# Proposing Measurements in Dynamic Systems

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## Abstract

Proposing measurements in diagnosis systems for static systems is a well understood task. Usually, entropy based approaches are used, sometimes extended by cost and other considerations. How to do the same task in dynamic systems is less clear, and so far measurement proposal algorithms have been ignored in the recent approaches advanced for dynamic systems. In this paper we will describe a set of techniques and algorithms suitable for measurement proposal in a temporal diagnosis formalism discussed in our previous work. This formalism is based on qualitative Allen constraints. The current paper introduces a measurement proposal algorithm and improves it in several ways. Finally an entropy-based computation method is described for this temporal setting.

## 1 Introduction

Most current diagnosis systems are able to propose useful/optimal measurements to discriminate among competing hypotheses. Several methods have been proposed, mainly based on entropy computations as described in [de Kleer and Williams, 1987]. Recently, some systems for diagnosis of dynamic systems have been proposed [Console *et* a/., 1992; Nejdl and Gamper, 1994]. [Console *et* a/., 199-2] is based on a time-slice approach, which considers the system dynamics as a sequence of static states. While this approach restricts expressiveness, it allows to use the same measurement proposal algorithms as in static systems. On the other hand, these static algorithms are insufficient to be applied in the more general approach described in [Nejdl and Gamper, 1994]. In [Gamper and Nejdl, 1994] we discussed first results on how to propose measurements in our diagnosis system, and this paper extends these results by proposing a set of techniques which propose measurements able to distinguish hypotheses described by qualitative temporal constraints.

In section 2 we summarize our diagnostic approach described in [Nejdl and Gamper, 1994J. Section 3 explains the concept of measurement proposals and discusses the general techniques used to incorporate measurements into temporal networks. We improve these techniques in section 4 by exploiting the concept of temporal difference and in section 5 by using the concept of predicted patterns. Section 6 generalizes the entropy computation framework in [de Kleer and Williams, 1987] for our approach. Section 7 discusses related work.

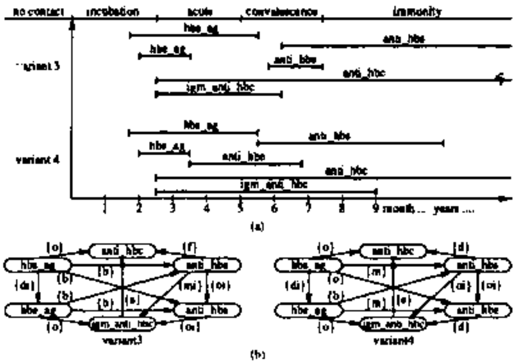

Figure 1: (a) Positive findings in *variant3* and *variant4* of hepatitis B, (b) IA-networks representing the temporal behavior of *variant3* and *variant4*.

## 2 Example and Basic Framework

Example 1 (Diagnosis of hepatitis B) In routine testing of hepatitis B the findings *hbs.ag, anti-hbs, hbe.ag, anti-hbe, anti-hbc* and *igm.anttJxbc* are tested, where each of them can assume the value *positive (p)* or *negative (n).* A hepatitis B virus infection is characterized by a typical sequence of these findings: 4 acute variants (two of them are shown in figure la where lines denote the periods of positive findings), and 4 persisting variants. In this paper we do not consider the different stages, e.g. *incubation.*

All variants look similar, involving basically the same findings. What distinguishes these variants is the order in which these findings occur. In [Nejdl and Gamper, 1994] we described a diagnosis framework which uses a subset of Aliens interval algebra to explicitly represent such temporal relationships.

**Temporal Framework.** A well known framework to represent qualitative temporal information is Allens *Interval Algebra IA* [Allen, 1983], which is based on 13 basic mutually exclusive relations between two intervals: *before*, *meets*, *overlaps*, *starts*, *during*, *finishes*, their inverses and *equal*, abbreviated as $I = \{b, m, o, d, s, f, bi, mi, oi, di, si, fi, e\}$. Indefinite knowledge is expressed as disjunction of basic relations and represented as a set, e.g. $(A\ before\ B) \vee (A\ meets\ B)$ is represented as $A\{b, m\}B$. We use a graphical representation for a set $R$ of assertions in IA, called *IA-network* and denoted by $\mathcal{R}$. The nodes represent intervals and the directed arcs are labeled with the relation between the connected intervals. A missing arc is implicitly labeled with $I$ (no knowledge). Given an IA-network, finding a consistent scenario and finding all feasible relations are basic temporal reasoning tasks. A *consistent scenario* is a labeling of the IA-network, where every label consists of a single basic relation and it is possible to map the nodes to a time line such that these relations hold. A basic relation is *feasible* iff there exists a consistent scenario containing this relation (for more details see [van Beek, 1992]).

**Qualitative Temporal Behavior in a Diagnosis Framework.** In [Nejdl and Gamper, 1994] we described a model-based framework for diagnosing dynamic systems by explicitly representing qualitative temporal information a la Allen. Given a system with components *COMPS* each component has associated a set of *behavioral modes*. The *temporal behavior* of each mode is described by a set $B$ of IA-relations between manifestations assuming that $B$ contains only the feasible relations. A manifestation $m(v, t)$ denotes the fact that the parameter $m$ assumes the value $v$ over time interval $t$. This allows to represent the consequences of a behavioral mode as a complex pattern of temporally overlapping manifestations. The temporal behavior $B$ can be represented as an IA-network $\mathcal{B}$. The nodes represent the manifestations, the arcs are labeled with the IA-relation between the connected manifestations. Figure 1b shows the network representation of the temporal behavior for *variant3* and *variant4* of hepatitis B. For clarity not all relations are shown. In the diagnostic process we generate an abductive and/or consistency-based explanation in terms of behavioral mode assumptions for a set of observations at arbitrary time points and the temporal relations between these observations. Usually, several explanations exists, and to distinguish between them we take additional measurements and continue the diagnosis process. As measurements produce costs we want to perform as few measurements as possible.

In this paper we will discuss how to propose measurements in our temporal diagnosis framework using the consistency based approach. We will refer to a set *HYP* of competing explanations as hypotheses $h_i$. We focus on their qualitative temporal behavior $B_i$, except when describing how quantitative constraints can improve measurement proposals. Each parameter has a finite set of possible values. All of them might be used in the description of the temporal behavior and a parameter assuming the same value might occur more than once.

## 3 Measurements in Temporal Networks

As dynamic systems are characterized by a complex pattern of manifestations over long time periods one might expect that measurements should be made in a similar way in a specific order at different times.

**Definition 1** (Measurement Proposal) A *Measurement Proposal* is defined as a formula $M \wedge C$, where $M$ is a conjunction of expressions $m(V, t)$ suggesting to measure parameter $m$ at time $t$, and $C$ is a conjunction of qualitative and quantitative temporal relations constraining time $t$ of the measurements.

A measurement proposal is a suggestion to perform the measurements in $M$ in the temporal order indicated by the relations in $C$. The above definition is very general including measurements at time points and measurements over time intervals. The relations in $C$ allow to specify the (possibly indefinite) temporal order in which the measurements should be performed, to constrain the measurements relative to the real time line as well as to constrain the duration of measurements. In particular measurements over time intervals are important in domains like medicine, e.g. did you have headache for at least two hours.

Performing a measurement proposal $M \wedge C$ results in an instantiation of the variables $V$ to the value actually observed, which we call the *Outcome* $O$ of a measurement proposal.

**Example 2** The measurement proposal $hbsag(V_1, t_1) \wedge hbe\_ag(V_2, t_2) \wedge hbs\_ag(V_3, t_3) \wedge t_1 < t_2 \wedge t_2 < t_3$ suggests to measure first *hbs_ag*, after some time *hbe_ag*, and later again *hbs_ag*. One of the 8 possible outcomes of this measurement proposal is $hbsag(p, t_1) \wedge hbeag(p, t_2) \wedge hbs\_ag(p, t_3) \wedge t_1 < t_2 \wedge t_2 < t_3$. Another example is $hbs\_ag(V_1, t_1) \wedge hbeag(V_2, t_2) \wedge t_1 = t_2$ suggesting to measure *hbs_ag* and *hbe_ag* at the same time point with 4 possible outcomes, e.g. $hbs\_ag(p, t_1) \wedge hbe\_ag(n, t_2) \wedge t_1 = t_2$. The measurement proposal $hbs\_ag(V, t) \wedge t = 2$ suggests to measure *hbs_ag* at time 2.

In this paper we only consider measurements performed at time points and restrict the relations in $C$ to the basic relations $\{<, =, >\}$ between two time points. In particular, we do not consider quantitative information except for a short discussion on how to improve measurement proposals by exploiting quantitative constraints. For clarity we will leave out the constraints $C$ in measurement proposals and their outcomes, use the same time parameter for all measurements related by "=" and assume $t_i < t_j$ if $i < j$. If a measurement proposal contains only concurrent measurements we leave out the time parameter as well. Then the first measurement proposal in the above example and its outcome are represented as $hbs\_ag(V_1, t_1) \wedge hbe\_ag(V_2, t_2) \wedge hbs\_ag(V_3, t_3)$ and $hbs\_ag(p, t_1) \wedge hbeag(p, t_2) \wedge hbs\_ag(p, t_3)$ respectively, the second one as $hbs\_ag(V_1) \wedge hbe\_ag(V_2)$ and the outcome as $hbs\_ag(p) \wedge hbe\_ag(n)$.

Depending on the outcome of a measurement proposal some hypotheses remain consistent and some other hypotheses are refuted. The aim of a measurement proposal algorithm in a diagnosis system is to find proposals which reduce the hypothesis space.

## 3.1 A Measurement Proposal Algorithm

We have to find a measurement proposal with a possible outcome which supports one hypothesis and refutes another one. The outcome of a measurement proposal supports a hypothesis if we can fit each observed value into the temporal behavior of the hypothesis and the temporal relations of the measurement proposal are satisfied; otherwise, the hypothesis is refuted. As we do not have time points in our temporal framework we assume that the measured value holds over a small time interval "around" the measurement time point. It is easy to show that this assumption does not influence the results.

The following procedure ME AS PRO implements a measurement proposal strategy. For each hypothesis *hi £ HYP* we maintain a set Si of IA-networks initially containing the corresponding temporal behavior network *Bi.* For each hypothetical measurement with outcome *m(v,t)* we insert a measurement outcome node *onode* representing the small time interval around *t* into each IA-network N in Si, and constrain it as follows:

- For each other measurement outcome node *onode'* at time point *t'* we add the IA-relation *onode*{b}*onode* if *t* < *t'*, *onode{e}onode'* if *t = t'* and *onode {bi} onode* if *t > t'.*
- We constrain the *onode* to occur within a manifestation node *mnode* in the temporal behavior representing the same parameter m and the same value *v* by adding the relation *onode{s,d,f,e] mnode.* If several such *mnodes* (same parameter and value) are related by *before (after)* the *onode* can only occur in one of them at a time. In this case we construct for each such *mnode* a copy of the actual N to represent the different possible constraints of *onode.*
- * We constrain *onode* to occur outside each *mnode* representing parameter m and value *v' not= v* by adding the relation *onode{b,* m, *bi, mi} mnode.*

Finally we test consistency for each IA-network in *Si* using known temporal reasoning algorithms [Allen, 1983; van Beek, 1992] and remove inconsistent networks. Situations, where we have *Si*, *Sj* such that Si, not= 0 and *Sj* = 0 characterize possibly discriminating measurement proposals for the hypotheses hi, and *hj*.

Example 3 We choose the measurement *hbs.ag(Vi,t)* with a positive outcome, insert the measurement outcome node *o-hbs.ag* into the IA-networks for *variant3* and *variant 4* and constrain it to be within the temporal extent of *hbs.ag,* i.e. *o-hbs-ag{s,d,j,e}hbs.ag.* The positive *hbs.ag* is consistent with both variants, and we continue with a new hypothetical measurement at the same time point, e.g. *anti*-hbe(V2,<) with a positive outcome. Adding *o-antLhbe{e}o-hbs.ag* and *o-anti.hbe{s,d,f,e}antiJibe* (figure 2) and propagating these relations yields an inconsistency in *variant3* (due to *hbs-ag{b}anti-hbe),* but it is consistent in *variant4-* Thus the measurement proposal *hbs.ag(Vi,t)* A *anti.hbe(V2,t)* has an outcome *hbs.ag(p,t)* A *anti.hbe(p,t)* which can distinguish between the two variants.

The algorithm ME AS PRO makes no assumptions about whether measurements are proposed in the past

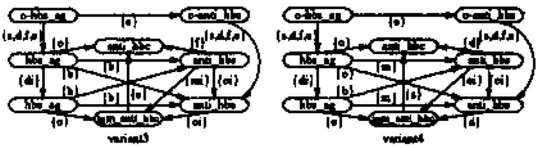

Figure 2: Proposing measurements using MEASPRO.

or in the future. In some domains measurements in the past make sense, e.g. in the medical domain.

**Definition 2** (Confirmation, Refutation) The outcome $O$ of a measurement proposal $M \wedge C$ *confirms* hypothesis $h$ iff the IA-network consisting of the temporal behavior $B$ extended by the outcome $O$ according to the strategy of algorithm MEASPRO is consistent. Otherwise, the outcome $O$ *refutes* hypothesis $h$.

**Definition 3** (Possibly-Discriminating) A measurement proposal $M \wedge C$ is *Possibly-Discriminating* for a set $HYP$ of hypotheses iff it has a possible outcome $O$ and there exists $h_i, h_j \in HYP$ such that $O$ confirms $h_i$ and refutes $h_j$.

This definition ensures that a possibly-discriminating measurement proposal has at least one possible outcome which allows to reduce the hypothesis space.

**Example 4** The measurement proposal $hbs\_ag(V_1) \wedge hbe\_ag(V_2)$ is not a possibly-discriminating one as the temporal relationship between $hbs\_ag$ and $hbe\_ag$ is $\{di\}$ in both hypotheses and thus the possible outcomes in both hypotheses are the same, i.e. $hbs\_ag(p) \wedge hbe\_ag(n)$, $hbs\_ag(p) \wedge hbe\_ag(p)$ and $hbs\_ag(n) \wedge hbe\_ag(n)$. Another measurement proposal which is not possibly-discriminating is $anti\_hbs(V_1) \wedge anti\_hbc(V_2)$ although the corresponding relation is $\{f\}$ in *variant3* and $\{d\}$ in *variant4*. Both relationships allow all possible outcomes except $anti\_hbs(p) \wedge anti\_hbc(n)$. An example of a possibly-discriminating measurement proposal is $hbs\_ag(V_1) \wedge anti\_hbe(V_2)$. As the temporal relationship is $\{b\}$ in *variant3* and $\{o\}$ in *variant4* the outcome $hbs\_ag(p) \wedge anti\_hbe(p)$ occurs only in *variant4*. Another possibly-discriminating measurement proposal is $hbs\_ag(V_1,t_1) \wedge anti\_hbs(V_2,t_1) \wedge hbs\_ag(V_3,t_2) \wedge anti\_hbs(V_4,t_2) \wedge hbs\_ag(V_5,t_3) \wedge anti\_hbs(V_6,t_3)$. The outcome $hbs\_ag(p,t_1) \wedge anti\_hbs(n,t_1) \wedge hbs\_ag(n,t_2) \wedge anti\_hbs(n,t_2) \wedge hbs\_ag(n,t_3) \wedge anti\_hbs(p,t_3)$ confirms *variant3*, which has $hbs\_ag$ $\{b\}$ $anti\_hbs$, and refutes *variant4*, which has $hbs\_ag$ $\{m\}$ $anti\_hbs$.

MEASPRO represents a very general method but it has some drawbacks. First, implementing a nondeterministic procedure represents some difficulties, and can be very inefficient if implemented as a simple generate and test procedure. Second, it performs temporal constraint propagation [Allen, 1983; van Beek, 1992] for each hypothetical measurement, which can be very costly since it depends on the whole network. Moreover, consistency testing in IA is exponential, and we have to use approximation algorithms which in general are not complete. In the following we will show how we can improve upon our measurement proposal algorithm.

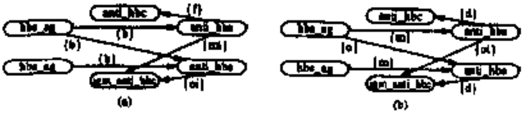

Figure 3: Difference graphs — (a) $DG_{v3,v4}$, (b) $DG_{v4,v3}$.

## 4 Exploiting the Temporal Difference

The generation of measurements can be focused by exploiting the difference in the predicted behavior of hypotheses. In the simplest case this can be a parameter predicted only in one hypothesis, which is trivial and thus not considered here. As each hypothesis is characterized by a typical sequence of temporally overlapped manifestations we will rather focus on the temporal difference of hypotheses (assuming that each hypothesis is described by the same parameters).

**Definition 4** (Temporal Difference) Given are 2 hypotheses $h_1$ and $h_2$ and their qualitative temporal behavior $B_1$ and $B_2$. The *Temporal Difference* between $h_1$ and $h_2$ is defined by the two sets of IA-relations

$$D_{1,2} = \{m_i R_{ij} m_j | \; m_i R^1_{ij} m_j \in B_1, m_i R^2_{ij} m_j \in B_2, R_{ij} = R^1_{ij} \setminus R^2_{ij}\}$$

$$D_{2,1} = \{m_i R_{ij} m_j | \; m_i R^1_{ij} m_j \in B_1, m_i R^2_{ij} m_j \in B_2, R_{ij} = R^2_{ij} \setminus R^1_{ij}\}$$

The difference set $D_{1,2}$ ($D_{2,1}$) between $h_1$ and $h_2$ contains those relations which occur only in $B_1$ ($B_2$) but not in $B_2$ ($B_1$). For each pair of hypotheses these two sets capture the difference in their temporal behavior. An empty relation $R_{ij} = \emptyset$ in $D_{1,2}$ means that $R^1_{ij} \subseteq R^2_{ij}$, i.e. all basic relations between $m_i$ and $m_j$ in $h_1$ are allowed in $h_2$. In $D_{2,1}$, however, we might still have a nonempty set. The temporal difference $D_{1,2}$ can be represented as a graph $DG_{1,2}$, called *Temporal Difference Graph*. The nodes represent manifestations, the directed arcs are labeled with the relations in $D_{1,2}$. Arcs labeled with the empty set need not be drawn. Figure 3 shows the temporal difference graphs for *variant3* and *variant4*.

As the two difference graphs for two hypotheses capture all differences in the temporal behavior we use them to focus the generation of measurement proposals. The following proposition helps to reduce the measurements to be considered.

**Proposition 1** Given are two hypothesis $h_1$ and $h_2$. A *necessary condition* for a possibly-discriminating measurement proposal $M \wedge C$ suggesting to measure $m_1$ to $m_n$ is that there exist $m_i, m_j$ such that $m_i$ and $m_j$ are connected in $DG_{1,2}$ or $DG_{2,1}$.

**Example 5** Let us consider the difference graphs for *variant3* and *variant4* in figure 3. Applying the above proposition we never propose measurements consisting of a subset of *hbs_ag*, *hbe_ag*, *igm_anti_hbc* and *anti_hbc*. Indeed, looking at figure 1 we recognize that the qualitative temporal relations between these manifestations do not differ from each other in both variants. For concurrent measurements of two parameters we have to consider only 6 measurement proposals instead of 15 without difference graphs. We have the 3 possibly-discriminating measurement proposals *hbs-ag($V_1$)* A *anti_hbe($V_2$)*, *anti_hbs($V_1$)* ∧ *igm_anti_hbc($V_2$)* and *anti_hbe($V_1$)* ∧ *igm_anti_hbc($V_2$)*. Examples of two connected manifestations in the difference graphs which do not provide a possibly-discriminating measurement proposal are *hbs-ag($V_1$)* A *anti_hbs($V_2$)* and *anti-hbs($V_1$)* ∧ *anti-hbc($V_2$)*. However, these parameters provide possibly-discriminating measurement proposals if measured 3 times as we have seen in the last example for *hbs.ag* and *anti_hbs.*

Temporal difference graphs tell us more. Suppose we can only perform measurements at the current time point and in the future. The temporal difference graph can be used to avoid measurements which only would provide discriminating information if performed in the past. For instance, if we measured *antLhbe* to be positive then it makes no sense to measure *hbe.ag* at the current time point or in the future. The relation between *hbe.ag* and *antLhbe* is *{b}* and {m} in the corresponding difference graphs indicating that the temporal extent of *hbe.ag* already terminated. Another example is when the difference graphs contain $m_1\{s\}m_2$ and $m_1\{o\}m_2$. If we measure $m_2$ to be positive then measuring $m_1$ at the same time or later provides no discriminating information.

## 5 Predicted Patterns

Difference graphs give us a means to reduce the number of measurement proposals which have to be considered as possibly-discriminating. However, the test for possibly-discriminating itself still has to be done using constraint propagation on temporal networks. To avoid this second source of complexity, we introduce the concept of *predicted pattern tables,* which basically compile the result of testing measurement proposals for consistency. The basic idea we use is that temporal relations can be characterized by predicted patterns. For the sake of simplicity we introduce an alternative representation for measurement proposals and their outcomes. As we allow only the basic point relations the manifestations in a measurement proposal can be partially ordered.

Definition 5 (Measurement Tuple/Sequence) A Measurement Proposal *M* ∧ *C* can be represented as a *Measurement Sequemts* = ≪ $mt_1(t_1)$,..., $mt_n(t_n)$ >>, where $t_i < t_j$ for $i < j$. Each $mt_1$ is called a *Measurement Tuple mt(t)* — (mi, ...,$m_n$) and is a suggestion to measure the parameters m,- at time point *t.*

Definition 6 (Pattern/Pattern Sequence) The outcome of a measurement tuple *mt(t)=(mi,..., $m_n$)* is called a *Measurement Pattern mp(t)=($v_1$...*, $v_n$ where each $V_1$ is a possible outcome for parameter $m_i$. The outcome of a measurement sequence is a sequence of measurement patterns $mps = \ll mp_1(t_1), ..., mp_n(t_n) \gg$, called a *Measurement Pattern Sequence.*

We will leave out the time parameter in measurement tuples and patterns. Measurement sequences suggesting to measure the same tuple *mt n* times are abbreviated as *$(mt)^n$*. For instance, the measurement proposal $hbs\_ag(V_1, t_1) \wedge anti\_hbe(V_2, t_1)$ A *hbs-ag($V_3 t_2$)*

| Basic rel. | Meaning | Predicted pattern sequence | | | | |
|---|---|---|---|---|---|---|
| $x$ before $y$ | | $\langle\bar{x},\bar{y}\rangle$ | $\langle x,\bar{y}\rangle$ | $\langle\bar{x},\bar{y}\rangle$ | $\langle\bar{x},y\rangle$ | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ meets $y$ | | $\langle\bar{x},\bar{y}\rangle$ | $\langle x,\bar{y}\rangle$ | | $\langle\bar{x},y\rangle$ | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ overlaps $y$ | | $\langle\bar{x},\bar{y}\rangle$ | $\langle x,\bar{y}\rangle$ | $\langle x,y\rangle$ | $\langle\bar{x},y\rangle$ | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ starts $y$ | | $\langle\bar{x},\bar{y}\rangle$ | | $\langle x,y\rangle$ | $\langle\bar{x},y\rangle$ | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ during $y$ | | $\langle\bar{x},\bar{y}\rangle$ | $\langle\bar{x},y\rangle$ | $\langle x,y\rangle$ | $\langle\bar{x},y\rangle$ | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ finishes $y$ | | $\langle\bar{x},\bar{y}\rangle$ | $\langle\bar{x},y\rangle$ | $\langle x,y\rangle$ | | $\langle\bar{x},\bar{y}\rangle$ |
| $x$ equal $y$ | | $\langle\bar{x},\bar{y}\rangle$ | | $\langle x,y\rangle$ | | $\langle\bar{x},\bar{y}\rangle$ |

Table 1: IA-relations and predicted pattern sequences.

| Relation tuples | Predicted Patterns | | | |
|---|---|---|---|---|
| $\langle b\rangle, \langle m\rangle, \langle bi\rangle, \langle mi\rangle$ | | $\langle p,n\rangle$ | $\langle n,p\rangle$ | $\langle n,n\rangle$ |
| $\langle o\rangle, \langle oi\rangle$ | $\langle p,p\rangle$ | $\langle p,n\rangle$ | $\langle n,p\rangle$ | $\langle n,n\rangle$ |
| $\langle s\rangle, \langle d\rangle, \langle f\rangle$ | $\langle p,p\rangle$ | | $\langle n,p\rangle$ | $\langle n,n\rangle$ |
| $\langle si\rangle, \langle di\rangle, \langle fi\rangle$ | $\langle p,p\rangle$ | $\langle p,n\rangle$ | | $\langle n,n\rangle$ |
| $\langle e\rangle$ | $\langle p,p\rangle$ | | | $\langle n,n\rangle$ |

Table 2: 2-tuple table.

A anti_hbe($V_4$,$t_2$) can be represented by the measurement sequence *(hbs.ag, anti-hbe)* . One possible outcome of this measurement sequence is the pattern sequence < (p, n),(p,p) >.

### 5.1 Predicted Pattern Sequences

Obviously, there is a strong relationship between pattern sequences and the qualitative temporal relations in I A. Let us consider the meets-relation between *hbs.ag* and *anti-hbs* in *variant4* (figure 1). It can be characterized by two consecutive time slices: in the first time slice *hbs.ag* is positive and *antt-hbs* is negative and in the second time slice *hbs.ag* is negative and *antLhbs* is positive.

Definition 7 (Predicted Pattern Sequence) Each basic relation in IA is characterized by a specific pattern sequence, called *Predicted Pattern Sequence.*

Table 1 shows the basic relations (without the inverse relations) in IA and their predicted pattern sequences, *x* and *y* denote that the property associated with the corresponding interval holds, ~*x* and *y* denote that the corresponding property does not hold. As the basic relations are mutually exclusive the predicted pattern sequences are too. We therefore get the following proposition:

Proposition 2 Given are two hypothesis *h1* and h2. A *sufficient condition* for a possibly-discriminating measurement sequence (mi, *rnj)*$^n$ for *h1* and h2 is that *n* > 5 and m, and m*j* are connected in both difference graphs.

It is important that the two parameters are connected in both difference graphs stating that both hypotheses have a unique relation, which can be recognized by a sequence of length 5. If we assume a measurement rate which recognizes each value change of the parameters we can give stronger conditions.

Proposition 3 Given are two hypothesis *h* and h2 and the assumption that each value change of a parameter is recognized. A *sufficient condition* for a possibly-discriminating measurement sequence {$m_i$,,mj)$^n$ for *hi* and $h_2$ is that *n* > 4 and that m, and $m_j$ are connected in at least one of the two difference graphs.

This proposition ensures that whenever two hypotheses differ in their qualitative temporal behavior there exists a possibly-discriminating measurement sequence suggesting to measure two parameters 4 times. Of course, in some cases even smaller sequences are sufficient. In the next section we look at sequences of length one, i.e. measurement tuples.

### 5.2 Predicted Pattern Tuples

Sequences of measurements postpone a distinction between several hypotheses into the future. Sometimes it is better to choose a measurement consisting of a set of concurrent measurements (even if they are more expensive) rather than a sequence of measurements.

A set $R$ of IA-relations between $n$ manifestations can be represented by two tuples: a tuple $\langle m_1, ..., m_n\rangle$ of manifestations and a relation tuple $R_{\langle m_1,...,m_n\rangle} = \langle R_{12}, ..., R_{1n}, R_{23}, ..., R_{2n}, ..., R_{(n-1)n}\rangle$ of length $n(n-1)/2$ corresponding to the number of manifestation pairs. For instance, the relations {*hbs_ag*($p$) $\{b\}$ *anti_hbs*($p$), *hbs_ag*($p$) $\{di\}$ *hbe_ag*($p$), *hbs_ag*($p$) $\{b\}$ *anti_hbe*($p$), *anti_hbs*($p$) $\{bi\}$ *hbe_ag*($p$), *anti_hbs*($p$) $\{oi\}$ *anti_hbe*($p$), *hbe_ag*($p$) $\{b\}$ *anti_hbe*($p$)}, between the positive *hbs_ag, anti_hbs, hbe_ag* and *anti_hbe* in *variant3* can be represented by $m = \langle$*hbs_ag*($p$), *anti_hbs*($p$), *hbe_ag*($p$), *anti_hbe*($p$)$\rangle$ and $R_m = \langle b, di, b, bi, oi, b\rangle$.

**Definition 8** (Predicted Pattern) Given is a set $R$ of IA-relations between the manifestations $m_1, \ldots, m_n$ assuming the values $v_1, \ldots, v_n$. A pattern $pp{=}\langle v'_1, \ldots, v'_n\rangle$, where each $v'_i$ is a possible value of $m_i$, is called a *Predicted Pattern* by the relation tuple $R_{\langle m_1,...,m_n\rangle}$ iff the following set of IA-relations is consistent:

$$R \;\cup\; \{pp\{s,d,f,e\}m_i | v_i = v'_i\}$$
$$\;\cup\; \{pp\{b,m,bi,mi\}m_i | v_i \neq v'_i\}$$

The basic idea of this definition is the same as the one used in algorithm MEASPRO. The pattern node is constrained to be within ($\{s,d,f,e\}$) or outside ($\{b,m,bi,mi\}$) the corresponding manifestation nodes. This tells us that the pattern $pp$ appears somewhere in the temporal behavior specified by $R$. More formally, there is a consistent scenario of $R$ containing a time slice corresponding to the pattern $pp$.

We use the definition above to generate all relation tuples which predict a specified pattern $\langle v_1, \ldots, v_n\rangle$. Given a measurement tuple $\langle m_1, \ldots, m_n\rangle$, we apply this definition for each possible pattern (outcome) $mp$ initializing the relations between the $m_i$'s in $R$ to the set $I$ of all basic IA-relations and construct a table with relation tuples and predicted patterns. Table 2 shows all relation tuples and the predicted patterns for a measurement tuple of length 2. In [Gamper and Nejdl, 1994] we describe in more detail how to construct the tables.

Using table 2 to find a measurement of length 2 turns out to be a simple table lookup. A measurement tuple is possibly-discriminating for two hypotheses $h_1$ and $h_2$ iff the corresponding relation tuples from $B_1$ and $B_2$ predict different sets of patterns. For disjunctions of basic

| Rel. tuples | Predicted Patterns | | | |
|---|---|---|---|---|
| $\langle b, si, bi\rangle$ | $\langle p, n, p\rangle$ | $\langle n, p, n\rangle$ | $\langle n, n, n\rangle$ | $\langle p, n, n\rangle$ |
| $\langle o, si, mi\rangle$ | $\langle p, n, p\rangle$ | $\langle n, p, n\rangle$ | $\langle n, n, n\rangle$ | $\langle p, p, n\rangle$ |

Table 3: Part of the 3-tuple table.

relations we take the union of the patterns predicted by the basic relations, e.g. for *{s, e)* we take the union of the patterns predicted by *(s)* and (e), which is the set of all possible patterns. Again we use the difference graphs to reduce the number of tuples to be considered.

**Example 6** Let us consider the measurement tuple *(hbs-ag, anti-hbe)* where we have a {b}-relation in *variants* and an {o}-relation in *variant4.* The corresponding relation tuples (6) and *(o)* predict different sets of patterns. The pattern *(p.p)* is only predicted by (o). Thus, *{hbs.ag, anti-hbe)* represents a possibly-discriminating measurement tuple. A measurement which is not a possibly-discriminating one is *(anti-hbs,.anti-hbc)* with the corresponding relations (/) and *(d).*

Obviously, if we use only the 2-tuple table, no inferences are compiled over more than two measurements. If we want to propose a measurement tuple of length 3, the table tells us only about the pairwise consistency of the measurements included in this tuple. If we want to get better discriminating power, we have to construct similar tables for larger tuples of relations and patterns. Table 3 shows a part of the relation tuples and the predicted patterns of length 3.

Example 7 In this example we assume that the positive *hbs-ag* and *hbe.ag* start at the same time, i.e. *hbs.ag{si}hbe.ag.* The measurement tuple *(hbs_ag, anti-hbe)* is a possibly-discriminating tuple, as it allows a distinction in the case of (p,p) as seen from table 2. If we look at the measurement tuple *(hbs-ag, anti-hbe, hbe-ag)* (i.e. adding the additional measurement *hbe.ag),* we get no additional distinguishing patterns from the 2-tuple table. However, the 3-tuple table gives us an additional distinguishing pattern for the whole 3-tuple measurement, i.e. (p, n, n), which excludes *variant4.*

Proposition 4 Given are two hypothesis *h1* and *h2* with temporal behavior *B* and *B2.* A *sufficient condition* for a possibly-discriminating measurement tuple $(m_i, \ldots, m_n)$ for h1, $h_2$ is that the corresponding relation tuples from *B1,* $B_2$ predict different sets of patterns.

Generating the predicted pattern tables requires to find all consistent scenarios for sets of IA-relations which is in general intractable. However, we can precompile these tables, and the set of relations depends only on the length of the measurement tuples and not on the size of the temporal behavior of the hypotheses as in MEASPRO.

## 6 Evaluating Measurements by Entropy

So far, we have only checked for possibly-discriminating measurement tuples. What we will do now is to generalize the entropy computation framework from [de Kleer and Williams, 1987] to our case. We have to use whole measurement tuples instead of single measurements **and** we have to deal with hypotheses which predict more **than** one value for a measurement tuple.

Given are a set *HYP* of hypotheses h1, and their temporal behavior B1,. For a measurement sequence *mts = (mt\,..., $mt_n$)* we represent the corresponding relations in *Bi* as relation tuple Ri mts and the set of predicted pattern sequences as *Pi* $_{mts}$. The set of possible pattern sequences of *mts* is *MPS mts.* We construct for each pattern sequence *mps* 6 $MPS_{mts}$ the set $HYP_{mps}$ of hypotheses which predict the pattern sequence *mps.* These sets provide the basis of our analysis. Obviously, each hypothesis *hi* appears in at least one $HYP_{mps}$. The measurement sequence *mts* is possibly-discriminating iff there are at least two different sets $HYP_{mps1}$ =/ $HYP_{mps2}$, where at least one of these sets is neither the empty set nor the set of all hypotheses.

Now the expected entropy for the hypothesis probabilities given a measurement sequence *mts* is given by

$$H_e(mts) = \sum_{mps \in MPS_{mts}} p(mts{=}mps) H(mts{=}mps)$$

*p(mts-mps)* is the probability that the outcome of *mts* is *mps* and *H(mts—mps)* is the entropy of the hypothesis probabilities given the hypothetical measurement result *mps.* The best measurement sequence *mts* is the one which minimizes the expected entropy $H_e(mts)$ of hypothesis probabilities [de Kleer and Williams, 1987].

The probability *p(mts=mps)* is given by the formula $p(mts{=}mps) = \sum_{h_i \in HYP} p(mts{=}mps/h_i)p(h_i)$, where *p(hi)* is the probability that *hi* is the actual hypothesis, and *p(mts=mps/hi)* is the conditional probability that the outcome of *mts* is *mps* assuming hi, is the actual hypothesis. In dynamic systems this probability depends on the duration of *mps* appearing in hi,-. If we do not have quantitative information concerning the temporal extent of intervals in B, we can use the approximation $p(mts{=}mps/h_i) = \sum_{mps' \in P^i_{mts}: mps \sqsubseteq mps'} \frac{1}{|P^i_{mts}|}$ assuming that each measurement pattern sequence predicted by *hi* has equal probability *(mps* C *mps'* means that *mps* is a subsequence of *mps').* Note, that this is a case where using quantitative information really helps.

The entropy *H(mts—mps)* of hypothesis probabilities under the assumption that *mts* is measured to be *mps* is given by the formula *H(mts=mps)* = $-\sum_{h_i \in HYP} p(h_i/mts{=}mps) \log p(h_i/mts{=}mps)$, where *p(hi/mts=mps)* is the probability that hi, is the hypothesis given that *mts* has been measured to be *mps,* and can be computed using the Bayes rule.

**Example 8** We consider the hypotheses *variants* and *variant4* and the measurement tuple *(hbs.ag, anti-hbe)* with the possible patterns *(p,p), (p,* n), *(n,p)* and *(n, n).* (p,p) is only predicted by *variant4* whereas all other patterns are predicted by both *variants* and *variant4.* In table 4 we summarize the results of calculating the expected entropy. As the pattern *(p,p)* distinguishes between both hypotheses its entropy is 0. For the other patterns the entropy is 0.98. The overall expected entropy for the measurement tuple *(hbs.ag, anti-hbe)* is approximately 0.87. For the other possibly-discriminating

| $mps$ | $p(mts = mps)$ | $p(v3/mts = mps)$ | $p(v4/mts = mps)$ |
|---|---|---|---|
| $(p,p)$ | 1/8 | 0 | 1 |
| $(p,n)$ | 7/24 | 4/7 | 3/7 |

Table 4: Calculation of the entropy.

measurement tuples of length 2 we get the same value, since they also have exactly one measurement pattern which allows to distinguish among the two variants. Hence, all three measurement tuples of length 2 are expected to distinguish equally well, if we cannot exploit quantitative information. If we have quantitative information as in figure 1, then the best measurement tuple would be *(anti-fibs^ igm-anti-hbc),* because the interval corresponding to the discriminating pattern is larger than for all other measurement tuples.

## 7 Discussion

Proposing measurements which discriminate between a set of hypotheses is an important issue in diagnostic reasoning. For model-based diagnosis of static systems several algorithms has been developed, mostly variations of the entropy-based algorithm described in [de Kleer and Williams, 1987]. Mcllraith [Mcllraith, 1994] recently examined the problem of test generation for hypothetical reasoning in general including diagnosis. While some of the concepts are similar to ours, e.g. a relevant test in [Mcllraith, 1994] corresponds to a possibly-discriminating test in our framework, Mcllraith characterizes test generation as an abduction problem, while we start with a nondeterministic algorithm and end up with efficient table lookup techniques. The major difference, however, is that we investigate measurement proposal for dynamic system and hence focus on the different temporal behavior of the hypotheses.

As far as we know no work has been done in proposing measurements in reasoning about dynamic systems. Most closely related to our work are situation recognition systems. Nokel [Nokel, 1989] describes a system for generating measurement sequences in order to recognize dynamic situations, which also uses a subset of Aliens interval algebra. The main difference to our approach is that we propose measurements to distinguish among several hypotheses characterized by a dynamic behavior which is different from recognizing such hypotheses. Distinguishing among several behaviors is based on their difference, while recognizing them involves observing all their characteristics. Indeed, Nokel proposes at least one measurement for each manifestation. While Nokel plans sequences of measurements allowing only one measurement at a single time point and does neither discuss entropy based algorithms nor efficiency improvements like we do, we investigate concurrent measurements as well as measurement sequences, including entropy based proposal algorithms and give a set of efficiency improving techniques to check for distinguishing measurement tuples. Another situation recognition system similar to Nokels approach is described in [Dousson *et al,* 1993].

## 8 Conclusion

In contrast to static model-based diagnosis systems, current model-based formalisms for temporal diagnoses have not yet investigated the issue of proposing measurements in a dynamic setting. In this paper we investigate techniques and algorithms for proposing new measurements in the temporal diagnosis framework described in [Nejdl and Gamper, 1994] which represents explicitly qualitative temporal behavior using a subset of Allen relations. We discuss a general (nondeterministic) algorithm based on constraint propagation for proposing possibly-distinguishing measurements. We show how the efficiency of this method can be improved by using temporal difference graphs and table lookup in precompiled predicted pattern tables. Finally, we generalize the entropy based measurement proposal algorithm defined in [de Kleer and Williams, 1987] to our temporal setting.

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