TWO RESULTS ON DEFAULT LOGIC

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ABSTRACT

We focus on default logic, a formalism introduced by Reiter to model default reasoning. The paper consists of two parts. In the first one a translation method of non-normal defaults into the normal ones is given. Although not generally valid, this translation seems to work for a wide class of defaults. In the second part a semantics for normal default theories is given and the completeness theorem is proved.

INTRODUCTION

[Reiter, 1980] introduces a formalism modelling default reasoning. An example of such a reasoning is the inference rule stating "Typically children have parents". In default logic this rule is represented as the following default

child(x) : M has-parents(x) /has-parents(x)

Formally, a default is any expression of the form $\alpha(\bar{x}) : M\beta(\bar{x})/w(\bar{x})$, where $\alpha(\bar{x})$, $\beta(\bar{x})$, $w(\bar{x})$ are all first-order formulas whose free variables are among those of $\bar{x}=(x1,...,xn)$. $\alpha(\bar{x})$ is called the prerequisite, $\beta(\bar{x})$ the justification, and $w(\bar{x})$ the consequent of the default. This default is interpreted as "for all individuals x1,...,xn, if $\alpha(\bar{x})$ is believed and $\beta(\bar{x})$ is consistent with our beliefs, then $w(\bar{x})$ can be believed. If $\alpha(\bar{x})$, $\beta(\bar{x})$ and $w(\bar{x})$ are all closed formulas, the default is said to be closed, otherwise it is open.

In default logic the knowledge about a world is represented as a default theory, i.e., a pair (W,D), where W is a set of first-order axioms and D is a set of defaults. Defaults extend the knowledge contained in axioms. Such an extension provides an acceptable set of beliefs about a world being modelled (see [Reiter, 1980] for details).

To assure the existence of extensions Reiter limits himself to normal theories, i.e., theories all of whose defaults are of the form $\alpha(\bar{x}) : M\beta(\bar{x})/\beta(\bar{x})$. Because the class of normal theories has turned out to be insufficient for practical applications, in [Reiter, Criscuolo, 1981] the more general class of semi-normal theories, i.e., theories all of whose defaults are of the form $\alpha(\bar{x}) : M(\beta(\bar{x})\wedge w(\bar{x}))/w(\bar{x})$, has been introduced. Although these theories seem to cover all practical applications, they need not have extensions.

In [Lukaszewicz, 1984] an alternative default logic, coinciding with that of Reiter for normal theories, has been specified. Although each theory has an extension in our approach, non-normal theories should be avoided if possible. The reason is that they are computationally more complex than the normal ones.

This paper consists of two parts. In the first one we give a translation of non-normal defaults into the normal ones. Although not generally valid, this translation seems to work for a wide class of defaults. In the second part a semantics for normal default theories is given.

TRANSLATION

The translation we propose is very simple. The first step is to replace any default $d=\alpha:M\beta/w$ by the semi-normal default $d1=\alpha:M(\beta\wedge w)/w$. Intuitively, this transformation seems to be incontroversial. The only distinction between d and d1 arises from the different applicability criteria for them. Because the criterion of applying d1 is stronger than that of d, any agent including d into his knowledge base, who does not accept to replace it by d1, considers the applicability criterion for d1 too strong. Thus he is prepared to apply d when the application of d1 is explicitely blocked. In other words, he considers as possible to apply d when the formula β is consistent with his beliefs, while the formula $w\wedge\beta$ is not. But in such a case, applying d contradicts its justification. This means that the agent is irrational. Thus any reasonable agent including d into his knowledge base should accept to replace it by d1.

The second step of our translation is to replace the semi-normal default $d1=\alpha:M(\beta\wedge w)/w$ by the normal default $d2=\alpha:M(\beta\wedge w)/\beta\wedge w$. This transformation is more controversial. Before giving its applicability criteria let us start by observing that it is often the case of "Typically if α then β". This observation underlay the method of replacing semi-normal theories by the normal ones in [Reiter, Criscuolo, 1981]. Consider for example the default stating "Typically adults are employed, except when they are high-school dropouts". Formally

$$d1 = adult(x) : M(\neg dropout(x) \wedge employed(x)) / employed(x)$$

Because "Typically adults are not high-school dropouts", we clearly can replace d1 by the normal default

$$d2 = adult(x) : M(\neg dropout(x) \wedge employed(x)) / \neg dropout(x) \wedge employed(x)$$

In [Reiter, Criscuolo, 1981] d1 is replaced by the pair of normal defaults

$$adult(x) : M \neg dropout(x) / \neg dropout(x)$$

$$adult(x) \wedge \neg dropout(x) : M\ employed(x) / employed(x)$$

If all we know about John is that he is an adult, both normal representations of d1 lead to the conclusion that John is employed not high-school dropout. The difference arises when we additionally know that John is unemployed. The representation of Reiter and Criscuolo forces John not to be a high-school dropout, while ours remains agnostic on this point.

Even if "Typically if α then β" does not hold, our normal representation works if the following weaker condition is satisfied: "Typically if α and w then β". To illustrate this point let us consider the default stating "Typically adults are married, except when they are 21 year olds". Formally

$$d1 = adult(x) : M(married(x) \wedge \neg 21\text{-}old(x)) / married(x)$$

If all we believe about John is that he is an adult, we do not assume that he is not 21 years old. But if we additionally believe him to be married, the conclusion that John is not 21 years old seems to be plausible. In other words, we accept the statement "Typically married adults are not 21 year olds". Note that applying d1 for John we start to believe that he is married. Thus we can plausibly conclude that John is not 21 years old. Exactly the same effect is achieved when we replace d1 by the normal default

$$d2 = adult(x) : M(married(x) \wedge \neg 21\text{-}old(x)) / married(x) \wedge \neg 21\text{-}old(x)$$

All the above discussion can be summarized as follows. If $d = \alpha : M\beta / w$ is any default, then it is always reasonable to replace it by the semi-normal default $d1 = \alpha : M(\beta \wedge w) / w$. If it is the case that "Typically if α and w then β", d1 can be further replaced by the normal default

$$d2 = \alpha : M(\beta \wedge w) / \beta \wedge w.$$

SEMANTICS FOR NORMAL DEFAULT THEORIES

We limit ourselves to closed theories, i.e., theories all of whose defaults are closed. Using a technique given in [Reiter, 1980] a generalization to open theories is straightforward.

The idea is to view defaults of a theory (W,D) as restricting the models for W in such a way that

(1) Any restricted set of models for W is the set of all models for some extension for (W,D).

(2) If E is any extension for (W,D) then there is some such restricted set of models for W which is the set of all models for E.

Some preliminary terminology. Let X be a set of first-order models. We say that a formula α is <u>X-valid</u> (<u>X-satisfiable</u>) iff α is true in all models (in some model) of X. We say that a closed normal default $\alpha : Mw/w$ is <u>X-applicable</u> iff α is X-valid and w is X-satisfiable.

We begin by observing that each closed normal default can naturally be regarded as a mapping from sets of models into sets of models. Formally, if X is a set of models and $d = \alpha : Mw/w$ is a closed normal default, then the set d(X) is given by

$$d(X) = \begin{cases} X - \{N \in X : N \models \neg w\} & \text{if d is X-applicable;} \\ X & \text{otherwise} \end{cases}$$

Intuitively, X-valid and d(X)-valid formulas can be interpreted as sets of beliefs before and after the application of the default d, respectively.

Imagine an agent reasoning on the basis of some closed normal default theory A=(W,D). His initial set of beliefs should be identified with Th(W) or, from the semantic perspective, with the set of all models for W. At each step he chooses a default and tries to apply it to the <u>current</u> set of his models X. The new set of the agent's models is d(X). The new set of his beliefs is the set of all X-valid formulas. Assume that the agent is able to repeat this process infinitely. It can happen, perhaps after applying infinitely many defaults, that the set of the agent's current models is <u>stable</u>, i.e. , for each $d \in D$, $d(X) = X$. As we shall see, each stable set of models resulting from such a process characterizes some extension for A. Moreover, each extension for A is characterized by some such stable set of models.

The formal details are these. Let $\langle d_i \rangle$ be a sequence of defaults, and suppose that X is a set of models. By $\langle di \rangle X$

we denote the set of models given by

$$\langle d_i\rangle X = \begin{cases} X & \text{if } \langle d_i\rangle \text{ is the empty sequence} \\ \bigcap_0^n X_i, \text{ where } X_0=X,\ X_i=d_i(X_{i-1}) & \text{if } \langle d_i\rangle = d_1,\ldots,d_n \\ \bigcap_0^\infty X_i, \text{ where } X_0=X,\ X_i=d_i(X_{i-1}) & \text{if } \langle d_i\rangle = d_1,d_2,\ldots \end{cases}$$

Note that if $\langle d_i\rangle = d_1,\ldots,d_n$, then $\langle d_i\rangle X = d_n(d_{n-1}(\ldots(d_1(X))\ldots))$.

Let A=(W,D) be a closed normal theory. Let X be the set of all models for W. We say that a set of models Y is stable with respect to A iff

(S1) $Y=\langle d_i\rangle X$ for some sequence $\langle d_i\rangle$ such that each $d_i \in D$.

(S2) For each $d\in D$, $d(Y)=Y$.

Theorem 1: Let A be a closed normal default theory, and suppose that a set of models Y is stable with respect to A. Then Y is the set of all models for some extension for A.

Proof (outline): $Y=\langle d_i\rangle X$ for some $\langle d_i\rangle$, where X is the set of all models for W. We can assume that $\langle d_i\rangle$ is infinite (otherwise, i.e., if $\langle d_i\rangle = d_1,\ldots,d_n$, define the infinite $\langle d_i'\rangle$ by $d_i'=d_i$ for $i=1,\ldots,n$, $d_i'=d_1$ for $i>n$).

It follows that $Y=\bigcap_0^\infty X_i$, where the sets X_i were defined earlier. Let F_i ($i\geq 0$) be the set of all X_i-valid formulas. Assume that $d_i=\alpha_i : Mw_i/w_i$. It is easily verified that

(1) $F_0=Th(W)$.

(2) $F_{i+1}=$ if $\alpha_i\in F_i$ and $\neg w_i \notin F_i$ then $Th(F_i \cup w_i)$ else F_i.

(3) $E=\bigcup_0^\infty F_i$.

It is easily checked that Y is the set of all models for E. Thus it remains to prove that E is an extension for A. Define

$E_0=W$

$E_{i+1}=Th(E_i)\cup\{w : (\alpha:Mw/w)\in D \text{ and } \alpha\in E_i \text{ and } \neg w\notin E\}$.

In view of Theorem 2.1 [Reiter, 1980] it is sufficient to show that $E=\bigcup E_i$. By induction on i, it is easily proved that for each $i\geq 0$ we have

(4) $E_i \subseteq E$ and (5) $F_i\subseteq \bigcup E_i$

From (3), (4), (5) we immediately have $E=\bigcup E_i$ what completes the proof of the theorem.

Theorem 2: Let E be an extension for a closed normal default theory A=(W,D), and suppose that X is the set of all models for W, Y is the set of all models for E. Then Y is stable with respect to A.

Proof (outline): Define

$GD=\{(\alpha:Mw/w)\in D : \alpha\in E,\ \neg w\notin E\}$,
$CONSEQUENTS(D)=\{w : (\alpha:Mw/w)\in D\}$.

In view of Theorem 2.5 [Reiter, 1980] we have $E=Th(W\cup CONSEQUENTS(GD))$. Consider two cases.

(1) $GD=\{\}$. Take the empty sequence of defaults. It is clear that $\langle\rangle X=Y$, and $d(Y)=Y$ for each $d\in D$. Thus Y is stable.

(2) $GD\neq\{\}$. Let $d1,\ldots,dn$ ($d1,d2,\ldots$) be any sequence of all elements of GD. Define $d1',\ldots,dn'$ ($d1',d2',\ldots$) by

$d1'=dj$ where j is the smallest integer such that dj is X-applicable

Given $d1',\ldots,di'$
$di+1'=dj$ where j is the smallest integer such that

(i) dj is $\langle d1',\ldots,di'\rangle X$-applicable
(ii) for each $1\leq k\leq i$, $dj\neq dk'$

It is readily verified that $\langle di'\rangle$ is well defined, and that the elements of $\langle di'\rangle$ are those of $\langle di\rangle$. Because for each $d\in D$ we clearly have $d(Y)=Y$, to complete the proof of the theorem it remains to show that $\langle di'\rangle X=Y$. It is easily proved that $\langle d1',\ldots di'\rangle X$ is the set of all models for $Th(W\cup\{w1',\ldots,wi'\})$ where wj' is the consequent of dj'. It follows therefore that $\langle di'\rangle X$ is the set of all models for $Th(W\cup CONSEQUENTS(GD))$. Thus $\langle di'\rangle X=Y$.

This completes the proof of Theorem 2.

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