# SPECULAR STEREO

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ABSTRACT

A glossy highlight, viewed stereoscopically, can provide information about surface shape. For example, highlights appear to lie behind convex surfaces but in front of concave ones.

A highlight is a distorted, reflected image of a light source. A ray equation is developed to predict the stereo disparities generated when a point source of light is reflected in a smooth, curved surface. This equation can be inverted to infer surface curvature properties from observed stereo disparities of the highlight. To obtain full information about surface curvature in the neighbourhood of the highlight, stereo with two different baselines or stereo with motion parallax is required

The same ray equation can also be used to predict the monocular appearance of a distributed source. A circular source, for instance, may produce an elliptical specular patch in an image, and the dimensions of the ellipse help to determine surface shape.

## 1 INTRODUCTION

When the reflectance of a surface has a specular as well as a diffuse component, the viewer may see highlights. Highlights can give extra information about surface shape Ikeuchi [8] uses photometric stereo with specular surfaces to determine surface orientation Beck [I] notes that stereo vision might be able to perceive highlights on a convex surface as lying beneath the surface Grirnson [7] incorporated lambertian and specular components of reflectance into stereo But he found that the computation of surface orientation could be numerically unstable.

Here a computation is proposed that is less ambitious than Crimson's, in that it. attempts to determine only local surface geometry, at specular points But it avoids relying on precise assumptions a b o u t surface reflectance

Instead, the only assumption is that a specularity can be detected in an image, and its position measured For instance a method like that of Ulltnan [ 15] could be used Thereafter specularities are matched in the same way as features in conventional stereo [6,9.10]. The disparity of a stereo-matched specular point is then compared with the disparity of any nearby surface features

The basic principle of the surface shape estimation relies on the properties of curved mirrors (fig 1 ).

To interpret, specular stereo, both horizontal and vertical disparities are used. Ideally, three non-collinear eyes are needed to obtain full information about local curvature. Alternatively, parallax from a known vertical motion of a viewer, combined with conventional stereo geometry, is just as good If only a static, stereo view is available then this still yields partial information This could be combined

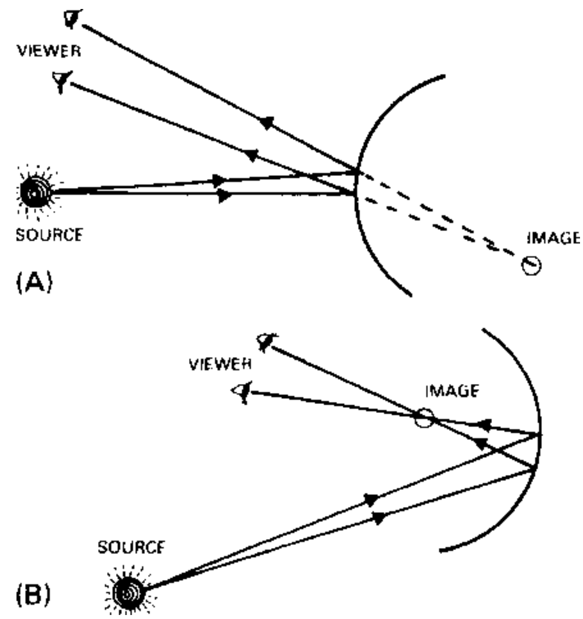


*Figure* 1: *Viewing geometry. In a convex mirror* (a) *the image of a distant point source appears behind the mirror surface In a concave one* (6) *the image may appear in front Study of the ordinary domestic soup spoon should confirm this*

with a *priori* knowledge or measurements from other sources (stereo, shape-from shading or specular reflection of a distributed source) to fully determine local surface shape

Finally, observe that the path of a light ray from source to viewer can be reversed Analysis developed to show the effect of moving the viewer also serves for movement, of the source The resulting equation is used to predict the appearance in an image of a distributed source under specular reflection in a curved surface. For instance a circular source generally produces an elliptical specularity in the image. The orientation and length of its major and minor axes, in principle, determine local surface shape.

## 2 IMAGING EQUATIONS

Equations are given to describe the process of formation of images of specular reflections Details of derivations are given in [3] These predict the dependence of observed stereo disparities on surface and viewing geometry Certain assumptions about the geometries are made, for the sake of mathematical simplicity Then the equations are inverted so that, given viewing geometry and disparities, local surface geometry can be inferred

### 2.1 Viewing, surface and reflection geometry

The stereo viewing geometry is shown in fig 2.

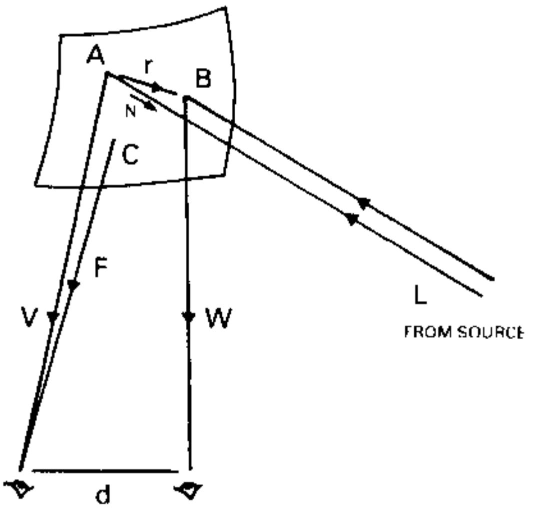


*Figure 2: Stereo viewing geometry. Illumination comes from a distant point source, in direction $\boldsymbol{L}$. Rays to left and right eyes lie along vectors $\boldsymbol{V},\boldsymbol{W}$, and strike the surface at points $A,B$ respectively. Surface normals at $A,B$ are $\boldsymbol{N},\boldsymbol{N}'$ respectively. The vector from $A$ to $B$ is $\boldsymbol{r}$, and the baseline lies along vector $\boldsymbol{d}$. A surface feature is assumed to be present nearby, at $C$, with position vector $\boldsymbol{F}$ relative to the left eye.*

It is assumed that the curved surface is locally well approximated by terms up to 2nd order in a Taylor series (see eq (4)). The vectors $\boldsymbol{V},\boldsymbol{d},-\boldsymbol{W},-\boldsymbol{r}$ form a closed loop, so that

$$\boldsymbol{V} + \boldsymbol{d} - \boldsymbol{W} - \boldsymbol{r} = 0. \qquad (1)$$

A coordinate frame is chosen with origin at $A$, with $\boldsymbol{N}=(0,0,1)$, and with $\boldsymbol{L},\boldsymbol{V}$ lying in the $x$-$z$ plane, so that

$$\boldsymbol{V} = (V\sin\sigma, 0, V\cos\sigma),\ \boldsymbol{L} = (-\sin\sigma, 0, \cos\sigma) \qquad (2)$$

where $\sigma$ is the slant of the tangent plane at $A$. Its tilt direction lies in the $x$-$z$ plane. Note that if viewing geometry and light source direction $\boldsymbol{L}$ are known (and the latter could be obtained as in [11]) then surface slant and tilt are known: the surface normal lies in the plane of $\boldsymbol{V},\boldsymbol{L}$ and bisects them.

It is assumed that some feature at point $C$ on the surface, is available near to the specular points $A,B$ (fig 2) and that stereo is able to establish the position of $C$. Its position vector $\boldsymbol{F}$ is used to estimate $V$, the length of the vector $\boldsymbol{V}$. Assuming that $C$ is not too far away from $A$, so that $C$ lies, approximately, in the tangent plane at $A$,

$(\boldsymbol{V}-\boldsymbol{F}).\boldsymbol{N} = 0$ so that

$$V\cos\sigma = \boldsymbol{F}.\boldsymbol{N} \qquad (3)$$

Since the choice of coordinate frame ensures that gradients vanish ($\partial z/\partial x = \partial z/\partial y = 0$), the surface, in the neighbourhood of $A$, is described by

$$z(x,y) = (1/2)\boldsymbol{x}.(H\boldsymbol{x}) + O(|\boldsymbol{x}|^3) \qquad (4)$$

where $\boldsymbol{r}=(x,y,z)$, $\boldsymbol{x}=(x,y)$ and $H$ is the (symmetric) hessian matrix [4] of the surface. Note that $\boldsymbol{r},\boldsymbol{d}$ etc are 3-dimensional vectors but $\boldsymbol{x}$ is a 2-dimensional vector, in the $xy$-plane. Similarly $H$ is a 2×2 matrix operating on $\boldsymbol{x}$.

The law of reflection at $A$ is that

$$2(\boldsymbol{V}.\boldsymbol{N})\boldsymbol{N} - \boldsymbol{V} \parallel \boldsymbol{L}. \qquad (5)$$

where $\parallel$ denotes "is parallel to". Similarly for the other eye, at $B$,

$$2(\boldsymbol{W}.\boldsymbol{N}')\boldsymbol{N}' - \boldsymbol{W} \parallel \boldsymbol{L}. \qquad (6)$$

Combining (1) (5) (6) gives (see [3] for details):

$$\boldsymbol{M}H\boldsymbol{x} = \boldsymbol{x} + \boldsymbol{w} \qquad (7)$$

where $w_x = -d_x + d_z\tan\sigma$, $w_y = -d_y$ and

$$\boldsymbol{M} = \begin{pmatrix} 2(V\sec\sigma + d_x + d_z\tan\sigma) & 2d_y\tan\sigma \\ 0 & 2(V\cos\sigma + d_z) \end{pmatrix} \qquad (8)$$

The approximations used above hold good provided $|\delta\boldsymbol{N}| \ll \cos\sigma$ and $|\boldsymbol{x}| \ll V\cos\sigma$. This means that surface slant $\sigma$ must not be close to 90°, and that both vergence angle and (angular) disparity should be small. It can be shown that these conditions will usually be satisfied when the stereo baseline is short, so that $|\boldsymbol{d}| \ll V\cos\sigma$.

To solve equation (7), we note also that

$$\det(\boldsymbol{M}) = 4(V\sec\sigma + d_x + d_z\tan\sigma)(V\cos\sigma + d_z)$$

so that, provided surface slant $\sigma$ is not near 90° as above, and provided $|\boldsymbol{d}| < (1/2)V$ (baseline length less than half viewing distance), then $\det(\boldsymbol{M}) \neq 0$. In that case, equation (7) can be inverted to give

$$H\boldsymbol{x} = \boldsymbol{v} \text{ where } \boldsymbol{v} = \boldsymbol{M}^{-1}(\boldsymbol{w}+\boldsymbol{x}) \qquad (9)$$

which, in general, can be expected to impose 2 constraints on the 3 variables of $H$. If something is known already about $H$ - say, that the surface is locally cylindrical at $A$ - then it might be possible to determine $H$ completely.

### 2.2 Disparity measurement

The equations just derived require the vector $\boldsymbol{x}$ to be determined from disparity measurements, as shown in fig 3.

Having obtained from the stereo images the angular disparity $\boldsymbol{\delta}$ of the specularity as in fig 3, it can be "back projected" onto the surface to obtain the length $\boldsymbol{x}$. The assumption that $|\delta\boldsymbol{N}|$ is small is used again to obtain

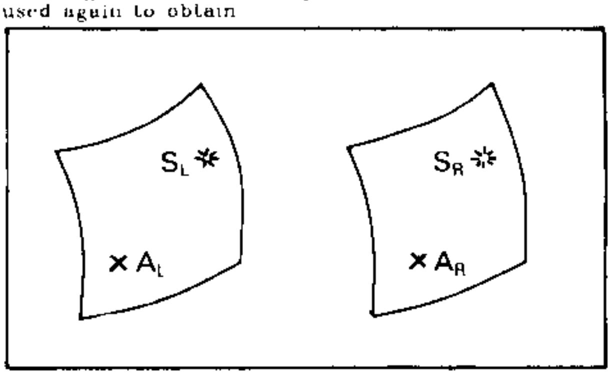


*Figure 3: Disparity measurements. The specular point is imaged at angular positions $S_L,S_R$ in the left and right images respectively. A nearby surface feature is imaged at $A_L,A_R$ and provides a disparity reference point on the surface. From these measured positions in the image, $\boldsymbol{\delta} = (S_R - A_R)-(S_L - A_R)$ is computed - the difference between the angular disparities of the 2 points. Then $\boldsymbol{x} = (x,y) = V(\delta_x\sec\sigma, \delta_y)$*

$$\boldsymbol{x} = (x,y) = VP\boldsymbol{\delta}, \text{ where} \qquad (10)$$

$$P = \begin{pmatrix} \sec\sigma & 0 \\ 0 & 1 \end{pmatrix}$$

### 2.3 Focusing effects

Equation (7) predicts that imaging of a specularity can become degenerate (fig 4). The equation can be rewritten as

$$(MH-I)x = w \qquad (11)$$

and the condition for degeneracy is that $\det(MH-I)=0$.

The focusing effect may produce either a line or a blob in the image:

1. If the rank of $(MH-I)$ is 1 the specularity will appear in the image as a line. Or else, there may be no solution for $x$ in (7), and nothing of the specularity will be visible. Stevens [13] observes that, with infinitely distant source and viewer, any line specularity must lie on a plane curve in the surface a special cose of the rank 1 focusing effect.
2. If the rank of $MH-I$ is zero, that is $MH-I=0$, then either the specular reflection is invisible as above, or it is focussed in 2 dimensions onto the imaging aperture, and appears in the image as a large bright blob
3. It can be shown that if the surface patch is convex, the effect cannot occur This corresponds to physical intuition Only concave mirrors focus distant light sources

### 2.4 A source at a finite distance

*If* the source is at a finite distance $L$, rather than infinite as assumed so far, the imaging equation (7) becomes

$$(MH-(1+p)I)x = w \qquad (12)$$

The constant $p = (d_2\sec\sigma + V)/L$ and clearly, as $L\to\infty$, $p\to 0$ which gives the infinite source equation (7)

Suppose the infinite source computation of surface shape is performed, when in fact the source is at a finite distance $L$, how great is the resulting error? The answer is that the error in the curvature (along a given direction in the xy-plane) is of the order of $\pm V/L$. The error is negligible if, assuming $\sigma$ not to be close to 90°, either

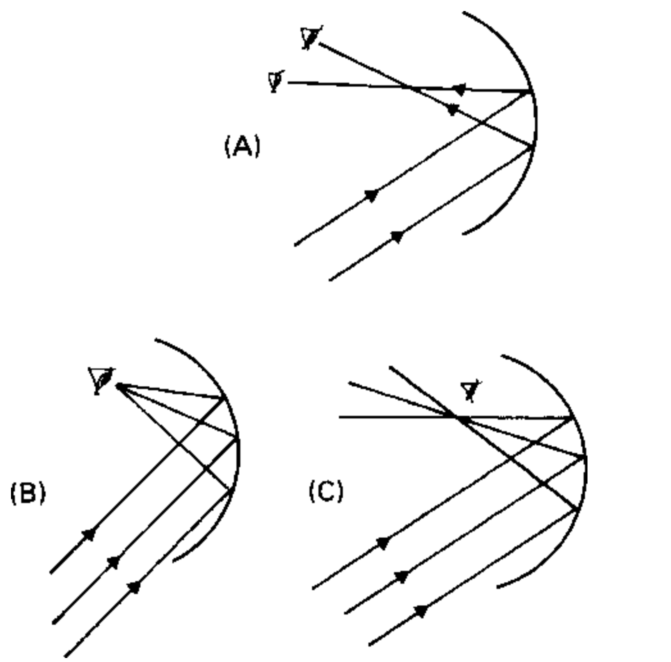


*Figure 4: Degenerate imaging of a specularity. Normal imaging of a specularity (a) becomes degenerate because of the focusing action of the curved surface. The surface may focus onto the viewing aperture (b) to produce a bright blob or line in the image, or away from the aperture (c) in which case nothing is visible in the image*

- the light source distance is large compared with the viewing distance: $L \gg V$, or
- the surface has *high curvature* for *both* principal curvatures, $\kappa_i \gg 1/L$, $i=1,2$

The first case is intuitively reasonable; if the light source is distant compared with the observer distance $V$, then equations for an infinite light source can be used with little error. What is perhaps less obvious is the second condition that, for highly curved objects, the source need *not* be further away than the observer.

### 2.5 Distributed sources

The mathematical model that has been used so far assumes a point source. In practice the source may be distributed, so that it subtends some non-zero solid angle, at the surface

Equation (12), for a source at a finite distance, is used but source and viewer positions are interchanged The light ray is reversed Vector $d$ now represents the movement of *source* for a fixed *viewer* position. After some rearrangement, this yields a new equation, looking rather like (12) but with a factor $V/L$ on the right hand side:

$$(MH-(1+p)I)x = (V/L)w. \qquad (13)$$

It would be most convenient to express the shape of the image specularity (using angular position in the image, $\delta$) in terms of source distribution (using a new angular variable $\alpha$). From (10) $x=VP\delta$, and it is straightforward to show that $w=LP\alpha$ So now

$$T\delta = \alpha \text{ where } T=P^{-1}MHP-(1+p)I. \qquad (14)$$

What equation (14) says is that the viewer sees an image of the source that has undergone a linear transformation $T^{-1}$. The effect of the transformation depends on surface shape. For a planar mirror for example, $H=0$ so that $\delta=\alpha/(1+p)$ an isotropic scaling that preserves the shape of the source Note that if the source is very distant, $p\approx 0$ and the scaling factor is unity.

If the angular dimensions of the source are known then, in principle, surface shape may be recovered completely by monocular observation. For a circular source with slant $\sigma=0$, the ellipse axes coincide with the principal curvature directions of the surface In general, when $\sigma\neq 0$, measuring the length and direction of ellipse axes enables $T$ and hence $H$ to be found from (14) Note that for a circular source, because of its symmetry, principal curvatures are determined only up to sign inversion (approximately)

## 3 INFERRING LOCAL SURFACE SHAPE

### 3.1 Locally cylindrical surface

On a surface that is *known* to be locally cylindrical, equation (7) is sufficient to recover both parameters of local surface shape. For instance, when the source is distributed, a strip shape image-specularity indicates that the surface may be cylindrical - or at least that one principal curvature may be much larger than the other (This can be deduced from (14) )

The parameters to be determined are the direction of the cylinder axis $\theta$ and the radius $R$ Using (9):

$$\tan\theta = v_y/v_x$$

$$R = (x\cos^2\theta + y\sin\theta\cos\theta)/v_x$$

### 3.2 Spherical surface

The knowledge that the surface is locally spherical could be derived monocularly, from (14), as in the cylindrical case except that rather more must be known about the source for example, that it is circular

On a spherical surface there is only one parameter to specify - the radius of curvature $R$, and from (9).

$$R = x/v_x = y/v_y$$

the second equality being available as a check for consistency of assumptions

### 3.3 Known orientation of principal axes

If the orientation of principal axes about the surface normal, is known then the complete local surface geometry can be obtained. Orientation could be derived monocularly (assuming source shape known) from (14).

Rotating coordinates about the $z$ axis, a primed (') frame can be obtained in which $H$ in (9) becomes diagonal:

$$H'x' = v'$$

Now, in general, the two diagonal components of $H'$ can be obtained immediately. Experiments with computer generated images have obtained curvature to an accuracy of 10%.

### 3.4 General case

In the general case, the surface curvature at the specular point is described by 3 parameters, but the specular stereo measurements yields only 2 constraints. However two additional constraints - 4 in all - are available if a second baseline is used. The extra baseline could be derived either from a third sensor, suitably positioned, or from known motion of the viewer (parallax).

Suppose now that there are 2 baselines $d^{(i)}, i=1,2$ with corresponding $x^{(i)}, M^{(i)}, w^{(i)}, v^{(i)}$. Now equation (9), applied once for each baseline, gives

$$HX = V \quad \text{where} \tag{15}$$

$$X = \begin{pmatrix} x^{(1)} & x^{(2)} \\ y^{(1)} & y^{(2)} \end{pmatrix} \quad V = \begin{pmatrix} v_x^{(1)} & v_x^{(2)} \\ v_y^{(1)} & v_y^{(2)} \end{pmatrix}$$

and $H$ can be recovered provided $X$ is non-singular.

It appears to be impossible to suggest baselines that guarantee to generate a non-singular $X$, for *all* viewing geometries and surfaces. This is because $\det(X)$ depends on the surface and the viewing geometry, as well as on the baselines. This is probably best achieved (see [3]) by making the baselines $d^{(i)}$ fairly near orthogonal, and certainly nowhere near collinear.

The disparity measurements give 4 constraints. If $H$ is the only unknown, it is now overdetermined. One could either

1. Test whether the $H$ obtained from (15) is indeed symmetric as a check on validity of assumptions (for example, the validity of the local approximation of (4), over the range of movement of the specular point on the surface)
2. Use a least-squares error method to find the symmetric $H$ that fits the data best. Then $H$ is the solution of linear equations:

   $$HXX^T + XX^TH = VX^T + XV^T.$$

   The error measure $\|HX-V\|$, if it is too large, indicates that some assumptions were not valid.

## 4 CONCLUSION

Is specular stereo actually useful? We argue that it is. Of course the presence of specularities in the image cannot be guaranteed; specular stereo is not an autonomous process in the sense that conventional stereo is. Indeed specular stereo itself *relies* on conventional stereo to provide a disparity reference. In the case of a densely textured surface, conventional stereo with surface fitting [2,5,6,12,14) would be able to give an accurate estimate of surface shape. But for a smooth surface, stereo features may be relatively sparse, and fitting a surface to disparity measurements may be difficult and inaccurate. Then, provided at least one nearby surface feature is available as a disparity reference, specular stereo, together with monocular analysis of specularity, provides valuable surface shape information

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