

# NONLINEAR PLANNING: A RIGOROUS RECONSTRUCTION

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The problem of achieving several goals simultaneously has been central to domain-independent planning research, the non-linear constraint-posting approach has been most successful. Previous planners of this type [4, 5] have been complicated, heuristic, and ill-defined. I have combined and distilled the state of the art into a simple, precise, implemented algorithm (TWEAK) which I have proved correct and complete. The simplicity and rigor of this algorithm illuminate the workings of previous planners, the range of applicability of current planning technology, and suggest future directions for research. This paper presents the mathematical foundations for non-linear planning; due to space limitations, I have omitted proofs, some detail, and much discussion. These appear in [1].

This paper begins by presenting a series of necessarily dry and obvious definitions, leading up to that of a plan that solves a problem. I present a "truth criterion" which provides an efficient means of analyzing a plan to determine when a proposition will be true in the world as the plan is executed. The truth criterion also provides a way of making a plan achieve a goal, and this is the basis of the TWEAK algorithm. Finally I state a completeness correctness theorem and present conclusions.

TWEAK is a constraint posting planner. Constraint posting is the definition of an object, a plan in this case, by successively specifying more and more partial descriptions it must fit. Alternatively, constraint posting can be viewed as a search strategy in which rather than generating and testing specific alternatives, chunks of the search space are progressively removed from consideration by constraints that rule them out, until finally every remaining alternative is satisfactory. The advantage of the constraint posting approach is that properties of the object being searched for do not have to be chosen until a reasoned decision can be made. This reduction of arbitrary choice often reduces the amount of backtracking necessary.

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As TWEAK works on a problem, it has at all times an incomplete plan, which is a partial specification of a plan that may solve the problem. This incomplete plan could be completed in many different ways, depending on what constraints are added to it. Thus it represents a class of complete plans. The incomplete plan supplies partial knowledge of the complete plan that will eventually be chosen; ideally all possible completions of the current plan should solve the given problem. I will say "necessarily foo" if foo is true of all possible completions of the current plan, and "possibly foo" if foo is true of some completion of the current plan.

A complete plan is a total "time" order on a set of steps, which represent actions. The plan is executed by performing the actions corresponding to the steps in the order given. A step has a set of preconditions, which are things that must be true about the world for it to be possible to execute the action. A step also has postconditions, which are things that will be true about the world after the corresponding action has been executed. Pre- and postconditions are both expressed as propositions. Propositions can be positive or negative, and have a content, which is a tuple of elements. Elements can be variables or constants. Functions, propositional operators and quantification are not allowed.

Plans in TWEAK can be incomplete in two ways; the time order may be incompletely specified, using temporal constraints, and steps may be incompletely specified, using codesignation constraints. A temporal constraint is a requirement that one step be before another; thus a set of temporal constraints is simply a partial order on steps. A completion of a set of temporal constraints  $C$  is any total order  $O$  on the same set of steps such that  $aCt$  implies  $sOt$ .

Codesignation is an equivalence relation on variables and constants. In a complete plan, each variable that appears in a pre- or postcondition must be constrained codesignating with a specific constant. In execution, that constant will be substituted for the variable when the action is performed. Distinct constants may not codesignate. Two propositions codesignate if both are positive or both are negative and if their contents are of the same length and if corresponding elements in the contents codesignate.

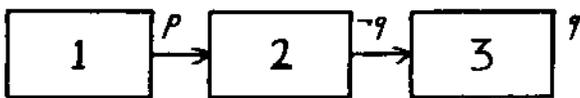
TWEAK represents the state of the world as a set of propositions. This set changes as steps are executed. A plan has an initial situation, which is a set of propositions describing the world at the time that the plan is to be executed. Associated with each step in a plan is its input situation, which is the set of propositions that are true in the world just before it, is executed,

and its output situation, which is the set of propositions that are true in the world just after it is executed. In a complete plan, the input, situation of each step) is required to be the same set as the output situation of the previous step. The final situation of a plan has the same set of propositions in it as the output situation of the last step. The time order extends to situations: the initial and final situations are before and after every other situation respectively. The input situation of a step is before the step and after every other situation that is before the step; the output situation of a step is after the step and before any other situation that is after the step.

Say that a proposition is true in a situation if it codesignates with a proposition that is a member of the situation. Say that, a step asserts a proposition in its output situation if the proposition codesignates with a postcondition of the step. Say that a proposition is asserted in the initial situation if it is true in that situation. A proposition is denied in a situation if another proposition with codesignating content but opposite truth value is asserted there. It's illegal for a proposition to be both asserted and denied in a situation.

A step can be executed only if all its preconditions are true in its input situation. In this case, the output situation is just the input situation minus any propositions denied the step, plus any propositions asserted by the step. This model of execution does not allow for indirect or implied effects of actions, any changes in the world must be explicitly mentioned as postconditions.

I will now sketch the derivation of a criterion for when a proposition is necessarily true in a situation. Of course a proposition is necessarily true in a situation if it is necessarily asserted in it. Once a proposition has been asserted, it remains true until denied. Thus a proposition  $p$  is necessarily true in a situation if there is some previous situation in which it is necessarily true, and there is no possibly intervening step that possibly denies it for if there is a step that is even possibly inbetween that even possibly denies  $p$ , there is a completion in which the step actually is inbetween and actually denies  $p$ . (A step possibly denies  $p$  by denying a proposition  $q$  which possibly codesignates with  $p$ ) The converse of this criterion is not true; this plan illustrates an exception:



If  $p$  and  $q$  are possibly codesignating, this plan has two classes of completions: one in which  $p$  and  $q$  actually codesignate, in which case  $p$  is asserted by step 3; and one in which  $p$  and  $q$  are noncodesignating, so that  $p$  is asserted by step 1, and is never denied. In either case,  $p$  is true in the final situation, even though no step necessarily asserts  $p$  without an intervening step possibly denying it. The complete criterion is as follows

Truth criterion: A proposition  $p$  is necessarily true in a situation  $s$  iff two conditions hold: there is a situation  $t$  necessarily equal or previous to  $s$  in which  $p$  is necessarily asserted; and for every step  $C$  possibly before  $s$  and every proposition  $q$  possibly codesignating with  $p$  which  $C$  possibly denies, there is a step  $W$

necessarily between  $C$  and  $s$  which asserts  $r$ , a proposition such that  $r$  and  $p$  codesignate whenever  $p$  and  $q$  codesignate. The criterion for possible truth is exactly analogous, with all the modalities switched (read "necessary" for "possible" and vice versa)

This criterion can be computed in polynomial time, though it does exponentially much "work" by describing properties of the exponentially large set of completions of a plan. The remainder of TWEAK depends heavily on this theorem; its proof is surprisingly complex. It can be usefully viewed as a completeness/soundness theorem for a version of the situation calculus.

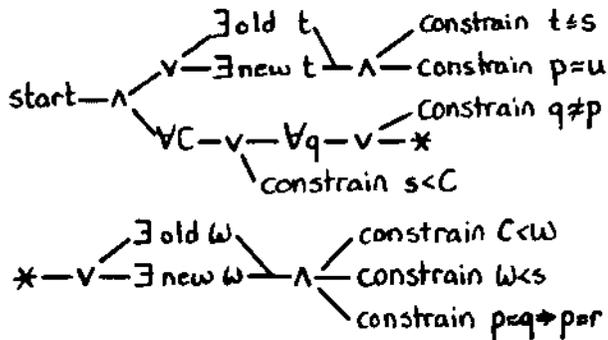
Now I will define problems and their solutions. A problem is an initial situation and a final situation, which are two sets of propositions. A plan for a problem is a plan every proposition of whose initial situation is true in the initial situation of the problem. A goal is a proposition which must be achieved (true) in a certain situation. The goals of a plan for a problem are defined to be the propositions in the final situation of the problem which must be true in the final situation of the plan, and the preconditions of steps in the plan, which must be true in the corresponding input situations. A complete plan for a problem solves the problem if all its goals are achieved. Thus, a complete plan solves a problem if it can be executed in the initial situation of the problem and if the final situation of the problem is a correct partial description of the world after execution. The aim of TWEAK is to produce a plan that, necessarily solves the problem it is given. This plan may be incomplete in this case any of its completions may be chosen for execution.

TWEAK's contract is to produce a plan for a specific problem it is given. TWEAK has at all times an incomplete plan, initially null, which is an approximation to a plan that solves the problem. The top-level loop of the planner nondeterministically chooses a goal that is not already achieved and uses a procedure which I will now describe to make the plan achieve that goal.

The goal-achievement procedure is derived by interpreting the truth criterion as a nondeterministic procedure. Universal quantification over a set becomes iteration over that set, existential quantification a nondeterministic choice from a set, disjunction a simple nondeterministic choice; and conjunction several things that must all be done. Also, an existentially quantified situation can be satisfied by nondeterministically choosing either an existing situation in the plan or a situation belonging to a newly added step. The newly added step must represent an action that is possible to execute in the domain in which the problem is specified, the choice is among those that are allowed in the domain and that possibly assert, the desired goal.

To make a situation be before another or to make two propositions codesignating or not codesignating, the procedure just adds constraints. These constraints may be incompatible with existing constraints; for example, you can't constrain a before / if you have already constrained / before s. The constraint maintenance mechanism signals failure in such cases, and the top-level control structure backtracks. Since the set of things of things possibly asserted in a situation can not be changed, to make a proposition necessarily asserted there, the procedure constrains codesignation of the given proposition with one of those asserted

The following diagram defines the nondeterministic procedure



Because the truth criterion is sufficient as well as necessary, this achievement procedure encompasses *all* the ways to make a plan achieve a goal. In this respect TWEAK can not be improved upon.

Step addition adds new preconditions to the plan that need to be achieved, and the added step may also deny, and so undo, previously achieved goals. Therefore, TWEAK tries to avoid step addition. This is not always successful. There are three possible outcomes to planning: success, in which a plan is found; failure, when the planner has exhaustively searched the space of sequences of plan modification operations, and every branch fails; and nontermination, when the plan grows larger and larger and more and more operations are applied to it, but it never converges to solve the problem.

The central theorem of this paper is the following

**Correctness/completeness theorem:** If TWEAK, given a problem, terminates claiming a solution, the plan it produces does in fact solve the problem. If TWEAK returns signalling failure or does not halt, there is no solution to the problem.

The theorem follows easily from the truth criterion.

The rigor of TWEAK's formulation has several uses. Previous planning research becomes substantially clearer when analyzed with the tools built in constructing TWEAK. In [1,] present a detailed history of planning, showing that the classic

planners are more similar than has previously been realized.

Another use of this rigor is that the range of applicability of state-of-the-art planning techniques becomes clear. The correctness of TWEAK and similar planners depends crucially on details of the representation of actions. Useful extensions to this representation, such as range restrictions for variables, non-atomic propositions, derived effects of actions, actions whose effects depend on the situation in which they are applied, and changes in the world due to agencies other than execution of the constructed plan, invalidate the truth criterion, and so the correctness of TWEAK. There seems to be no simple way to extend the criterion to accommodate these effects. The difficulty is just that of the McCarthy frame problem 2; my thesis suggests an approach to its solution.

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