reiter [S] and Lukaszewicz [9] or the different possible definitions of extensions of the default theory in which $W = \{A \vee B\}$ and $D = \{\frac{A:M(C)}{C}, \frac{B:M(D)}{D}\}$ where $C \vee D$ may (logic of McDermott and Doyle) or may not be implied (Reiter's default theory). We do not want in our study to comit ourselves to any of these possible semantics of the combination of defaults, since there is no agreement about them. Instead, we rather want to have a formal apparatus to express and study the problem of translatability for all these alternative definitions of combinations of defaults. Besides we can also define the ordered (prioratised) combination of defaults through the operation of composition applied to the default rules r_1 and r_2-

Now let us define what does it mean for the general inference rule to be represented by the partial order between the structures of L.

<u>Definition</u> 5

The inference rule r over L is representable (translatable) by the partial order $\leq$ defined over the set of truth assignments of some extended language L iff there exist a formula Ab such that

for every W and σ from L $<W,\sigma> \in r$ iff $W \cup \{Ab\} \models_{\leq} \sigma$

Notice that if r is the syntactical consequence of propositional (predicate) logic than the corresponding partial order relation is trivial. It is defined over truth assignments of L in such a way that any two different assignments are incomparable. This definition is a generalization of the previous one in a sense that we can now talk about various combinations of defaults. Let us now present the main positive results of this section:

Theorem 5: If r_1 and r_2 are representable by the partial order so are $r_1 r_2$ and $r_1 \backslash r_2$-

The proof follows by the construction of the order relation corresponding to $r_1 \backslash r_2$) from the order relations corresponding to r_1 and r_2.

The operations of composition and parallel composition of the inference rules are important - they correspond to the (ordered and unordered) composition of the set of defaults. We have here as the direct consequence of the theorem 2 and preceding lemmas the following result

Theorem 6: Arbitrary combination (ordered and unordered) of the set of seminormal defaults without prerequisites is representable by the circumscription.

The combination of defaults which we have in mind can have both the semantics in the Lukasiewicz and Reiter sense. In fact showing the representability for Lukasiewicz semantics is easier - we can apply directly theorem 2 (for parallel combination of the inference rules). To show it for Reiter's definition of extension requires transformation of the set of defaults to the equivalent set for which the parallel composition can be used. The main reason for why we have to do this is that in Reiter's definition of extension it is not sufficient to apply applicable defaults - the consequents of defaults should not contradict the premises of the other defaults.

In fact we have even the stronger result - the complete characterization of the expressive power of the order semantics in terms of default theories.

Theorem 7: The default theory is modularly translatable to circumscription (simple abnormality theory) iff it is equivalent to the combination (ordered, unordered) of the seminormal defaults without prerequisites

Two sets of defaults are considered equivalent here if they lead to the same sets of conclusions for the same set of facts

In one direction the theorem is the direct consequence of the previous one (soundness) Completeness follows from the ''reconstruction" of the set of defaults from the partial order relation as described in the first section The above result provides therefore a precise limit to the expressive power of the order semantics, if we restrict ourselves to the classical models

IV Conclusions

It is important to establish the formal criteria of translatability between different nonmonotonic systems. There are many potential advantages, inter alia, the possibility of determination of limits of the systems under consideration. In this paper we have provided such a criterion for two important nonmonotonic systems and we have established a number of positive and negative results The negative result obtained here has rather disappointing consequences for a perspective of "clean" modular translations from the default theories to circumscription as long as circumscription is based on one fixed partial order relation induced over the set of models. In fact we proved that there is no partial order between the models which could capture the important class of normal defaults with prerequisites. One way out is to make a circumscription "conditional" by changing the circumscription axiom to the conditional formula "if C then <Circumscription axiom > In this way instead of one single order relation we would have rather a family of order relations (possible semantic orders), and depending on the theory which is going to be circumscribed the particular circumscription axiom would be selected (or in other words the particular order relation would be chosen) This will lead however to even more complex proof theory and may be not worth the effort overall

The order semantics considered here is interesting by itself as an intuitive and attractive formally candidate for semantics of nonmonotonic logic. It remains to be investigated what are the limits of this semantics especially if w reconsider the order semantics also for nonclassical models like for example Kripke structures. As the semantical tool order semantics can be used to establish various equivalence results For example one may be interested in determining whether one set of defaults is equivalent to another We may also be interested in a more restricted question whether two sets of defaults are equivalent modulo particular sublanguage, i.e. the set of conclusions if restricted to this sublanguage are the same. The other question which can be successfully answered on the basis of order semantics is the characterization of the class of formulas with respect to which the given system is monotone.

These topics and the extension of the order semantics to capture more expressive power are the subject of a forthcoming paper.

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References

[1] McCarthy,J "Circumscription - a form of nonmonotonic reasoning" *AJ* 13 1 (1980) 27-39

[2] McDerrnott,D. and Doyle,J "Nonmonotonic logic 1." *AI* 13.1 (1980) 41-72

|S| Reiter,R "A logic for default reasoning." *AJ. 13*.I (1980) 81-132

|4| Grosof.Benjamin "Default reasoning as circumscription " In *Workshop in Nonmonotonic Logic.* AAA1, Oct, 1984.

|5| McCarthy,John "Application of circumscription to formalize common sense knowledge " In *Workshop in Nonmonotonic Logic.* AAAI, Oct, 1984

|6) Lifschitz,Vladimir "Some results on circumscription " In *Workshop in Nonmonotonic Logic* AAAI, Oct, 1984

|7] Reiter,R. and Criscuolo,G. "On interacting defaults" In *International Joint Conference on Artificial Intelligence* AAAI, 1981, 270-276.

|8| Reiter, R , "On closed world databases." In *Logic and databases* Plenum Press, 1978, .

|9] Lukaszewicz,Witold "Considerations on default logic." In *Workshop in Nonmonotonic Logic* AAAI, Oct, 1984