On Semantics of TMS

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ABSTRACT

In this paper we first give a formal semantics of non-monotonic TMS theory with CP justifications. Then we prove that the model of a theory J is also a model of theory J*(I). Next we conclude thai for every TMS theory J, there must be a theory J* such that J* has no CP justifications and all the models of J is also J*'s. Finally we prove that the concept of extension defined by U. Junker and Kun Konolige is also correct under our definition.

Keywords: TMS, Extension, Semantic, Non-Monotonic, CP, SL, Justification

1. Introduction

TMS (Truth Maintenance System) is first proposed by Doyle in 1979, which is a technique to maintain the consistency of belief and find the source of the contradiction (Dependency-Directed Backtracking). It is more proper to call TMS by BMS (Belief Maintenance System) or RMS (Reason Maintenance System). Traditionally we still call it TMS in this paper. Because TMS plays an important rule in KB, non monotonic reasoning, pattern recognition, and all the other fields in which the knowledge will be adjusted because of the new knowledge's accumulation or introspection, there has been much researches work on TMS in this decade. In this paper we do not intend to introduce the results in (his respect. What we want to point out is that (here has been less research on the TMS's basic theory, especially on its formal semantics. This phenomenon can be explained as the following: In this decade, TMS is always studied as a technique, and it seems to be a practical non-monotonic belief maintenance technique by birth. Since TMS is NP-hard, much attention has been attracted to how to make it more practical. The computation complexity of non-monotonic theory has been known since the beginning of 80s, there is still more researches on the beautiful form of non-monotonic reasoning and on the theoretically formal proofs of non-monotonic theory. Although there are some comparison between TMS and non-monotonic reasoning, the results in mis field art few and not deep after all.

When we study how to implement a practical non-monotonic reasoning system, we find that it must contain at least two basic pans: one is the non-monotonic inference mechanism, the other is TMS. This opinion complies with the one in reference [B. Smith and G. Kelleher, 1988]. When the default reasoning based on Horn logic is combined with TMS fWang Xianchang and Chen Huowang, 1990(a),(c)l, we find that the first problem we meet is to get the formal semantics of TMS. Only by this way can we ensure the consistency between the non-monotonic inference mechanism and the belief maintenance system. But many of the concepts in TMS are not clear. There are different understandings for these concepts among different people. For example what is the correct meaning of CP justification? What should be a premise node? When we replace a valid CP justification by the so called equally SL justification , is the new TMS theory equal the original theory? We notice that in resent years, there are some researches on the formalization of TMS, but what they study is the TMS with no CP justification. For example. Ulrich Junker and Kurt Konolige make comparison between the relations among TMS which have no CP justification with AEL and DL. They discuss how to translate the AEL, DL to TMS [U. Junker and K. Konolige. 1990]. This kind of work is also very important in our research work [Wang Xianchang and Chen Huowang, 1990(a), (b), (c)].

In this paper, we first define the TMS language and the TMS theory. Then we define the formal semantics of TMS. Next we prove the valid transformation of the CP justification to SL justifications. Finally we discuss the semantic relation between ours and U Junker semantics of TMS with no CP justification.

2. TMS Theory

The basic language of TMS is a prepositional language. In TMS a statement with variables is not allowed.

Def 1. p,q,p1,q1,...,pi,qi,... are propositions.

Def 2. Suppose r is a proposition, then r and (~r) are nodes.

In general, a TMS theory contains finite nodes. Suppose NS is a set of nodes, here NS is always finite. Node r, (~r) have no semantics relation in TMS. They are formally non-related statements , although we always regard them as excluding each other.

Def 3. SL (Support List) justification

The SL justification on node set NS is a formula as following:

p <-- in(I),out(O)

Here p is in NS, I and O are subset of NS, I∩O= {}. p is called the consequence of above SL justification, (in(I),out(O)) is called one of p's reason. The node in I is called p's premise node, the node in O is called p's default premise node.

Def 4. CP(Conditional Proof) justification

The CP justification on node set NS is a formula as following:

p <-- q, in(I),out(O)

Here p,q are in NS, I and O are subset of NS, I∩O={}.

The informal meaning of above CP justification is:

If there is a non-monotonic proof of q under premise (in(I),out(O)), then p is believable.

Doyle has not discussed the meaning of CP justification in the case O=/= {}. In this paper we mainly discuss the general case. We ensure that when O=={} the meaning of our definition of CP justification complies with Doyle's original meaning.

Def 5. TMS theory.

A TMS theory J on node set NS is based on finite SL justifications or CP justifications.

Def 6. J(l)

Suppose J is a TMS theory on node set NS, l is a CP justification, l = p <-- q, in(I), out(O), Then we define a TMS theory J(l) as:

J(l)= (J - {l} + { p'<-- l for every p' in I}) - { l1 | l1 is a SL justification of J, and p' is l1's consequence, p' is in O }

Example 1. Consider the following five TMS theories. It is not necessary to list the corresponding node sets of each theory.

J1= { p<-- in(q), q<-- in(p) }

p, q are the premise of each other.

J2= { p<-- out(q), q<-- out(p) }

p, q are the default premise of each other,.

J3= { p<--q, in(q1), q<--in(q1) }

If q1 is a belief of theory J3, then q is also a belief of theory J3; *Suppose q1 is believable, if we can get that q is also believable, then p* is a belief.

J4 = { p2<--in(p1,p4), p3<--in(p2,p5), p4<--, p5<--, p6<--p3, *in(p1)* }

This theory can be referred to in [J. Doyle, 1979].

J5 = { p<--q1, q1<-- out(q2),q2<--out(q1) }

If there is a non-monotonic proof of q1 then p is believable.

3. The Semantic Study on TMS Theory.

3.1 Definitions.

Def 1. Assignment f.

Suppose J is a TMS theory on node set NS, then an assignment f of J is a map, f: NS ---> {in out}, such that:

For every node d in NS, f(d)=in iff

Either J has a SL justification, l= d<-- in(I),out(O) such that for every p in I,f(p)=in, and for every node in O, f(p)=out, l is called a valid SL justification.

Or J has a CP justification l = d <-- p, in(I),out(O) such that J(l) has a model f' such that f'(p)=in, l is called a valid CP justification.

Def 2. In assignment f of theory J,we say node p is an in-node if f(p)=in, else we call it out-node.

Suppose p is an in-node, if it has a valid SL justification p<--in(I),out(O), then (I + O) is called p's support set, denoted by Sup(J,f,p). Else,p has an valid CP justification l=p<--q,in(I),out(O) then Rant(J(l), f', I + O, q) is Sup(J, f, p), here f' is the theory J(l)'s model which make q believable. Rant is defined as following.

Def 3. Suppose p is an in-node of theory J in assignment f. We define p's ancestor set, Ant(J,f,p) which is the minimal set satisfying the following conditions:

1. Sup(J,f,p) is a subset of Ant(J,f,p)
2. If in-node q is in Ant(J,f,p), then Sup(J,f,q) is a subset of Ant(J,f,p)

Def 4. Repercussions set.

Suppose J is a TMS theory on node set NS, f is a model of J, for every p in NS we define p's repercussions set , Rep(J,f,p) :

Rep(J,f,p)= {q | q is an in-node and p is in Ant(J,f,q)}

Obviously any node whether it is in-node or out-node,can be one of the reason of an another in-node.

Def 5. Suppose S is a subset of NS, f is an assignment of theory J on node set NS. Then we define Rep(J,f,S) as

1. if S={} then Rep(J,f,S)={}
2. if S= S1 + {p} then Rep(J,f,S)= Rep(J,f,p) + Rep(J,f,S1)

Def 6. Related-ancestors set

Suppose p is an in-node of theory J in assignment f, then the related-ancestors set under premise set S, Rant(J,f,S,p) is the maximal set which satisfies the following conditions:

1. Rant(J,f,S,q) is a subset of Ant(J,f,q)
2. Rant(J,f,S,q) ∧ S = {}
3. Rep(J,f,S) ∧ Rant(J,f,S,p) = {}

The meaning of Rant(J, f, S,q) would be used to define what the premise argument of an valid CP justification.

Suppose CP justification l= p<--q,in(I),out(O) is valid in assignment f of theory J, and f' is a model of theory J(l) such that f'(q)= in. q's related-ancestors set under premise set S= I + O is Rant(J(l), f', S, q). Then we define p's support node set under f is Rant(J(l), f', S, q).

The concept of assignment has involved an undefined concept "model", in the following we will define it recursively.

Def 7. An assignment f of TMS theory J is a model iff

1. For every in-node p of J, p is not in Ant(J, f, p)
2. For every valid CP justification l, J' is defined as def 2 in section 3.4, f is an assignment of J'.

Def 8. We call TMS theory J's assignment f is well-defined iff every in-node p of f, p isn't in set Ant(J, f, p). Obviously the definition of model of TMS theory is very complicated , but it will became very simple and easily understand if the theory has no CP justification. In the following we will discuss the way to translate the CP justification to SL justification, and discuss the correctness of this translation.

3.2 Example 2.

According to the above definitions, we consider the theories of example 1.

J1 = { p<--in(q), q<--in (p)}

J1 has only one model: f1= {p--->out, q--->out}

J2 = { p<--out(q), q<--out(p)}

J2 has only two models:

f2,1 ={ p--->out, q--->in }

f2,2 ={ p--->in, q--->out}

J3 = {p<--q,in(q1), q<--in(q1)}

J3 has only one model: f3 = {p--->in, q--->out, q1--->out }

J4 = {p2<--in(p1,p4), p3<--in(p2,p5),p4<--, p5<--, p6<--p3, in(p1) }

J4 has only one model: f4= {p1--->out,p2--->out, p3--->out, p4---> in, p5--->in, p6--->in}

J5 = {p<--q1,q1<--out(q2), q2<--out(q1) }

J5 has two models:

f5,1 = {p--->in, q1--->in, q2--->out }

f5,2 = {p--->in, q1--->out, q2--->in }

3.3 The Semantics Consideration on CP Justification.

In section 3.1, the meaning of a CP justification is as following:

A theory J's CP justification l= p<--q,in(I),out(O) is valid in model f iff there is a model f' of theory J(l) such that f'(q)=in.

Obviously

(a) Whether J's CP justification l is valid in model f has nothing to do with f, its validity only depend upon J.

(b) Suppose |—— is a relation :

J |——p iff TMS theory J has a model f such that f(p)=in.

Suppose ||—— is a relation:

J ||——p iff for every model f of TMS theory J f(p)=in.

Then TMS has the following properties:

1. J + {p<--p} ||—— p iff J |——p
2. Suppose I is a subset of node set NS, then J + {p<-- | p is in I} |——q iff J + {q<--q,in(I)} ||——q.
3. The validity of CP justification express the "non-monotonic provableness"

For example. a CP justification l = p<--q,in(I),out(O) is valid in theory J iff J(l) |——q.

Non-Monotonic properties:

From J |——q we can not conclude J + {p<--p} |——q

For example. J2 |——q, but J2 + {p<--p} |-\-q.

4. if J |——q then we have J + { p<--q } ||——p.

(c) A TMS theory may have no model.

Example 3.

J6 = {p<--out (p)}

J7 = {p1--out(p2), p2<--out(p3), p3<--out(p1)}

The above two theories have no model. This kind of theory is called paradox theory.

(d) We can modify the semantics definition of *CP justification.* For example we can modify the "J(l) has a model " of def.1 in section 3.1 by "for all model of J(l)".

Under this interpretation J5 in example 2 will have other two different models:

f'5,1 = {p--->out, q1--->in, q2--->out }

f'5,2 = {p--->out, q1--->out, q2--->in }

(e) It may be very difficulty to give a definition by which the validity of CP justification depends upon both the theory and the present model. *The importance to find such definitions is not clear now, but it can be* studied further.

3.4 Translation of CP to SL Justification.

In order to simplify the complex of TM processing, Doyle gives a method to translate a valid CP justification to so called "equally" SL justification, maybe it is this unproved "equality" that makes the followed researchers turn to the TMS theory with no CP justification. But our research shows that this "equality " is false.

First we define what is the translated SL justification in theory J under model f through a valid CP justification.

Def 1. Suppose CP justification l= p<--q,in(I),out(O) is valid in model f of theory J, f' is a model of theory J(l) such that f'(q)= in. Suppose Sup(J,f,p) is Rant(J(l), f', I + O, q) then we define the translated SL justification of CP justification under J,f,f' is:

p<-- in(I'),out(O') such that

1. I' + O' = Rant(J(l), f', I + O, q)
2. I'∧O'={}
3. For every r in I', f(r)=in;
   For every r in O', f(r)=out.

Example 4. (Continue example 2)

First we consider J3.

J3'=J3(p<--q,in(q1)) = {q<--in(q1), q1<--}, it has only one model f3'= {q1--->in, q--->in}, Rant(J3', f3', {q1}, q) = {}. So the translated SL justification of p<--q,in(q1) under model f3 is:

p<--

Second we consider J4.

J4'=J4(p6<--p3,in(p1)) has only one model f4'={ p1--->in, p2--->in, p3--->in, p4--->in, p5--->in, p6--->out}. Rant(J4', f4', {p1}, p3) = {p4,p5}, so the translated SL justification of p6<--p3,in(p1) is:

p6<--in(p4,p5)

J5'= J5(p<--q1)={q1<--out(q2),q2<--out(q1)}, it has one model f5' = {q1--->in, q2-->out}. Since Rant(J5',f5',{q1},p) = {q2}, the translated SL justification of p<--q1 under (f5,1), f5'is:

p<--out(q2)

The translated SL justification of p<--q1 under (f5,2), f5' is p<-- in(q2)

Def 2. Suppose f is a model of TMS theory J. CP justification l = p<--q,in(I),out(O) is valid in f. Suppose p<--in(I'),out(O') is the translated SL justification of l, then theory J'= J - { l } + { p<--in(I'), out(O') } is called one replaced theory of J under model f.

Obviously we have following theorem.

Theorem 1. Suppose f is a model of theory J, J' is one replaced theory of J under model f. Then f is still a model of J'.

In general some models of J may be not the models of J', and some models of J' may be not J's models.

Example 5. Consider Theory J5 of example 4.

Obviously one replaced theory of J5 under model f5,1 is J'5,1 = { p<--out(q2), q1<--out(q2), q2<--out(q1) }. This theory has two models: one is f5,1 , the other is f'5,2 = {p-->out, q1-->out, q2--->in }. Obviously f'5,2 is not J5's model and the model f5,2 of J5 is also not the model of J'5,1.

Def 3. Suppose J has a valid CP justification l = p<--q, in(I), out(O), we define a translated theory of J as J*(l), which is: J*(l) = J - {l} + { p<--in(I'),out(O') | there is a model f of J such that p<--in(I'),out(O') is a translated SL justification of l under model f}

It is easy to prove

Theorem 2. Suppose l is a valid CP justification in theory J then for every model f of J, f is still a model of J*(l).

Example 6.

Consider TMS theory J4 of example 3. CP justification l4= p6<--p1,in(p3) is valid, it is easy to get that J4*(l4)= { p2<--in(p1, p4), p3<--in(p2, p5), p4<--, p5<--, p3<--in(p4, p5)}. Obviously every model of J4 is also the model of J4*(l4).

Consider TMS theory J5 of example 3. l5= p<--q1 is valid in J5,J5*(l5)={ q1<--out(q2), q2<--out(q1), q<--out(q2), q<--in(q2) }. Obviously all the models of J5 are also J5*(l3)'s models.

Whether the model of J*(l) is the model of J? the answer is not clear now.

From theorem 1 and theorem 2, we can conclude that:

Theorem 3. Suppose J is a TMS theory, then there must be a TMS theory J* such that

1. J* has no CP justifications.
2. Every model of J is also the model of J*.

4. Comparison With U. Junker and Kurt Konolige's Work

U. Junker and Kurt Konolige give a Semantics of TMS theory with no CP justification. Here we do not want to introduce their work, the concepts appeared bellow can be referred in [U. Junker and K. Konolige, 1990]. The following are our main conclusions.

Def 1. Suppose J is a TMS theory on node set NS, f is J's model, then we define T(f) = {c | c is in NS, and f(c)=in } as an extension of J.

Now is U. Junker's extension the same as ours?

In the following all the TMS theories we consider have no CP justifications.

Lemma 1. f is a model of TMS theory J iff

$$apply_{J,T(f)} \quad (T(f)) = T(f)$$

Proof: We can get it from the definition of model f and the function apply.

Lemma 2. For any model f of theory J, we have

$$apply^{*}_{J,T(f)} \quad (\{\}) \subseteq T(f)$$

Proof: Because

$$apply^{*}_{J,T(f)} \quad (\{\}) = \bigcup_{i=0}^{\infty} apply^{i}_{J,T(f)} \quad (\{\})$$

So we can get lemma 2 after we prove the following by inductive method:

for every i=0,1, ..., n,...

$$apply^{i}_{J,T(f)} \quad (\{\}) \subseteq T(f)$$

Lemma 3. If f is a model of TMS theory J, then

$$apply^{*}_{J,T(f)} \quad (\{\}) = T(f)$$

Proof: we can construct a function t:NS--->{0,1,...} such that

1. if p is an out-node of f then t(p) is 0.
2. if SL justification p<--in(I),out(O) is in-node p's valid support justification, then t(p) is defined as
   1 + Max(0, { t(q) | q is in I}

Because every in-node p is not in Ant(J, f, p), the above function t is well defined. It is easy to proof that for every in-node p

$$p \text{ is in } \quad apply^{t(p)}_{J,T(f)} \quad (\{\})$$

so T(f) is a subset of $apply^{*}_{J,T(f)} \quad (\{\})$

From lemma 2 and lemma 3 we can get lemma 4.

Lemma 4. f is a model of TMS theory J iff

$$T(f) = apply^{*}_{J,T(f)} \quad (\{\})$$

Lemma 5. Suppose J is a TMS theory on node set NS. T⊆NS, if

$$T= apply^{*}_{J,T} \quad (\{\})$$

Then the following three properties hold.

a. $T = apply_{J,T} \quad (T)$
b. $T = apply^{*}_{J,T} \quad (\{\})$
c. There is no T0⊂T such that
   $T0 = apply_{J,T0} \quad (T0)$

Proof: The properties a,b can be found in [U. Junker and K. Konolige, 1990]]. Now we prove c. Suppose c is not hold then there must be a T0 such that

T0⊂T and

$T0 = apply_{J,T0} \quad (T0)$

Because $T0 \subset T$, $\{\} \subseteq T0$ so

$apply_{J,T0} \quad (T0) \supseteq apply_{J,T} \quad (T0) \supseteq apply_{J,T} \quad (\{\})$

Suppose for i

$T0 = apply_{J,T0} \quad (T0) \quad \supseteq \quad apply^{i}_{J,T} \quad (\{\})$

then

$$T0 = apply_{J,T0} \ (T0) \supseteq apply_{J,T} \ (apply_{J,T0} \ (T0))$$
$$\supseteq apply_{J,T} \ (apply^{i}_{J,T} \ (\{\}))$$
$$= apply^{i+1}_{J,T} \ (\{\})$$

So we have proven the following by inductive method

for every $i=0,1,\ldots,n,\ldots$

$T0 \supseteq apply^{i}_{J,T} \ (\{\})$

$$\text{So } T0 \supseteq \bigcup_{i=0}^{\infty} apply^{i}_{J,T} \ (\{\}) = T$$

So $T0 \supseteq T$, this is contrary to $T0 \subset T$.

c is proved. Q.E.D

From above three lemmas, we can get.

Theorem 6. J is a TMS theory with no CP justification, then E is an extension of what U. Junker and K. Konolige define iff E is also an extension of what we define. In other words, J must have a model f such that T(f) = E .

This theorem shows that the extension concept defined by U. Junker and Kurt Konolige is correct in our semantics frame.

5. Conclusion

In (his paper we first give a format semantics of nonmonotonic TMS theory with CP justifications. Then we prove that the model of a theory J is also a model of theory J*(I). Next we conclude that for every TMS theory *J,* there must be a theory J* such that J* has no CP justifications and all the model of J is also J*s; Finally we prove that the concept of extension defined by 11 Junker and Kurt Konolige is the same under our definition. *It is* necessary *to point* out that there are more *properties* of CP justification left for us to study. From our discussion, it seems that the CP justification has a close relation with "non-monotonia provenance".

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