LOCK, LINEAR λ-PARAMODULATION IN OPERATOR FUZZY LOGIC

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ABSTRACT

The author proposed concepts of Operator Fuzzy Logic and λ-Resolution in **1984.** He and his cooperaters have obtained some theoretical results. This paper introduces λ-paramodulation to handle a set of clauses with the predicate of equality, and thus the equality substitution can be used in fuzzy reasoning. Then the λ-paramodulation method is proved to be complete with λ-lock-semantic method. Finally, Yang Fengjie and I have improved the λ-paramodulation, and proved that the linear λ-paramodulation is complete too.

I. INTRODUCTION

The author proposed concepts of Operator Fuzzy Logic(or OFL for short) and *K*~Resolution in 1984. During the past few years (1985-1987), he and H. Xiao, then he, K.Y.Fang, Carl.K.Chang and Jeff. J-P Tsai working together on the λ-resolution method obtained a series of results. We proved that a λ-resolvent of two fuzzy clauses Ci and C2 is a λ-logical concequence of C1 and C2, and that λ-resolution method is complete for λ-inconsistent set of clauses. We explained the practical meaning of fuzzy reasoning based on λ-resolution method in OFL. From the above results, we can see that some theorems which can not be proved with the traditional logic, can be proved fuzzily by using λ-resolution method in our logic system.

Equality relation is a very important relation in mathematics. This equality relation lias some important special properties: it is transitive and can substitute equals for equals. Thus it is natural that the transitive and substitutive properties of fuzzily equality are needed in proving fuzzy theorems.

This paper proposed a λ-paramodulation to handle fuzzy equality. In conjunction with λ-resolution, λ-paramodulation can be used to prove fuzzy theorems in OFL. We proved that λ-paramodulation is complete for the λ-inconsistent set of clauses in conjunction with λ-resolution.

II. FUNDAMENTAL CONCEPTS AND PROPERTIES

From [1-3], we know that Operator Fuzzy Logic is built on operator lattice and interval [0,1] is an operator lattice if we define:

$$x*y=\min\{x,y\},$$
$$x\oplus y=\max\{x,y\},$$
$$x\bullet y=(x+y)/2,$$
$$x'=1-x,$$

for any x,y [0,1].

in the following discussion, we assume that OFL is built on interval [0,1] and the amount of operator λ is finite.

These concepts such as operator lattice, fuzzy literal, formula, interpretation, truth-value T1(G) of formula G under 1, λ-complemented literal, λ-identical literal, can be found in Refs. [1-3].

Definition 1. Choose arbitrarily $\lambda\in[0,1]$. Formula G is called λ-valid if and only if for every I there exists T1 (G)$\geq\lambda$; G is called λ-inconsistent if and only if for every 1, there exists T1(G)$\leq\lambda$.

Obviously, Formula G is λ-valid if and only if Formula (~ G) is (1-λ) -inconsistent .

Definition 2. Let C1 and C2 be two clauses without the same variables and $\lambda_1 L_1$ and $\lambda_2 L_2$ be two literals of Ci and C2 respectively. If L1 and L2 have a Most General Unifier (MGU for short) σ, and $\lambda_1 L_1^\sigma$ and $\lambda_2 L_2$ are λ-complemental, then

$$(C_1^\sigma - S_1)\cup(C_2^\sigma - S_2)$$

is the binary λ-resolvent for C1 and *C2*, denoted by R_A(**C1**, C2), where

$S_1=\{\lambda^*L^r \mid (\lambda^*L^r \in C_1^r)\wedge(\lambda^*L^r$ and $\lambda_1 L_1^r$ are λ-identical$)\}$,

$S_2=\{\lambda^*L^r \mid (\lambda^*L^r \in C_2^r)\wedge(\lambda^*L^r$ and $\lambda_2 L_2^r$ are λ-identical$)\}$.

The following properties are simple, and so their proofs are omitted.

Property 1. Let P be an atom, then $\sim(1P)=0P$.

Property 2. Let P be an atom, then $\sim(0P)=1P$.

Henceforth, IP can be denoted by P in OFL.

Property 3. Let P be an atom, then $\sim(\lambda P)=(1-\lambda)P$.

Property 4. Let P be an atom, then

$$\sim(\lambda_1 \ldots \lambda_n P)=(1-\lambda_1)\ldots(1-\lambda_n)P.$$

Property 5. Let G be formula, then $\sim G \ne 0G$. For example, Let G=0.3P, I={P}, then

$T_I(\sim G)=T_I(0.7P)=0.7$.

$T_I(0G)=T_I(0(0.3P))=0.15$.

Property 6. Let G be a formula, then

$$\sim\lambda G=(1-\lambda)(\sim G).$$

Property 7. Let G be a formula, then

$$\sim(\lambda_1 \ldots \lambda_n G)=(1-\lambda_1)\ldots(1-\lambda_n)(\sim G).$$

Property 8. Let P be an atom, then

$$\lambda_1\lambda_2 P \ne (\lambda_1 * \lambda_2)P$$

For example, $0.8(0.6P) \ne (0.8*0.6)P=0.7P$.

Property 9. Let G be a formula, then

$$\lambda_1\lambda_2 G \ne (\lambda_1 * \lambda_2)G.$$

Definition 3. Let G and H be two formulas, I be any interpretation. If $T_I(H) \geq \lambda$ for $T_I(G) > 1-\lambda$, then it is said that G λ-implies H, denoted by $G \Rightarrow H$.

Definition 4. Let G and H be two formulas and I be any interpretation. If $T_I(H)>\lambda$ for $T_I(G)>\lambda$, then it is said that G λ-strong implies H, denoted by

Property 10. Let A be formula, then

1) $A \Rightarrow A$ for $\lambda \leq 0.5$.

2) $A \Rrightarrow A$.

Property 11. Let A,B and C be formulas. Then

1) if $A \Rightarrow B, B \Rightarrow C$ then $A \Rightarrow C$ for $\lambda > 0.5$;

2) if $A \Rrightarrow B, B \Rrightarrow C$ then $A \Rrightarrow C$.

Property 12. Let C_1 and C_2 be two clauses and $R_\lambda(C_1, C_2)$ be a λ-resolvent of C1 and C2. Then

1) $C_1 \wedge C_2 \Rightarrow R_\lambda(C_1, C_2)$ for $\lambda=0.5$;

2) $C_1 \wedge C_2 \Rrightarrow R(C_1, C_2)$ for $\lambda \geq 0.5$.

The proof of this property will be given by another paper (to appear).

Property 13. Let A,B and C be three formulas. Then

1) if $A \Rightarrow B, A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$;

2) if $A \Rrightarrow B, A \Rrightarrow C$, then $A \Rrightarrow (B \wedge C)$.

Definition 5. A clause C is called a λ-empty clause (denoted by λ-$\square$), if for any literal $\lambda^*L \in C$, it satisfies the condition:

$1-\lambda \leq \lambda^* \leq \lambda$

for $\lambda \geq 0.5$.

Property 14. Let $\lambda \geq 0.5$. A set of clauses is λ-inconsistent if and only if there is a resolution deduction of the λ-empty clause from S.

This property is an important theorem in Ref. [1] and another paper is waiting for publishing. Its proof is omitted here.

III. THE SET OF FUZZY EQUALITY AXIOMS

Definition 6. An E-interpretation IE of set S of clauses is an interpretation of S satisfying the four following conditions. Let α, β and γ be any terms in the Herbrand universe of S, and let $\lambda P(\ldots\alpha\ldots)$ be any fuzzy literal in S. Then

1. $T_{IE}((\alpha=\alpha))=T$;
2. if $T_{IE}((\alpha=\beta))=T$, then $T_{IE}((\beta=\alpha))=T$;
3. if $T_{IE}((\alpha=\beta))=T$, if $T_{IE}((\beta=\gamma))=T$, then $T_{IE}((\alpha=\gamma))=T$;
4. if $T_{IE}((\alpha=\beta))=T$, then $T_{IE}(P(\ldots\alpha\ldots))=T_{IE}(P(\ldots\beta\ldots))$.

Definition 7. Let S be a set of clauses, $\lambda \in [0,1]$ and $\lambda \geq 0.5$. Then the set K_λ of λ-equality axioms for S is the set consisting of the following clauses: for any $\lambda^* > \lambda$

1. $\lambda^*(x=x)$;
2. $(1-\lambda^*)(x=y) \vee \lambda^*(y=x)$;
3. $(1-\lambda^*)(x=y) \vee (1-\lambda^*)(y=z) \vee \lambda^*(x=z)$;
4. $(1-\lambda^*)(x_j=x_0) \vee (1-\lambda^*)P(\ldots x_j \ldots) \vee \lambda^* P(\ldots x_0 \ldots)$,

for any atom $P(x_1,\ldots,x_n)$ occurring in S, $j=1,\ldots,n$;

5. $(1-\lambda^*)(x_j=x_0) \vee \lambda^*(f(\ldots x_j \ldots)=f(\ldots x_0 \ldots))$,

for any function symbol $f(x_1,\ldots,x_n)$ occurring in S, $j=1,\ldots,n$.

Definition 8. A set S of clauses is called λE-inconsistent if and only if $T_{IE}(S) \leq \lambda$ for any E-interpretation IE ; S is called λE-valid if and only if $T_{IE}(S) \geq \lambda$, for any IE.

If IE is an E-interpretation of S, we can obtain results obviously as follows:

1. $T_{IE}(\lambda^*(x=y))=\lambda^*>\lambda$.
2. $T_{IE}((1-\lambda^*)(x=y) \vee \lambda^*(y=x))=\lambda^*>\lambda$.
3. $T_{IE}((1-\lambda^*)(x=y) \vee (1-\lambda^*)(y=z) \vee \lambda^*(x=z))=\lambda^*>\lambda$.
4. If $T_{IE}((x_j=x_0))=T$, because IE is an E-interpretation, then $T_{IE}(P(\ldots x_j \ldots))=T_{IE}(P(\ldots x_0 \ldots))$, and therefore,

$$T_{IE}((1-\lambda^*)(x_j=x_0) \vee (1-\lambda^*)P(\ldots x_j \ldots) \vee \lambda^*(P\ldots x_0\ldots))=\lambda^*>\lambda.$$

5. If $T_{IE}((x_j=x_0))=T$, because I_E is an E-interpretation, then

$T_{IE}(f(\ldots x_j \ldots))=T_{IE}(f(\ldots x_j \ldots))$
$=T_{IE}(f(\ldots x_j \ldots)=f(\ldots x_0 \ldots))$., and

since $T_{IE}((\alpha=\alpha))=T$ for any α, we have

$T_{IE}(f(\ldots x_j \ldots)=f(\ldots x_0 \ldots))=T$.

Thus

$$T_{IE}((1-\lambda^*)(x_j=x_0) \vee \lambda^*(f(\ldots x_j \ldots)=f(\ldots x_0 \ldots))) = \lambda^*>\lambda.$$

From the above discussion, we can easily see that $T_{IE}(K_\lambda)>\lambda$, where K_λ is a set of λ-equality axioms of S.

If I is an interpretation of S and $T_I(K_\lambda)>\lambda$, we can obtain results obviously as follows:

1. $T_I((\alpha=\alpha))=T$ for any term α in S.

2. If $T_I((\alpha=\beta))=T$, because $T_I(K_\lambda)>\lambda$, thus $T_I((1-\lambda^*)(\alpha=\beta)V\lambda^*(\beta=\alpha))>\lambda$, therefore, there must be $T_I(\beta=\alpha))=T$.

3. Let $T_I((\alpha=\beta))=T$, $T_I((\beta=\gamma))=T$. According to $T_I(K_\lambda)>\lambda$, thus $T_I((1-\lambda^*)(\alpha=\beta)V(1-\lambda^*)(\beta=\gamma)V\lambda^*(\alpha=\gamma))>\lambda$, there must be $T_I((\alpha=\gamma))=T$

4. Suppose $T_I((\alpha=\beta))=T$, $T_I(P(\ldots\alpha\ldots))=T$. Since $T_I((1-\lambda^*)(\alpha=\beta)V(1-\lambda^*)P(\ldots\alpha\ldots) V\lambda^*P(\ldots\beta\ldots))>\lambda$, there must be $T_I(P(\ldots\beta\ldots))=T$. Similarly, we can easily see that if $T_I((\alpha=\beta))=T$, $TI(P(\ldots\beta\ldots))=T$, then there must be $TI(P(\ldots\alpha\ldots))=T$, namely, $TI(P(\ldots\alpha\ldots))=TI(P(\ldots\beta\ldots))$.

From the above discussion, we can easily see that 1 is an E-interpretation of S.

Therefore, we can obtain a theorem as follows:

***Theorem** 1. Let S be a set of clauses and K_λ be the set of λ-equality axioms for S. For any interpretation I of S, I is an E-interpretation if and only if*

$$T_I(k_\lambda)>\lambda.$$

***Theorem** 2. Let S be a set of clauses and K_λ be the set of λ-equality axioms for S. Then S is λE-inconsistent if and only if $(S\cup K_\lambda)$ \- inConsistent.*

($\Rightarrow$) Suppose S is λE-inconsistent, but $(S\cup K_\lambda)$ is not λ-inconsistent. Then there exists an interpretation I such that $T_I(S\cup K_\lambda)>\lambda$. Thus $T_I(S)>\lambda$ and $T_I(K_\lambda)>\lambda$.

From theorem 1, we know that I must be an E-interpretation. From $T_I(S)>\lambda$, we know that S is not λE -inconsistent, which contradicts the assumption that S is λE -inconsistent.

($\Leftarrow$). Suppose $(S\cup K_\lambda)$ is λ-inconsistent, but S is not λE -inconsistent. Then there exists an E-interpretation IE such that $T_{IE}(S)>\lambda$. Clearly, $T_{IE}(K_\lambda)>\lambda$. Hence $T_{IE}((S\cup K_\lambda))>\lambda$. This contradicts the assumption that $(S\cup K_\lambda)$ is λ- inconsistent.

IV.PARAM0DULAT10N

Definition 9. Let ^>O.5 and C1 , *C2* be two clauses without any variables in common such that

Ki L[t]VC , XJ >x or x <1-x,

x2 (r=s)VC2 ' , x2>x,

where x1L[t] is a fuzzy literal containing the term t and C1' and C2' are clauses. If t and r have MGU <r , then

is called a binary λ-paramodulant of C1 and C2, $\lambda_1 L[t]$ and $\lambda_2(r=s)$ are called λ-paramodulated literals where $L^\sigma[s^\sigma]$ denotes the result obtained by replacing one single occurrence of t^σ in L^σ by s^σ ----

$$\lambda^* = \{(\lambda_1+\lambda_2)/2 \text{ when } \lambda_1>\lambda,$$

Definition 10. A λ-paramodulant of C1 and C2 is a binary λ-paramodulant of C1 or a x -factor of Ci and C2 or a λ-factor of C2 , denoted by $P_\lambda(C_1,C_2)$.

Definition 11. Let S be a set of clauses; G and H be two clauses in S. [f $T_{IE}(H)>\lambda$ when $T_{IE}(G)>\lambda$ for every E-interpretation 1E of S, then it is said that G AE-strongly implies H, denoted by G=>H.

***Theorem 3. Let C1 and C2 be two clauses,** λ >0- 5, and suppose $P_\lambda(C_1,C_2)$ is a λ-paramodulant. Then*

$$(C_1 \wedge C_2)\Rightarrow P_\lambda(C_1, C_2).$$

Without loss of generality, we may assume that C1 is $\lambda_1 L[t]VC_1'$, C2 is $\lambda_2(r=s)VC_2'$, $P_\lambda(C_1, C_2)$ is a binary λ-paramodulant, w i $\lambda_2>\lambda$. Let MGU of t and r be σ , IE be any one of E-interpretation such that

$$T_{IE}(C_1)>\lambda \text{ and } T_{IE}(C_2)>\lambda.$$

Then, Obviously, we have

$$T_{IE}(C_1^\sigma)>\lambda \text{ and } T_{IE}(C_2^\sigma)>\lambda.$$

(Notice: every variable in Ci and *C2* is considered to be governed by a universal quantifer). Choosing an arbitrarily ground instance of c1 and a ground instance of *C2* , we have the following considerations:

1. For $\lambda_1>\lambda$

If $T_{IE}(L^\sigma[t^\sigma])=F$, Then $T_{IE}(C_1'^\sigma)>\lambda$, thus $T_{IE}(P_\lambda(C_1,C_2))>\lambda$.

If $T_{IE}(L^\sigma[t^\sigma])=T$, we have $T_{IE}(L^\sigma[s^\sigma])=T$; because $t^\sigma=r^\sigma$ and $r^\sigma=s^\sigma$, we know $\lambda^*>\lambda$ (λ^* occurring in the definition of $P_\lambda(C_1, C_2)$), therefore $T_{IE}(P_\lambda(C_1,C_2))>\lambda$.

2. For $\lambda_1<\lambda$

similarly to the above proof, we can also obtain

$$T_{IE}(P_\lambda(C_1, C_2))>\lambda.$$

According to the above proof, we have the following fact:

$$(C_1 \wedge C_2)\Rightarrow P_\lambda(C_1, C_2).$$

Q.E.D.

Definition 12. Let C_1 and C_2 be two clauses, where every literal is locked with an integer and $\lambda\geq 0.5$ and let $P_\lambda(C_1, C_2)$ be a λ-paramodulant. Suppose

1. C_1 and C_2 are λ-positive clause, namely, every fuzzy literal λ^*L in clauses satisfies $\lambda^*\geq 1-\lambda$.

2. the paramodulated literal of C_i contains the smallest lock in C_i (i=1,2).

Then $P_\lambda(C_1, C_2)$ is called a λ-lock-hyperparamodulant, or a λ-LH-paramodulant for short.

Definition 13. Let $\lambda\geq 0.5$. A finite set of clauses $\{E_1,\ldots,E_q,N\}$ where every literal is locked with an integer is called a λ-lock-hypersemantic clash (or λ-LH-Clash for short) if and only if

$E_1,\ldots,E_q$ satisfies the following conditions:

1. $E_1,\ldots,E_q$ are λ-positive clauses.
2. Let $R_1=N$. For each $i(1<i<q)$, there is a λ-resolvent R_{i+1} of E_i and R_i.
3. The resolved literal of E_i contains the smallest lock in E_i.
4. R_{q+1} is a λ-positive clause, where R_{q+1} is called a λ-LH-resolvent.

Theorem 4. *let S be a set of clauses, where every literal is locked with an integer and $\lambda \geq 0.5$. If S is λ-inconsistent, then there is a deduction of λ-empty clause from S by the λ-LH-resolution method.*

proof. see [3].

Definition 14. Let S be a set of clauses and $\lambda \geq 0.5$. The set F_λ of λ-functionally reflexive axioms for S is the set defined as

$$F_\lambda=\{\lambda^*(f(x_1,\ldots,x_n)=f(x_1,\ldots,x_n))\},$$

where $\lambda^* \in [0,1]$ and $\lambda^*>\lambda$ and f is any function symbol occurring in S.

Definition 15. Let $\lambda \geq 0.5$. The fuzzy literal $\lambda^* L$ is called a λ-irrelevant literal if $1-\lambda \leq \lambda^* \leq \lambda$.

Definition 16. let C_1 and C_2 be two clauses. If we can build the one-to-one correspondence between the literals of C_1 and of C_2 such that the two corresponding literals are either λ-identical or λ-irrelevant, then C_1 is called λ-identical with C_2.

Theorem 5. *Let $\lambda \geq 0.5$, C_1 and C_2 be two clauses, C be a λ-lock-resolvent or a λ-lock-paramodulant of C_1 and C_2. If $C_1{}^*$ and $C_2{}^*$ are λ-identical with C_1 and C_2 respectively, then there is a λ-lock-resolvent C^* or a λ-lock-paramodulant C^* such that C^* is λ-identical with C.*

Proof 1. Let C_1 be $\lambda_1 L_1 \vee C_1{}'$ and C_2 be $\lambda_2 L_2 \vee C_2{}'$, where $\lambda_1>\lambda$ and $\lambda_2<1-\lambda$. Let

$$C=(C_1^\sigma-\lambda_1 L_1^\sigma)\cup(C_2^\sigma-\lambda_2 L_2^\sigma),$$

where σ is an MGU of L_1 and L_2.

It is clear that $C_1{}^*$ and $C_2{}^*$ are two clauses as follows:

$$\lambda_1{}^* L_1 \vee C_1{}^{*\prime},\ \lambda_2{}^* L_2 \vee C_2{}^{*\prime},$$

where $\lambda_1{}^*>\lambda$, $\lambda_2{}^*<1-\lambda$, $C_1{}^{*\prime}$ and $C_2{}^{*\prime}$ are λ-identical with $C_1{}'$ and $C_2{}'$ respectively. Let

$$C^*=(C_1{}^{*\sigma}-\lambda_1{}^* L_1^\sigma)\cup(C_2{}^{*\sigma}-\lambda_2{}^* L_2^\sigma).$$

Obviously, C^* is a λ-resolvent of $C_1{}^*$ and $C_2{}^*$ and is also a λ-lock-resolvent, C^* being λ-identical with C.

2. We might as well assume that C_1 is $\lambda_1 L[t] \vee C_1{}'$ and C_2 is $\lambda_2(r=s) \vee C_2{}'$, where $\lambda_1>\lambda$ and $\lambda_2>\lambda$. Let

$$C=\lambda_3 L^\sigma[s^\sigma] \vee C_1{}'^\sigma \vee C_2{}'^\sigma,$$

where $\lambda_3=(\lambda_1+\lambda_2)/2$ and σ is an MGU of t and r.

Then, $C_1{}^*$ and $C_2{}^*$ are two clauses as follows:

$$\lambda_1{}^* L[t] \vee C_1{}^{*\prime},$$

$$\lambda_2{}^*(r=s) \vee C_2{}^{*\prime},$$

where $\lambda_1{}^*>\lambda$, $\lambda_2{}^*>\lambda$, $C_1{}^{*\prime}$ and $C_2{}^{*\prime}$ are λ-identical with $C_1{}'$ and $C_2{}'$ respectively. Let

$$C^*=\lambda_3^* L^\sigma[s^\sigma] \vee C_1{}^{*\prime\sigma} \vee C_2{}^{*\prime\sigma}.$$

where $\lambda_3{}^*=(\lambda_1{}^*+\lambda_2{}^*)/2$. Obviously, C^* is a λ-lock-paramodulant and is λ-identical with C. Q.E.D.

Theorem 6. *Let $\lambda \geq 0.5$ and S be a set of clauses, where every literal is locked with an integer. Then, S is λE-inconsistent if and only if there is a deduction of λ-empty clause from $(S\cup\{\lambda^*(x=x)\}\cup F_\lambda)$ by λ-LH-resolution method and λ-LH-paramodulation method, where $\lambda^*>\lambda$ and F_λ is the set of λ-functionally reflexive axioms for S. (λ-empty clause is composed of λ-irrelevant literals, denoted by λ-$\square$.)*

($\Rightarrow$). Suppose K_λ is the set of λ-functionally reflexive axioms for S. Because S is λE-inconsistent, we know from theorem 2 that $(S\cup K_\lambda)$ is λ-inconsistent. From theorem 4, we know that there is a λ-LH-resolution deduction D of λ-empty clause from $(S\cup K_\lambda)$. We now show that D can be transformed into the deduction satisfying this theorem.

For each λ-LH-clash $(E_1,\ldots,E_q,N)$ in D, since $E_1,\ldots,E_q$ are λ-positive clauses, $E_1,\ldots,E_q$ must be in S or $\lambda^*(x=x)$. If $N \notin S$, then the clash is the desired one. If $N \in K_\lambda$, N is clearly not $\lambda^*(x=x)$. Then we have the following four possible cases:

1. N is $(1-\lambda^*)(x=y) \vee \lambda^*(y=x)$ and $\lambda^*>\lambda$. Obviously, there must be q=1 and $E_1=(\lambda_1(t_1=t_2) \vee E_1{}')$, where $\lambda_1>\lambda$. Therefore,

$$(E_1,N)=(\lambda^*(t_2=t_1) \vee E_1{}').$$

In addition,

$$P_\lambda(\lambda_1(x=x),\lambda_1(t_1=t_2) \vee E_1{}')$$
$$=(\lambda_1(t_2=t_1) \vee E_1{}').$$

Because (E_1, N) is a λ-LH-clash, $P_\lambda(\lambda_1(x=x), E_1)$ must be a λ-LH-paramodulant and (E_1, N) is λ-identical with $P_\lambda(\lambda_1(x=x), E_1)$.

2. N is $(1-\lambda^*)(x=y) \vee (1-\lambda^*)(y=z) \vee \lambda^*(x=z)$ and $\lambda^*>\lambda$. Obviously, we have q=2 and two clauses E_1 and E_2 as follows:

$$\lambda_1{}^*(t_1=t_2) \vee E_1{}',\lambda_2{}^*(s_1=s_2) \vee E_2{}',$$

where $\lambda_1{}^*>\lambda$, $\lambda_2{}^*>\lambda$. Thus,

$$(E_1, E_2, N)=(\lambda^*(t_1^\sigma=s_2^\sigma) \vee E_1{}'^\sigma \vee E_2{}'^\sigma),$$

σ being an MGU of t_2 and s_1. Clearly,

$$P_\lambda(E_1,E_2)=(\lambda_1{}^*+\lambda_2{}^*)/2(t_1^\sigma=S_2^\sigma) \vee E_1{}'^\sigma \vee E_2{}'^\sigma.$$

We can see that $P_\lambda(E_1, E_2)$ is a λ-LH-paramodulant and λ-identical with (E_1, E_2, N).

3. N is $(1-\lambda^*)(x_j=x_0) \vee (1-\lambda^*)P(\ldots x_j \ldots) \vee \lambda^* P(\ldots x_0 \ldots)$, with $\lambda^*>\lambda$. Obviously, we have q=2 and E_1 and E_2 are two clauses as follows:

$$\lambda_1(t_j=t_0) \vee E_1{}',\ \lambda_2 P(\ldots s_j \ldots) \vee E_2{}',$$

where $\lambda_1>\lambda$ and $\lambda_2>\lambda$. Thus

$$(E_1, E_2, N)=(\lambda^* P(s_1^\sigma \ldots t_0^\sigma \ldots s_n^\sigma) \vee E_1{}'^\sigma \vee E_2{}'^\sigma),$$

where σ is an MGU of $(x_1 \ldots t_j \ldots x_n)$ and $(s_1 \ldots s_j \ldots s_n)$.

Clearly,

$$P_\lambda(E_2, E_1)=(\lambda_1+\lambda_2)/2P(s_1^\sigma \ldots t_0^\sigma \ldots s_n^\sigma) \vee E_1{}'^\sigma \vee E_2{}'^\sigma.$$

We can see that $P_\lambda(E_2, E_1)$ is a λ-LH-paramodulation and λ-identical with (E_1, E_2, N).

4. N is $(1-\lambda^*)(x_j=x_0) \vee \lambda^*(f(\ldots x_j \ldots)=f(\ldots x_0 \ldots))$, where $\lambda^* > \lambda$.

Obviously, we have q=1 and E_1 is a clause as follows:

$$\lambda_1(t_j=t0) \vee E_1',$$

where $\lambda_1 > \lambda$. Thus,

$(E_1, N)=$

$(\lambda^*(f(x_1 \ldots t_j \ldots x_n)=f(x_1 \ldots t_0 \ldots x_n)) \vee E_1')$

Clearly, λ-paramodulant of $\lambda_1(f(x_1 \ldots x_j \ldots x_n)=f(x_1 \ldots x_j \ldots x_n))$ and E_1 is a clause as follows:

$P_\lambda(\lambda_1(f=f), E_1)=$

$(\lambda_1(f(x_1 \ldots t_j \ldots x_n)=f(x_1 \ldots t_0 \ldots x_n)) \vee E_1')$

We can see that $P_\lambda(\lambda_1(f=f), E_1)$ is a λ-LH-paramodulant and λ-identical with (E_1, N).

From Theorem 5, we know that each λ-LH-Clash in D can be transformed into a λ-LH-paramodulant of two clauses which belong to the set $(S \cup \{\lambda^*(x=x)\} \cup F_\lambda)$.

Therefore, we obtain a new deduction D' of λ-empty from $(S \cup \{\lambda^*(x=x)\} \cup F_\lambda)$ by using -LH-resolution method and λ-LH -paramodulation method.

(⇐). If there is a deduction of λ-empty clause from $(S \cup \{\lambda^*(x=x)\} \cup F_\lambda)$ by using λ-LH -resolution and λ-LH -paramodulation, but S is not λE-inconsistent, then there is an E-interpretation I_E such that $T_{I_E}(S) > \lambda$. Thus, for each clause C in $(S \cup \{\lambda^*(x=x)\} \cup F_\lambda)$, there must be $T_{I_E}(C) > \lambda$. From Property 11, Property 12 and theorem 3, we can easily see that $T_{I_E}(\lambda\text{-}\square) > \lambda$, which contradicts the definition of λ-□.

Q.E.D.

V. LINEAR λ-PARAMODULATION

Definition 17. Given a set S of clauses and a clause C_0 in S, a linear λ-deduction of C_n from S with top clause C_0 by λ-resolution and λ-paramodulation is a deduction of the form shown in Fig. 1, where

1. For i=1,...,n-1, C_{i+1} is a λ-resolvent or a λ-paramodulation of C_i (called a center clause) and B_i (called a side clause).
2. Each B_i is either in S, or is a C_j for some j<i.

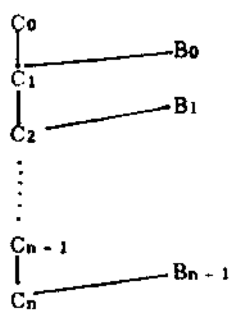

Figure 1

A Linear λ-refutation by λ-resolution and λ-paramodulation is a linear λ-deduction of λ-□ by λ-resolution and λ-paramodulation.

Definition 18. Let $\lambda \geq 0.5$, and S be a set of clauses. S_R is said to be a λ-reduction of S if and only if S_R is obtained by the following replacing: for any literal $\lambda^* L$ S,

1. if $\lambda^* \leq \lambda$ and $1-\lambda \leq \lambda^*$, delete $\lambda^* L$.
2. if $\lambda^* > \lambda$, $\lambda^* L$ is replaced by L.
3. if $1-\lambda > \lambda^*$, $\lambda^* L$ is replaced by $\sim L$.

Let S_R denote the λ-deduction of S, C_R denote the λ-deduction of C.

Theorem 7. ***Let*** C_{1R} ***and*** C_{2R} ***be the λ-reducing clauses of*** C_1 ***and*** C_2 ***respectively. If C' is a resolvent (or a paramodulant) of*** C_{1R} ***and*** C_{2R}***, then there is a λ-resolvent (or a λ-paramodulant) C of*** C_1 ***and*** C_2 ***such that*** $C_R = C'$***.***

Proof. If C' is a resolvent of C_{1R} and C_{2R}, we know that this theorem is correct. [1]

If C' is a paramodulation of C_{1R} and C_{2R}, without loss of generality, we may assume that C' is a binary paramodulant, let

$C_1 = \lambda_1 L[t] \vee C_1'$, $\lambda_1 > \lambda$ or $\lambda_1 < 1-\lambda$,

$C_2 = \lambda_2 (r=s) \vee C_2'$, $\lambda_2 > \lambda$.

Then
$$C_{1R} = \begin{cases} L[t] \vee C_{1R}', & \lambda_1 > \lambda, \\ \sim L[t] \vee C_{1R}', & \lambda_1 < 1-\lambda, \end{cases}$$
$$C_{2R} = (r=s) \vee C_{2R}'.$$

Therefore
$$C' = \begin{cases} L^\sigma[s^\sigma] \cup C_{1R}'^\sigma \cup C_{2R}'^\sigma, & \lambda_1 > \lambda, \\ \sim L^\sigma[s^\sigma] \cup C_{1R}'^\sigma \cup C_{2R}'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$$

Since
$$C = \begin{cases} [(\lambda_1+\lambda_2)/2] L^\sigma[s^\sigma] \cup C_1'^\sigma \cup C_2'^\sigma, & \lambda_1 > \lambda, \\ [(\lambda_1+1-\lambda_2)/2] L^\sigma[s^\sigma] \cup C_1'^\sigma \cup C_2'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$$

But when $\lambda_1 > \lambda$, $(\lambda_1+\lambda_2)/2 > \lambda$ and when $\lambda_1 < 1-\lambda$, $(\lambda_1+1-\lambda_2)/2 < 1-\lambda$.

Then
$$C_R = \begin{cases} L^\sigma[s^\sigma] \cup C_{1R}'^\sigma \cup R_{2R}'^\sigma, & \lambda_1 > \lambda, \\ \sim L^\sigma[s^\sigma] \cup C_{1R}'^\sigma \cup C_{2R}'^\sigma, & \lambda_1 < 1-\lambda. \end{cases}$$
$$= C'.$$

Q.E.D.

Theorem 8. ***If C is a clause in an λE -inconsistent set S of clauses including*** $\lambda^*(x=x)$ ***and the set*** F_λ ***of λ-functionally reflexive axioms for S and if S-{C} is λE-satisfiable, then S has a linear λ-refutation by λ-resolution and λ-paramodulation with top clause C.***

Proof. According the theorem above we know that S is λE -inconsistent if and only if $(S \cup K_\lambda)$ is λ-inconsistent if and only if $S_R \cup K$ is inconsistent. (K is the λ -reduction of K_λ).

Since S-{C} is λE--satisfiable, clearly, S_R-{C_R} is E~satisfiable, then SR has a linear refutation D by resolution and paramodulation with top clause CR [4]. By theorem 7, for every resolvent R{C_{1R}, C_{2R}} or paramodulant P{C_{1R}, C_{2R}} in D, there is a λ- resolvent R {C_1, C_2} or λ- paramodulation P_λ(C_1, C_2), and

$$R(C_{1R}, C_{2R})=(R_\lambda(C_1, C_2))_R$$
$$P(C_{1R}, C_{2R})=(P_\lambda(C_1, C_2))_R.$$

Therefore, we can obtain a linear deduction of λ-empty clause from S, by A~resolution and x-paramodulation.

Up to now we have obtained λ-paramodulation method in operator fuzzy logic. This is an inference rule for the fuzzy equality relation. λ-paramodulation is essentially an extension of the fuzzy equality substitution.

Because OFL describes fuzzy propositions naturally [6], the combination of λ-resolution and λ-paramodulation is convenient in doing fuzzy reasoning.

References

[1] Liu, X. H. & Xiao, H., Operator fuzzy logic and fuzzy resolution, Proc. of the 15th ISMVL, Canada, 5(1985),68-75

[2] Liu, X.H. & Fang, K.Y., Fuzzy reasoning on λ-horn set, Proc. of the 16th ISMVL, U.S.A., 5(1986), 248-251

[3] Liu, X.H., Chang, Carl K. & Jeffery J-P Tsai, Fuzzy reasoning base on λ- LH-resolution, Proc. of the *IEEF* 10th International Computer Software Application Conference, U.S.A., 10(1986), 154-157

[4]Chang, C. L. & Lee, R.C.T., Symbolic Logic and Mechanical Theorem Proving, Academic Press, New York, 1973.

[5] Lee, R.T., Fuzzy Logic and the Resolution Principle, Journal of ACM, 19(1972), 1:109-119

[61 liu, X.H., Fang, K.Y. & Jeffery J-P Tsai, Fuzzy reasoning in operator fuzzy logic, Proc. of 19th IMSVLS-89,China, 5, 1989.

[71 Liu, X.H. & Yang, F.J. Linear λ-paramodulation" (to appear).