W — A Logic System Based on Shared Common Knowledge Views

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ABSTRACT

In this paper, we give a logic system — W based on the view of shared common knowledge, and prove some properties of W. By W, we effectively describe and solve the Conway Paradox, the typical multi-agent problem involving common knowledge.

Keywords: Common Knowledge, Agent, Proof, Logic, Reasoning.

1. Introduction

Undoubtedly, the representation of and reasoning of common knowledge is a typical problem in KR (Knowledge representation and Reasoning) research; It is typical because it plays an important role in multi-agent reasoning, communication, distributed computing system, etc.. But what is common knowledge? How do we use common knowledge? Just as knowledge, there are different opinions about these questions among different people. But everyone agree that the representation of and reasoning of common knowledge depends on the representation of and reasoning of knowledge in which common knowledge exists. To put it briefly, there arc three opinions about common knowledge [Barwise 1988]. One is "infinite level" or "iterate level" opinion [Lewis 1969] Say ϕ is common knowledge for the agent group G, if

1. each agent in group G knows ϕ;
2. each agent in group G knows the fact " 1 " ;

n+1. each agent in group G knows the fact "n";

It is shown that [Halpern & Moses 1984], this kind of knowledge can not be gained in a synchronous and most distributed processing systems.

The second is the so called "fixed point" opinions [Harman 1977]. By introducing an operator C_G, we get a complete axiomatization to express the common knowledge's properties. The axioms and inference rules involved are follows [Halpern and Moses 1985]:

C1. $E_G \phi \equiv \& \{ Ki \phi \mid i \in G\}$
C2. $C_G \phi \& (C_G (\phi \rightarrow \varphi)) \rightarrow C_G \varphi$
C3. $C_G \phi \equiv E_G (\phi \& C_G \phi)$
RC1. $\phi \rightarrow E_G \phi \Rightarrow \phi \rightarrow C_G \phi$

C3 is called C_G's fixed point axiom, and RC1 is the common knowledge inductive inference rule. Obviously, this opinion of common knowledge characterizes the properties of common knowledge, but it gives no hint on how to use it.

The last is the shared environment common knowledge opinion where every knowledge is associated itself with a situation and each situation is also every agents' cognizable object [Clark and Marshall 1981]. For example, let p, q be two agents, we have three statements under situation S:

1. $S \models \phi$
2. $S \models KpS$
3. $S \models KqS$

Then ϕ is p and q's common knowledge. This opinion of common knowledge leads to the outcome of PROSIT system [Nakashima, et al 1991]. Undoubtedly, situation logic can express and apply common knowledge effectively; Can other logics, for example classical modal logic, express and apply the shared common knowledge effectively? In this paper, the answer is "Yes".

The paper is organized as follows: In section 2, we discuss the characteristics of logic under the shared common knowledge, we introduce the conception of "common knowledge agent" (we simply call it fool reasoner). Subsequently, an axiom logic system W based on knowledge language is given, it characterizes the multi-agent reasoning system based on shared common knowledge. In section 3, we briefly describe the proof properties of W, and conclude that: W is monotonic, powerful and can do common sense reasoning; W has four levels of contradictions. This means that local contradiction can not spread into global contradiction, one agent's contradiction can not lead other agents to be contradictive. In section 4, we prove that Conway Paradox can be easily solved in W. At last we give the future work under development.

2. Logic Under Shared Common Knowledge Views

In order to express shared common knowledge in logic, one of the obvious methods is to introduce a "virtual agent" in logic. This agent's knowledge is common knowledge, so it is also be called "fool reasoner" This agent must have following properties:

1. Common knowledge agent (denoted by 0) must believe all tautology instance statement.
2. If 0 believes p, then 0 also believes that every agent believes p Here 'believe' has the same meaning as 'know'.
3. If 0 believes p, then p must necessarily be true.
4. If Kip is common knowledge, then p is also common knowledge.
5. How to use 0's knowledge is also common knowledge. For example, "if agent i believes p and $p \rightarrow q$ is common knowledge, then agent i believes q" is also common knowledge.

Every agent can have his own axioms and inference rules. The inference rules can be nonmonotonic (Wang 1990, Wang and Chen 1990]. In this paper, we only consider the simple case where every agent (apart from 0 agent) are same, they have no local axioms and inference rules. We especially consider how common knowledge can be used by each agent. Considering the logic be constructed for multi-agent reasoning system, we do not need the knowledge axiom: $Kip \rightarrow p$.

Definition 1. Let F0 be basic proposition set, $AG=\{0,1,\dots,n\}$ is die set of agents in which 0 is called common knowledge agent, then the well-founded formula based on F0 and AG is defined as follows:

a. Basic formula

1. If $p \in F0$, then p is basic formula.
2. If p, q arc basic formulas, then $p \,\&\, q$, $\neg\, p$ are basic formulas.

b. Formula

1. Basic formula is formula.
2. If p and q are formulas, then $\neg\, p$, $p \,\&\, q$, Kip are formulas, here $i=0,1,\dots n$.

Define $p \vee q \equiv \neg(\neg\, p \,\&\, \neg\, q)$, $p \rightarrow q \equiv \neg(p \,\&\, \neg\, q)$.

***Definition* 2. Define formula Ki1...Kilp to be simple formula, if $l \geq 0$, $i1,\dots,il \in AG$, p is basic formula.**

***Definition* 3. Let T be a set of formula, we call T to be a view of the world. In general we divide T into (n+2) parts.**

$T = Ts \vee K0T0 \vee \dots \vee KnTn.$

Here, $Ti = \{p \mid Kip \in T\}$ for $i=0,\dots,n$.

$Ts = T - (\vee \{Kip \mid p \in Ti\} \mid i=0,\dots,n\})$

Ts is called system knowledge, and Ti is called agent i's view.

***Example* 1. Conway Paradox**

During a card game both Max and Pat have an ace. If asked whether they have any knowledge about the other person's cards they will answer no and this answer won't be different if the question repeated ; But if someone tells them "At least one of you has an ace", a fact they can infer from their own cards, the answer will be 'no' for the first answering (Max) and 'yes, he/she has an ace' for the second of the two (Pat). Suppose p be the statement 'Max has an ace', q 'Pat has an ace', then the Conway Paradox's formal description is a view:

$T1 = \{p, q, K0(p \vee q), K0(K1p \vee K1 \neg\, p), K0(K2q \vee K2 \neg\, q), K1p, K2q\}$

$= \{p,q\} \vee$;system knowledge

$K0\{p \vee q, K1p \vee K1 \neg\, p, K2q \vee K2 \neg\, q\} \vee$; at least one has an ace, each man knows whether he has an ace.

$K1\{p\} \vee$;agent 1 (Max) knows p

$K2\{q\}$;agent 2 (Pat) knows q.

Obviously system's knowledge p, q can not be knowledge of every agent, so the knowledge generalization rule: $\phi \Rightarrow Ki\, \phi$ does not hold in our logic system. In W this rule is replaced by a weaker rule: $K0\, \phi \Rightarrow K0Ki\, \phi$.

***Definition* 4. W logic system.**

W's axioms:

A1. For every tautology p, we have K0p.
A2. $K0((Ki(p \rightarrow q) \,\&\, K0p) \rightarrow Kiq)$
A3. $K0((K0(p \rightarrow q) \,\&\, Kip) \rightarrow Kiq)$
A4. $K0p \rightarrow p$
A5. $K0Kip \rightarrow K0p$
A6. $K0(Kip \rightarrow KiKip)$

W's inference rules:

R1. $K0p \Rightarrow K0Kip$
R2. Suppose $l \geq 0$, $i1,\dots,il \in AG$ then

$Ki1\dots Kil\; p,\; Ki1\dots Kil\; (p \rightarrow q) \Rightarrow Ki1\dots Kil\; q$

When n=0, R2 become general modus ponens rule p, $p \rightarrow q \Rightarrow q$. R2 is called multilevel modus ponens rule (MMP). Now we explain what these axioms mean.

A1 says that every tautology is common knowledge.

A2 says that Tf p is common knowledge and agent i knows $p \rightarrow q$, then agent i knows q' is common knowledge. This means that every agent can do common sense reasoning by using common knowledge and its own knowledge. This axiom is called common knowledge's distributed axiom 1.

A3 says that 'if $p \rightarrow q$ is common knowledge, and agent i knows p, then agent i knows q' is common knowledge. It is called common knowledge's distributed axiom2.

A4 says that common knowledge must be system [4]s knowledge, it must be true.

A5 says that if agent i knows p is common knowledge, then p is also common knowledge*.

A6 says that [4]If agent i knows p, then agent i knows that it knows p' is also common knowledge. It means that every agent knows that each agent has positive introspective ability. In section 3, we will prove some basic properties of W

3. Basic Properties of W

Definition 5. Suppose T is a view, we define T $\vdash \varphi$, if there is a sequences of formulas: f0,...,fx, $x \geq 0$ such that:

1. $fx = \varphi$
2. For every i: $x \geq i \geq 0$,

fi is an axiom; or

fi is a formula of T; or

fi is a formula of the form K0Kip, such that there a fj, $i \geq j \geq 0$, fj is K0p, we say we have got fi by using rule R1 on fj; or

fi is in the form of Ki1...Kil q, and there is fj1,fj2 such that $i > j1,j2 \geq 0$, fj1 is Ki1...Kil($p \rightarrow q$), fj2 is Ki1...Kilp. We say fi has been got by using rule R2 on formulas fj1,fj2.

Call f0,...,fn a proof sequence of φ, and we denote {} $\vdash \varphi$ by $\vdash \varphi$.

Suppose T={f1,...,fn} (n>=0), we sometimes denote T $\vdash \varphi$ by f1,...,fn $\vdash \varphi$.

Lemma 1. For every tautology p, $\vdash$ Kip

Proof: Kip's proof sequence is: K0p, K0Kip, K0Kip $\rightarrow$ Kip, Kip.

Lemma 2. For every tautology p, i1, ..., il $\in$ AG, $\vdash$ Ki1...Kilp

Proof:

We first prove that for any $l \geq 0$ we have:

$\vdash$ K0Ki1...Kilp

We inductively prove it.

When l=0 , it obviously holds.

Suppose l=t, it also holds.

When l=t+1, Since $\vdash$ K0Ki2...Kit+1p,K0Ki2...K0Kit+1p $\Rightarrow$ K0Ki1Ki2...Kit+1p

So by using rule R1, we get $\vdash$ K0Ki1...Kilp.

Since $\vdash$ K0Ki1...Kilp K0Ki1...Kilp $\rightarrow$ Ki1...Kilp

So, $\vdash$ Ki1...Kilp.

Lemma 3. $p \rightarrow q, \neg q \vdash \neg p$

Proof:

$\neg$ p's proof sequences is:

$K0((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p))$,

$K0((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)) \rightarrow ((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p))$,

$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$,

$p \rightarrow q$,

$\neg q \rightarrow \neg p$,

$\neg q$,

$\neg p$.

Lemma 4. Let i1, ... ,il $\in$ AG then

Ki1...Kil($p \rightarrow q$),Ki1...Kil $\neg$ q $\vdash$ Ki1...Kil $\neg$ p

Proof:

Omitted.

Lemma 5.let i1,...,il $\in$ AG,p,q are formulas then

Ki1...Kil($p \rightarrow q$),K0p $\vdash$ Ki1...Kilq

K0($p \rightarrow q$),Ki1...Kilp $\vdash$ Ki1...Kilq

Ki1...Kil($p \rightarrow q$),K0 $\neg$ q $\vdash$ Ki1...Kil $\neg$ p

K0($p \rightarrow q$) ,Ki1...Kil $\neg$ q $\vdash$ Ki1...Kil $\neg$ p

Proof:

Omitted.

Lemma 6. Kip,Kiq $\vdash$ Ki(p & q), Ki(p & q) $\vdash$ Kip, Ki(p & q) $\vdash$ Kiq

Proof:

Omitted.

A general result is:

Theorem 7. Suppose i1,...,il $\in$ AG, q is a tautology consequence of p1,p2,...,pm, then Ki1...Kilp1,..., Ki1...Kilpm $\vdash$ Ki1...Kilq

Proof:

As q is a tautology of p1,p2,...,pm

so $(p1 \rightarrow (p2 \rightarrow ... \rightarrow (pm \rightarrow q))$ ...) is a tautology , so

$\vdash$ Ki1...Kil($p1 \rightarrow (p2 \rightarrow ...(pm \rightarrow q))$...)

Inductively we can prove that:

Ki1...Kilp1, ... , Ki1...Kilpm $\vdash$ Ki1...Kilq

The logic system W is strong: Every agent has enough ability to do reasoning. It should be noticed that W itself is a monotonic logic. Because there is a common knowledge agent, we can not use Kripke model to define W's model semantics directly. Its proper model semantics is under development.

Since every agent's knowledge is closed , so there are four sorts of contradictive concepts in W.

Definition 6. We define a view T is hidden contradictive, if there are i1, ... , il $\in$ AG ($l \geq 1$),formula φ such that:

$T \vdash Ki1...Kil\ \varphi$, $T \vdash Ki1...Kil \neg \varphi$

We define a view T is contradictive about agent i if there is a formula φ such that:

$T \vdash Ki\ \varphi$, $T \vdash Ki \neg \varphi$

We define a view T is commonly contradictive if there is a formula φ such that:

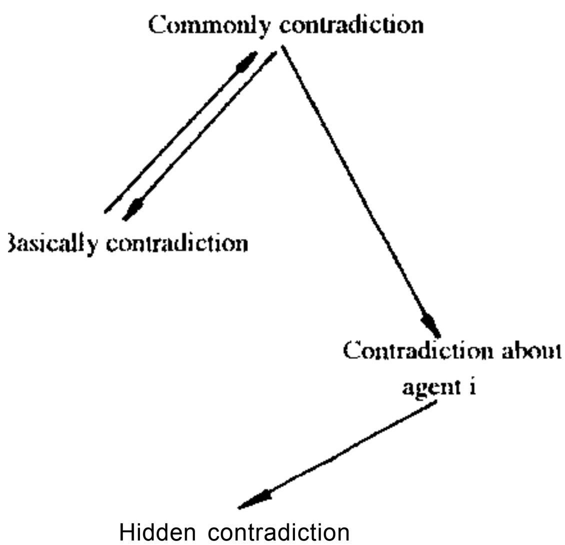
$T \vdash K0\ \varphi$, $T \vdash K0 \neg \varphi$

We say view T is basically contradictive, if there is a formula φ such that:

$T \vdash \varphi$, $T \vdash \neg \varphi$.

We call a view T is locally contradictive if it is hidden contradictive or contradictive about some agent i, here i>0.

There four sorts of contradictions have the following relation:



That is: If view T is commonly contradictive then it must be basically conlradicuve, hidden contradictive and contradictive about any agent; if view T is contradictive about agent i then it must be hidden contradictive.

4. How W Solve The Conway Paradox.

Continuing Example 1, we inquire " Can agent 1 (Max) know q?" to view T1, agent 1 will answer "No**. So we have not ($T1 \vdash K1q$).

Now we put $\neg$ K1q as common knowledge into T1, and get a new view T2, T2 is:

{p, q} v
K0{p v q, K1p v K1 $\neg$ p, K2q v K2 $\neg$ q, $\neg$ K1q} v
K1{p} v
K2{q}

We can prove $T2 \vdash K2p$.

The proof is following:

01. K0(K1 $\neg$ p & K0($\neg$ p $\rightarrow$ q) $\rightarrow$ K1q)	A3
02. K0 $\neg$ K1q	T1
03. K0($\neg$ K1 $\neg$ p v $\neg$ K0($\neg$ p $\rightarrow$ q))	1,2 theorem7
04. K0(p v q)	T1
05. K0($\neg$ p $\rightarrow$ q)	4 theorem7
06. K0K0($\neg$ p $\rightarrow$ q)	5, R1
07. K0 $\neg$ K1 $\neg$ p	3,6 theorem7
08. K0(K1p v K1 $\neg$ p)	T1
09. K0 K1 p	7, 8 theorem7
10. K0K1p $\rightarrow$ K0p	A5
11. K0p	9,10, R2
12. K0K2 p	11, R1
13. K0K2p $\rightarrow$ K2p	A4
14. K2p	12,13,R2

Henceforth, $T2 \vdash K2p$

5. Conclusion

In this paper, we propose a logic system W based on the idea of shared common knowledge views. It characterizes shared common knowledge under a multi-agent reasoning system. W's axioms and inference rules describe the properties and usage of common knowledge. These axioms and inference rules are primitively computable, so W has good computability. W has the following characteristics:

1. The fool reasoner is used to describe the common knowledge of all agents.
2. The knowledge in view is in different levels. For example let f $\in$ T, but f may be not known by agent i; This means that W is not a knowledge system which has the knowledge generalization inference rule: f $\Rightarrow$ Ki f; Also what agent knows may be not true, so knowledge axiom Ki f $\rightarrow$ f does not hold.
3. Every agent's inference ability is not weaker than propositional inference.
4. Every agent has enough power to utilize common knowledge.

5. As regards to 2, a view can not be basically contradictive caused by local contradiction

Though with such features, W is still an initial logic system, there are many unsolved aspects, including:

1. W's model theory.
2. W's consistence.
3. W's effective proof algorithms.

All these three aspects are very important. The results presented in this paper can serve as a standpoint for the research on common knowledge. Further research work on the formalism of common knowledge is expected to enrich our understanding of common knowledge and to obtain more fruitful results.

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