of 0.7 for generations 13 to 19 (after creation of a perfect-scoring cubic polynomial). The fitness level again abruptly dropped to virtually 0 for generation 20 when the environment again changed. However, by generation 22, a fitness level again stabilized in the neighborhood of 0.7 after creation of a new perfect-scoring quadratic polynomial.

6 Theoretical Discussion

Hierarchical genetic algorithms employ the same automatic allocation of credit inherent in the basic string-based genetic algorithm described by Holland (1975) and inherent in Darwinian reproduction and survival of the fittest amongst biological populations in nature. In hierarchical genetic algorithms, the individuals in the population are LISP S-expressions (i.e.rooted point-labeled trees in a plane) instead of linear character strings. The set of similar individuals sharing common features (i.e.the schemata) is the hyperspace of LISP S-expressions sharing common features. This infinite set can be partitioned into finite subsets by using the number of points as the partitioning parameter. If the subset sharing common features with a specified value of this parameter is considered, fitness proportionate reproduction causes growth or decay in the size of that subset in the new population in accordance with the relative fitness of the subset to the average population fitness in the same way as it does for string-based linear genetic algorithms (with the associated approximately near optimal allocation of trials). The deviation from this approximately near optimal rate of growth or decay is relatively small if the number of points defining the common feature is relatively small and to the extent that the points defining the common feature are coextensive with one subtree. Thus, the overall effect of fitness proportionate reproduction and crossover is that subprograms (i.e. sub-trees, sub-lists) from relatively high fitness individuals are used as "building blocks" for constructing new individuals and the search is concentrated for successive populations into sub-hyperspaces of S-expressions of ever decreasing dimensionality and ever increasing fitness.

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