

LEARNING ABOUT MOMENTUM CONSERVATION

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ABSTRACT

A system has been designed in which *explanation-based learning* is applied to classical mechanics. An overview of the fully-implemented system and an example run by it are presented. From the example the system acquires a general formula describing how the momentum of a collection of objects changes over time. The derivation of this formula is prompted by the analysis of a human's solution to a simple two-body, one-dimensional collision. The human's solution is based on the principle of momentum conservation. The acquired formula serves to solve a three-dimensional collision in the presence of external forces, a situation where momentum is *not* conserved.

I INTRODUCTION

Explanation-based learning [DeJong81] is a knowledge acquisition method that utilizes deep domain models. This type of learning involves generalizing a single problem solution into a form that can be later used to solve conceptually similar problems. The generalization process is driven by the *explanation* of why the solution worked. It is the deep knowledge about the domain that allows the explanation to be developed and then extended. This approach to learning has much in common with [Mitchell85] [Silver83] and [Winston83]. See [DeJong85] for a full discussion.

A system has been designed and implemented in which explanation-based learning is applied to classical mechanics. Initially this system is capable of performing many of the mathematical manipulations expected of a college freshman and is provided a representation of Newton's laws. Physical concepts taught in a first semester physics course are to be acquired; hence the name of the system, Physics 101. Newton's laws - which constitute the initial domain model - suffice to solve all problems in classical mechanics, but the general principles that are consequences of Newton's laws are interesting for their elegance as well as their ability to greatly simplify the solution process. Since the system's physical knowledge rests on the strong foundation of Newton's laws, only its mathematical sophistication will limit the physical concepts it can acquire.

System Overview

The operation of the system can be seen in figure 1. After a physical situation is described and a problem is posed, Physics 101 attempts to solve the problem, reporting its solution if one can be found. When its knowledge is not adequate to solve a problem, the system requests a solution from the human user. It then verifies the correctness of the answer (relative to its current knowledge level). The human-provided solution need not fully specify the solution steps; some "reading between the lines" is possible. If the system cannot verify the human's solution, or if an erroneous step was detected, more details are requested. Once a satisfactory answer to the posed problem is obtained, the system determines why the

human's solution worked and attempts to extend the solution technique. Any resulting generalizations are recorded and can be used to solve future problems.

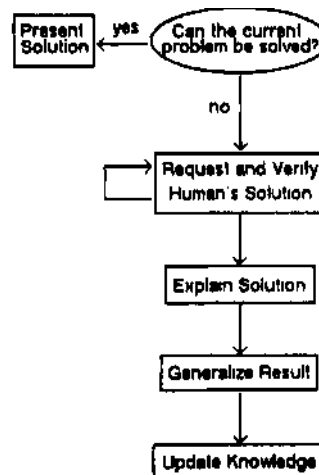


Figure 1. The Operation of Physics 101

Section II contains an example of this process. Conservation of momentum is used (by a human) to solve a one-dimensional, two-body collision problem that the system could not solve. The human's solution is verified and then generalized, during which the system discovers that momentum is only conserved in the absence of any net external force. A general equation describing how external forces affect momentum is derived by the system. A second problem, which involves three bodies under the influence of an external force, has been solved by Physics 101 using this generalized result

Explaining Human Solutions

The system classifies human solution steps into one of four categories

- (1) A known formula could have been used; $force = mass \times acceleration$ is an example of this type.
- (2) New variables can be defined in order to shorten later expressions. A formula such as $momentum = mass \times velocity$ would fall into this class.
- (3) Equations can be algebraic variants of previous steps. The replacement of variables by their values also falls into this category.
- (4) The user can specify an equation that states a relationship among known variables, yet the system knows of no algebraically equivalent formula. These steps require full justification, which the system does by using its abilities to symbolically reason with calculus. Only the equations falling in this category are candidates for generalization.

Representing Physical Situations

Physical situations are represented as *worlds*, which comprise a number of *objects*. Objects have four measurable attributes; mass, position, velocity, and acceleration. All but mass are time-dependent, three-dimensional vectors. The relationships among these attributes, with respect to symbolic differentiation and integration, are known to the system.

Newton's second and third laws are also known to the system. (Newton's first law is a special case of his second law.) His second law says that an object's acceleration is determined by the net force acting on the object, divided by its mass. The net force can be decomposed into two components; the external forces (due to sources outside the physical system), and the internal forces (due to each of the other objects within the system). These inter-object forces are constrained by Newton's third law, which says that every action has an equal and opposite reaction.

When solving physics problems it often proves useful to represent the instantaneous state of a world at two separate times (eg., the initial and final states). For this reason all variables in the world can have known values at two specific times, as well as a value (which may also be unknown) for all time. A constraint system [Shavlik84] maintains the known algebraic relationships among these values. Several of these values - for object attributes or any of the forces - are set by the user to characterize a physical situation.

II ILLUSTRATIVE EXAMPLE

In the one-dimensional problem shown in figure 2, there are two objects moving in free space, without the influence of any external forces. (Nothing is known about the forces between the two objects. Besides their mutual gravitational attraction, there could be a long-range electrical interaction and a very complicated interaction during the collision.) In the initial state (state A) the first object is moving toward the second, which is stationary. Some time later (state B) the first object is recoiling from the resulting collision. The task is to determine the velocity of the second object after the collision.

As there is insufficient information for the constraint sub-system to determine this velocity, Physics 101 attempts to determine an expression for object two's velocity as an explicit function of time. The resulting expression could then be evaluated in state B. Progress past the last line in figure 3 can not be made though, as the time-dependence of the inter-object force is not known. (All of the calculus in figures 3 and 5 is generated by Physics 101. Several minor intermediate expressions have been excised, for the sake of brevity.)

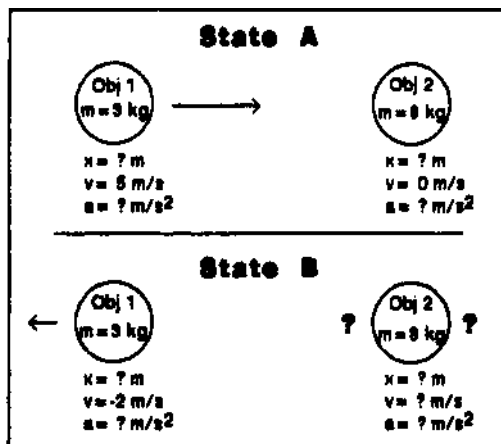


Figure 2. A Two-Body Collision Problem

$$\begin{aligned}
 \text{velocity}_{obj2,x}(t) &= \int \text{acceleration}_{obj2,x}(t) dt \\
 &= \int (\text{force}_{net,obj2,x}(t) / \text{mass}_{obj2}) dt \\
 &= 1 / \text{mass}_{obj2} \int \text{force}_{net,obj2,x}(t) dt \\
 &= 1 / \text{mass}_{obj2} \int (\text{force}_{external,obj2,x}(t) + \text{force}_{internal,obj2,x}(t)) dt \\
 &= 1 / \text{mass}_{obj2} \int (\text{force}_{external,obj2,x}(t) + \text{force}_{obj1,obj2,x}(t)) dt \\
 &= 1 / \text{mass}_{obj2} \int (\text{force}_{external,obj2,x}(t) - \text{force}_{obj2,obj1,x}(t)) dt \\
 &= - 1 / \text{mass}_{obj2} \int \text{force}_{obj2,obj1,x}(t) dt
 \end{aligned}$$

Figure 3. The Computer's Failed Solution Attempt

At this point the system requests a solution from the user. The solution provided can be seen in figure 4. Without explicitly stating it, the user took advantage of the principle of conservation of momentum, as the momentum of the world at two different times was equated. After that, various algebraic manipulations lead to the answer. In order to accept the answer, the system has to verify each of the steps in this solution.

The last three steps are easily verified, as they are simple algebraic manipulations. The hard part is determining a justification for the first equation in figure 4. Physics 101 must ascertain that this equation is consistent with its physical knowledge. Since the two sides of the equation only differ as to the state in which they are evaluated, an attempt is made to determine a time-dependent expression describing the general form of one side of the equation. Figure 5 presents the calculus involved in determining how the general expression explicitly depends on time. This calculation can go to completion because the unknown force that was an obstacle in figure 3 is now cancelled by the other inter-object force. Even though the values of these two forces are unknown, Newton's third law constrains them so that they cancel. Since the expression is constant, its value in any two states can be equated. The initial step has been validated.

Next, the system attempts to generalize the initial equation in the human's solution. In the initial equation of figure 4, four variables are used to determine the value of object two's velocity. The system ascertains that the first object's velocity produces the inter-object force that cancels the force that had been an obstacle in figure 3. Physics 101 also determines that the two mass terms are needed for this additive cancel to occur. Now that the system has explained why the human's attempt worked, it can generalize the technique to situations with more than two objects. Any object in a world may exert a force on any other object. So to cancel the total inter-object force in the general case, a *mass x velocity* contribution must come from every object in the world. Physics 101 produces the left-hand side of equation 1, and then uses its knowledge of physics and calculus to arrive at the right*

$$\begin{aligned}
 &\text{mass}_{obj1,stateA} \text{velocity}_{obj1,stateA,x} \\
 &+ \text{mass}_{obj2,stateA} \text{velocity}_{obj2,stateA,x} \\
 &= \text{mass}_{obj1,stateB} \text{velocity}_{obj1,stateB,x} \\
 &+ \text{mass}_{obj2,stateB} \text{velocity}_{obj2,stateB,x} \\
 3 \text{ kg } 5 \text{ m / s } &= 3 \text{ kg } -2 \text{ m / s } + 8 \text{ kg } \text{velocity}_{obj2,stateB,x} \\
 15 \text{ kg m / s } &= -6 \text{ kg m / s } + 8 \text{ kg } \text{velocity}_{obj2,stateB,x} \\
 \text{velocity}_{obj2,stateB,x} &= 2.63 \text{ m / s}
 \end{aligned}$$

Figure 4. The Human's Solution

$$\begin{aligned}
& \text{mass}_{\text{obj1}} \text{ velocity}_{\text{obj1},x}(t) + \text{mass}_{\text{obj2}} \text{ velocity}_{\text{obj2},x}(t) \\
&= \text{mass}_{\text{obj1}} \int \text{acceleration}_{\text{obj1},x}(t) dt + \text{mass}_{\text{obj2}} \int \text{acceleration}_{\text{obj2},x}(t) dt \\
&= \text{mass}_{\text{obj1}} \int (\text{force}_{\text{net,obj1},x}(t) / \text{mass}_{\text{obj1}}) dt + \text{mass}_{\text{obj2}} \int (\text{force}_{\text{net,obj2},x}(t) / \text{mass}_{\text{obj2}}) dt \\
&= \int \text{force}_{\text{net,obj1},x}(t) dt + \int \text{force}_{\text{net,obj2},x}(t) dt \\
&= \int (\text{force}_{\text{external,obj1},x}(t) + \text{force}_{\text{internal,obj1},x}(t)) dt + \int (\text{force}_{\text{external,obj2},x}(t) + \text{force}_{\text{internal,obj2},x}(t)) dt \\
&= \int (\text{force}_{\text{external,obj1},x}(t) + \text{force}_{\text{obj2,obj1},x}(t)) dt + \int (\text{force}_{\text{external,obj2},x}(t) + \text{force}_{\text{obj1,obj2},x}(t)) dt \\
&= \int (\text{force}_{\text{external,obj1},x}(t) - \text{force}_{\text{obj1,obj2},x}(t)) dt + \int (\text{force}_{\text{external,obj2},x}(t) + \text{force}_{\text{obj1,obj2},x}(t)) dt \\
&= \int (\text{force}_{\text{external,obj1},x}(t) + \text{force}_{\text{external,obj2},x}(t) + \text{force}_{\text{obj1,obj2},x}(t) - \text{force}_{\text{obj1,obj2},x}(t)) dt \\
&= \int (\text{force}_{\text{external,obj1},x}(t) + \text{force}_{\text{external,obj2},x}(t)) dt \\
&= \int (0 \text{ kg m / s}^2 + 0 \text{ kg m / s}^2) dt \\
&= \text{constant, kg m / s}
\end{aligned}$$

Figure 5. Verifying the First Step in the Human Solution

hand side. In the general case, all the internal forces cancel but the external ones remain.

$$\sum_{i=1}^N \text{mass}_i \text{ velocity}_i = \int \sum_{i=1}^N \text{force}_{\text{external},i} dt \quad (1)$$

This formula says *The total momentum of the objects in a world is determined by the integral of the sum of the external forces on those objects.*

III CONCLUSION

By analyzing a worked example, Physics 101 is able to derive a formula describing the temporal evolution of the momentum of any arbitrary collection of objects. This formula can be used to solve a collection of complicated collision problems.

Others [Forbus83] [Langley81] [Larkin81] have investigated the learning of physical concepts. Larkin's ABLE system learns how to algebraically apply principles of physics. Langley's work involves the analysis of multiple data measurements and does not utilize deep knowledge about the physical world. Forbus and Gentner consider learning *qualitative* physics by analogical reasoning. Unlike any of these systems, Physics 101 reasons with calculus to explore the implications of Newton's laws.

There are several possible long-term applications of this research. From the viewpoint of the development of expert problem solvers, the techniques outlined here could be used to "train" programs in domains that are based on the selection and manipulation of formulae. Their analogs of Newton's laws could be encoded and then sample solutions to various problems could be presented, much in the way technical textbooks are organized. This system should also prove applicable to cognitive modelling and intelligent computer aided instruction, as it is able to recognize incorrect applications of physical principles (e.g., assuming momentum is conserved in the presence of an external gravitational field).

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TRAINING AND TRACKING IN ROBOTICS*

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ABSTRACT

We explore the use of learning schemes in training and adapting performance on simple coordination tasks. The tasks are 1-D pole balancing. Several programs incorporating learning have already achieved this [1, S, 8]: the problem is to move a cart along a short piece of track to at to keep a pole balanced on its end; the pole is hinged to the cart at its bottom, and the cart is moved either to the left or to the right by a force of constant magnitude. The form of the task considered here, after (3), involves a genuinely difficult credit-assignment problem. We use a learning scheme previously developed and analysed [1, 7] to achieve performance through reinforcement, and extend it to include changing and new requirements. For example, the length or mast of the pole can change, the bias of the force, its strength, and so on; and the system can be tasked to avoid certain regions altogether. In this way we explore the learning system's ability to adapt to changes and to profit from a selected training sequence, both of which are of obvious utility in practical robotics applications.

The results described here were obtained using a computer simulation of the pole-balancing problem. A movie will be shown of the performance of the system under the various requirements and tasks.

1 INTRODUCTION

The importance of good training experience it well recognised in pattern classification and inductive inference, where careful choice of rule exemplars and counter-exemplars clearly affects learning progress (e.g., ref. [9]). It is also important for learning in symbolic problem solving as illustrated by the problem generation component of LEX [5]. Here we show that similar pedagogic care can be significant in non-symbolic problem solving of the kind that is important in robotics; namely, problems of learning to control physical dynamical systems.

It is sometimes faster for a system to learn to solve a different problem from the one assigned and then to adapt that solution once it it learned. And it is usually far faster for a system to adapt to new requirements by modifying old solutions than for it to start again from scratch. In the case of learning to control a physical system, it may be much easier, for example, to first learn to control a related system with simpler dynamics, and then to continue to learn at that system is deformed, continuously or by a sequence of steps, into the system required. This is not an approach that lends itself to orthodox control theory due to the difficulty of adjusting to unforeseen changes in dynamics and task requirements.

A related issue is that the ability of a system to adjust to unforeseen changes in circumstances and requirements is important even if it

does not lead to increased learning speed through training. Although an obvious role of learning is to construct a knowledge base through experience in domains where there is little a priori knowledge, another role that receives less attention is to track a moving optimum, either incrementally or non-incrementally, as unforeseen changes take place.

Here we show how both training and tracking can be done with a learning system that was described by Barto, Sutton, and Anderson [1]. Our domain is the classic problem of balancing a pole in one dimension. This problem has the advantage that it has been considered by several researchers [3, 8], and it can be made quite difficult by adopting certain assumptions. Specifically:

A rigid pole is hinged to a cart, which is free to move within the limits of a 1-D track. The learning system attempts to keep the pole balanced and the cart within its limits by applying a force of fixed magnitude to the cart, either to the left or to the right.

This task is susceptible to a number of different learning schemes depending on the quality of the evaluative feedback provided to the learning system. Here we consider a version of the task similar to the one studied by Michie and Chambers [3] in which the only evaluative feedback occurs on failure—hitting the stops with the cart, or having the pole fall over. This sparsity of evaluative feedback creates a genuinely difficult credit-assignment problem. Since a failure usually occurs only after a long sequence of individual control decisions, it is difficult to determine which decisions were responsible for it. To solve this problem, the system identifies certain regions as being in some sense "close" to failure, and will try to avoid them as well. The back-propagation method for generating this internal evaluation is like Samuel's method (6), but refined and improved by Sutton [7].

The learning system usually starts with an empty experiential knowledge base, and it takes some time to fill it with enough data for the system to perform well. We first show that to learn a new task—for example, one with very different parameters, like a vastly heavier pole—can be easier when the system adjusts its knowledge base from a successful state than from scratch. While it is obvious that a suitably primed knowledge base ought to facilitate learning compared to "tabula rata" learning, our point is that the initial knowledge can be that acquired from learning to solve an easier task, which may itself have been learned at a result of facing an even easier task. Other results show this explicitly.

We also consider tasks that add a constraint, such as avoiding particular pole positions. In these tasks, the learning system must adapt to the constraint without explicit instruction as to how to do it. Systems capable of discovering "how" to accomplish a goal, specified only in terms of "what", are more powerful than those that require

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more explicit specifications. Such "operationalization" capabilities are shown by systems like ours that modify behavior without performing adaptive model reference system identification, as is usual in adaptive control theoretic techniques. That is, what is used here is a "learning by discovery" method that owes more to AI than to conventional adaptive control.

II TECHNICAL APPROACH AND EXPERIMENTAL RESULTS

Although the state space of the cart-pole system is continuous—that is, the state variables are continuous—we divide the four of them discretely, as in [3]:

x : 3 ranges [the position of the cart]
 θ : 6 ranges [the angular position of the pole]
 \dot{x} : 3 ranges [$\dot{x} = dx/dt$]
 $\dot{\theta}$: 3 ranges,

which yield together 162 regions corresponding to the combinations. The system is always in just one region. The job of the learning system, then, is to assign the proper action to each region, so that the system will act correctly. The learning algorithm is given in the appendix and is discussed in detail in refs. [1] and [7].

The cart-pole system was simulated on a VAX-11/780, using the equations of motion given in [2] and the following initial parameter values:

cart mass: 1 kg
track length: 4.8 m
pole mass: 0.1 kg
pole length: 1 m
pole stops: $+12^\circ$, -12°
applied force: 10 newtons, left or right.

There are two small coefficients of friction, one for the cart and the other for the pole. In the initial state the pole is stationary and upright, and the cart is stationary in the middle of the track. Since there is always a force, the pole will not long stay upright. The time from initial position to failure—the cart or pole hitting a stop—is considered one learning trial. Time was discretised to a fiftieth of a second. The system was judged to have succeeded in balancing the pole when it achieved a trial that continued without failure for 10,000 time steps, corresponding to some 3.3 minutes real time. We used the number of failures before reaching this criteria of success as some measure of the power and efficiency of learning. A run using the standard parameters listed above typically might have 70 trials before a criterion trial is achieved.

In the first set of tasks the system was required to adapt to changes in the parameters of the cart-pole system. One task required adaptation to bias in the force, which changed from +10 and -10 newtons to +12 and -8, and to +8 and -12. This can be considered an (inaccurate) approximation to tilting the track right and left. That problem was solvable: the control surface learned was specific to the direction of the tilt, and a system trained for one tilt did not work well for another. However, after several switches of the direction of tilt, with training to criterion for each, the system was eventually able to balance the pole each way without failure. That is, the system had generalised its solution to satisfy all the problems simultaneously.

The second task required adaptation to an increase in the mass of the pole. The initial training used the standard .1kg pole to the criterion of 10,000 steps without failure. Then the mass of the pole

was increased one order of magnitude (to 1kg). This made the problem much harder, and performance dropped accordingly. However, criterion performance was soon regained: initial training took an average of 68 failures; subsequent adaptation to the heavier pole took only 20. On the other hand, without pre-training, learning to balance the heavier pole took an average of 86 failures. It is clearly more efficient to use a previous solution as the starting place for this new task than to start from scratch.

The system finds it harder to learn to handle shorter poles. If we started training with a pole reduced in length and mass to two-thirds of the original 1m/.1kg pole, it took 119 failures on average to reach criterion, compared to 67 with the full-size pole. When the system was first trained to criterion on the full-size pole and then switched to the short pole, however, only 6 additional failures were incurred on average after the switch, for a total of 73 failures overall. This result shows not only adaptability to changing requirements, but also improvement in learning rate with "directed" training.

Directed training resulted in a larger advantage in switching to a shorter track. With the track length 2 meters (instead of 4.8m), learning took more than 250 failures on the average. But training first at 4.8m, then at 4m, 3m, and finally at 2m (each to criterion) took merely $64 + 4 + 9 + 5 = 82$ failures.

The final task required learning to avoid some region of state space; namely, the region in which the pole is near vertical ($\pm 1^\circ$). We added a penalty, equal to 1/10 the penalty for failure, for being within this region. Despite the fact that remaining in this state is the "natural" way to balance the pole, the learning system reduced the proportion of time spent there from 22% to 8%. Increasing the penalty reduced the fraction still further, but then hitting the stops became more attractive relatively, and balancing failures occurred more often.

III ANALYSIS AND DISCUSSION

The training illustrated in these results consists of selecting a sequence of control tasks that ends with the required task and that presents a graded series of difficulties. Since this kind of sequence consists of entire problem-solving tasks, rather than exemplars and counter-exemplars of a pattern class, it is quite different from the usual training sequence in supervised learning pattern classification or concept formation. The potential utility of this type of training seems clear; these results show that the learning system considered here is in fact able to benefit from it. The learning system treated here is, moreover, a simple one; it does not deal with hierarchical control at all, nor does it take advantage of a specific accessible knowledge base.

Part of the utility of the training in our examples is due to the fact the external evaluative feedback is in the form of a binary signal—failure or not. Such systems are always subject to what has been termed the "mesa phenomenon" [4]. What is needed is a more continuous feedback so that the system can become better and better, rather than merely satisfy some set of binary constraints. One observes this constantly in people performing physical tasks; it would obviously be desirable in robotics as well. One mechanism used by the learning system to provide a more continuous evaluation is the part of the algorithm that constructs an internal evaluation signal. Training provides another way of reducing the effect of the mesa phenomenon by effectively leading the system to better regions of the solution space. Significantly, this can be done by manipulating aspects of the task without using detailed knowledge of what the performance surface is

like.

Another way to ease the problems caused by the mesa phenomenon is to provide our robotics systems not with just one purpose, but with a structure of purposes: the first purpose to be satisfied is to fulfill the constraints and avoid outright failure—that is, to keep the pole upright and to avoid allowing the cart to crash into the ends of the track. Next the system might try to keep the pole upright, by reducing the average magnitude of δ ; after that, it might try to minimize the angular velocity, so as to keep the pole still. Beyond that, we can imagine a universal purpose of wanting to do all the above with minimum work, just as people's physical efforts become more and more efficient with practice.

Related to the issue of training, and equally important, is the ability of the learning system to track a changing optimum for the control parameters. In robotics, there are practical reasons for this: the dynamics change with time, sizes and viscosities change with temperature, bearings and surfaces suffer wear, and so on. Such problems are clearly not feasible to solve analytically, and in any case, the point is to provide systems whose behavior is robust enough to handle unexpected as well as expected situations.

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APPENDIX I: The Learning Algorithm

The action of applying a positive force at time t is denoted $y_t = 1$; that of a negative force is denoted $y_t = 0$. The probability π_t that $y_t = 1$ is

$$\pi_t = \sigma(w_t \cdot x_t) = \frac{1}{1 + e^{-w_t \cdot x_t}},$$

where x_t is the received feature vector (indicating the current region of the state space) and w_t is a modifiable parameter vector the same length as x_t . The probability that $y_t = 0$ is $1 - \pi_t$. $\sigma(\cdot)$ is the sigmoidal logistic distribution with the properties that $\sigma(0) = .5$, $\sigma(\infty) = 1$, and $\sigma(-\infty) = 0$. Initially $w_0 = 0$, so that $\pi_0 = \sigma(0) = .5$. w_t is updated according to

$$w_{t+1} = w_t + \alpha \hat{r}_{t+1} e_t,$$

where α is a positive learning-rate parameter, and e_t is a modifiable vector, initially zero, and updated:

$$e_t = \lambda_w e_{t-1} + (1 - \lambda_w)(y_t - 1/2)x_t.$$

λ_w , $0 \leq \lambda_w < 1$, is a trace-decay parameter.

\hat{r}_t is an internal reinforcement signal generated from the external reinforcement signal, sensory input, and past experience. It is defined by

$$\hat{r}_{t+1} = r_{t+1} + \gamma v_t \cdot x_{t+1} - v_t \cdot x_t,$$

where r_{t+1} is the external reinforcement at time step $t+1$, $0 \leq \gamma < 1$ is a scalar discount-rate parameter, and v_t is a vector recursively generated by

$$v_{t+1} = v_t + \beta \hat{r}_{t+1} x_t, \quad v_0 = 0.$$

β is a second positive learning-rate parameter, and

$$x_t = \lambda_v x_{t-1} + (1 - \lambda_v)x_t,$$

where $0 \leq \lambda_v < 1$ is a second trace-decay parameter. See [1, 7] for discussion of these equations. All the results reported here were obtained using the values $\alpha = 1000$, $\beta = 0.5$, $\gamma = 0.95$, $\lambda_w = 0.9$, and $\lambda_v = 0.8$.

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