A default interpretation of defeasible network

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Abstract

This paper studies the semantics for the class of *all* defeasible (inheritance) networks, including *cyclic* and in *consistent* networks using a transformation approach. First we show that defeasible networks can be translated, tractably, to default theories while preserving Horty's path-off credulous semantics for all consistent networks. Using the existing methods in dealing with the semantics of default logic, we are able to provide a tractable skeptical semantics, the *well-founded semantics^* and a new credulous semantics, the *regular semantics^* both of which are defined for *any* defeasible network. Furthermore, we show that these semantics are based on the same principle of specificity used by Horty in defining his credulous semantics of defeasible networks.

1 Introduction

Two fundamental problems are to be addressed in this paper. First, the semantics of defeasible networks has previously been studied mainly under the assumption that such a network is acyclic and consistent. There is little understanding of the semantics for the class of *all* defeasible networks. Researchers in the field have not been able to provide an acceptable semantics for any defeasible networks that may involve cycles and/or that may be inconsistent.

Second, although a significant body of knowledge has been accumulated providing us with a good understanding of general nonmonotonic formalisms, such as default logic, autoepistemic logic, circumscription, and logic programming with negation, and their relationships, little is known about how path-based reasoning is related to other forms of nonmonotonic reasoning. For this, Horty raised the question of whether it is possible, and if yes, how to specify the consequences of a network by interpreting it in some more standard nonmonotonic formalism [Horty, 1994].

A number of transformations from defeasible network to a more general form of nonmonotonic reasoning have been proposed [Etherington and Reiter, 1983; Gelfond and Przymusinska, 1990; Gregorie, 1989; Haugh, 1988; Lin, 1991; Reiter and Cirscuolo, 1987]. None of these proposed transformations preserves the path-off credulous semantics even for consistent, acyclic networks. Only recently there had been some breakthrough. E.g. by transforming consistent and acyclic defeasible network into an abstract argumentation framework [Dung and Son, 1995], Dung and Son argue that the credulous semantics of consistent and acyclic network can be expressed in Dung's argumentation framework. In the same paper they show that the answer set semantics of extended logic programming can be used to express the credulous semantics of consistent and acyclic networks. However, if a network is cyclic, their transformation could generate an infinite extended logic program. Later, they reformulated a new semantics for default logic based on the idea of path-based defeasible reasoning and provided a translation from consistent and acyclic networks to their default logic [Dung and Son, 1996].

Cycles in a network are indispensable in representing certain concepts.

The first three nets in Figure 1 summarize some of the situations where a cycle in a network may be formed. E.g. *Net 1* is the case where two properties may lead to each other. In the strict sense, it describes an if and only if relation. A simple case of this relation is that two different names describe the same property. In addition, since we are dealing with *defeasible* networks, a link from p to q could mean, *normally* the property p leads to the property q. For example, a professor who teaches a course on logic programming usually also teaches a course on AI, and vice versa.

Net 2 is about two properties being mutually exclusive; e.g. a male is not a female and vice versa.

Net 3 shows a case where one concept leads to another which, usually, leads to the negation of the former. An example is that an Edmontonian is a North American

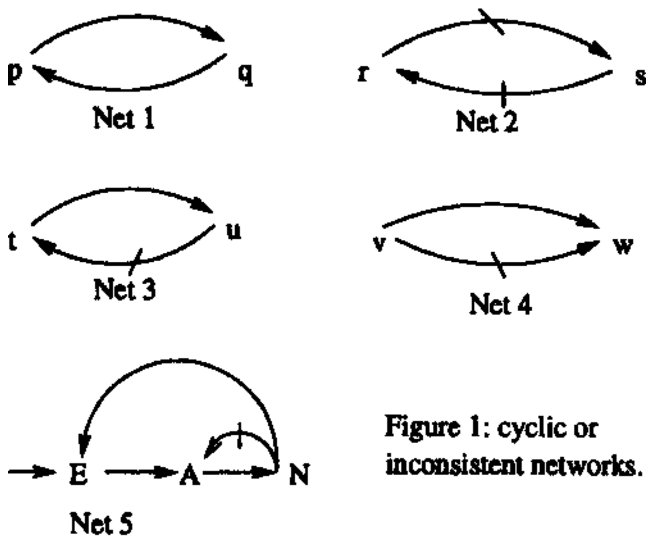


Figure 1: cyclic or inconsistent networks.

but a North American is usually not an Edmontonian. An extreme example is that every lottery winner is a person but a person is usually not a lottery winner.[1]

Note that, if a piece of information (such as, a person is usually not a lottery winner) is not presented in a network, no derivation is possible. This is because defeasible networks do not directly implement the Closed World Assumption.

Net 4 shows the situation where inconsistency arises in Horty's definition of *defeasible inheritability.* Such a definition allows an extension to include contradictory conclusions (thus allowing derivations from contradicted information). Alternative definitions could avoid this problem; e.g. Touretzky's [Touretzky, 1986] allows two extensions, in this case, with consistent conclusions.

We stress that it is the cycles, not the inconsistency described above, that causes a network to lose all its extensions: Horty showed that any acyclic network possesses at least one credulous extension, but a network involving cycles may not have any credulous extension [Horty, 1994].

The problem that a network has no credulous extension is caused by the presence of *conflicting information:* there is a part of the network, no matter how it is interpreted there is another part that contradicts it.

Net 5 illustrates a cyclic network that has no credulous extension. This could be understood when *a* is an individual, *E* is interpreted as Edmontonian, *A* as Albertan, and *N* as North American. Then, it is correct to say that a North American is usually not an Albertan *(N --> A).* However, this way of interpretating the network makes it erroneous to say that a North American is usually also an Edmontonian *(N --> E).*

The network could be interpreted differently; e.g. *E, A,* and *N* are different names of the same property. Then, it is the link *N-->A* that is erroneous.

In many application domains, the presence of only consistent information could be considered an exception rather than the norm. For example, in medical diagnosis, contradictions arise from many forms of knowledge incompleteness, e.g., lack of medical knowledge, lack of individual patient symptoms, and error or misinterpretation of both collective and individual data.

It has been argued by many authors that inconsistent information should not result in one of the two extremes: anything or nothing. In particular, inconsistent information should be localized. For example, the contradiction out of passing and failing a student should not affect the derivation that Houston is a city in Texas, and should not allow us to derive that Houston is a city in California. Though defeasible networks seem to provide a particularly suitable form of reasoning to accommodate a notion of local inconsistency, so far there has been no investigation into this possibility.

When we study the semantics of networks with cycles, what principle(s) should we follow? An important insight provided by this paper is that no new principle needs to be proposed. All we need is the principle of specificity that has been used all along in defining credulous semantics. *Net 6* illustrates this principle for an acyclic network: since the property *y* has conflicting inheritance from the two nodes *u* and v, the application of the path fl*(x,Ti,t;,T2,ti) blocks the path *u* —► *y.*

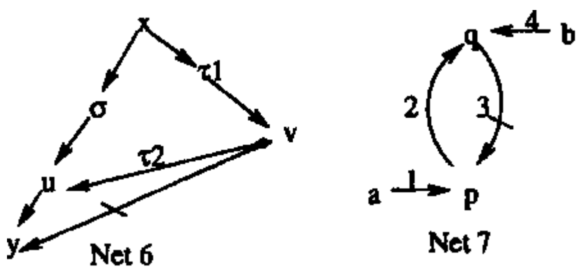


Figure 2

Under this principle, let us consider the network *Net 7* of Figure 2. From a conservative point of view, one may not be able to conclude anything from this net, since if we have *p{a),* then we will get *q(a)* and thus -»p(a). However, under the above principle, this net has a conservative extension {1,12,4,43} which concludes p(a), q(a), q(6), ¬p(6). This is because the path 12 blocks the path 3 on object node a. That is, *p(a)* is more specific than *¬p(a).* It would be easy to understand what the net could mean in a realistic situation if we represent p as *Lottery-winner*, *q* as *Person, a* and 6 as two individuals.

[1]Some of these examples can be better represented using both *defeasible* and *strict* links. The work has been extended to networks of mixed links in a forthcoming paper by the authors.

We choose a transformation approach to the semantics of all defeasible networks. First, we translate a network to a default theory and show that the translation preserves Horty's credulous semantics for all consistent networks (even if cyclic). This gives a default interpretation of defeasible networks. Then, we use the existing methods in dealing with the semantics of default logic to define a conservative, skeptical semantics, called the *well-founded semantics,* and a new credulous semantics, called the *regular semantics,* for arbitrary defeasible networks. The two semantics are named after their counterparts in logic programming [Van Gelder *et al.*, 1991; You and Yuan, 1995].

This paper is organized as follows: the next section describes the transformation from defeasible network to default theory, followed by a section on the new semantics for defeasible networks.

2 Prom Network to Default

In this section we give a translation from any defeasible network to a default theory and show a one-to-one correspondence between the credulous extensions of a consistent defeasible network [Horty, 1994] and Reiter's extensions of the translated default theory.

To simplify our discussion, we assume that a default theory is a set of defaults. A default d is of the form $\frac{A:B_1,\ldots,B_n}{C}$, where $n \geq 0$ and A, B_i, C are formulas of the underlying language $\mathcal{L}$. We denote A by $Pre(d)$, $\{B_1, ..., B_n\}$ by $Just(d)$, and C by $Cons(d)$. The definition of a default extension is standard [Reiter, 1980].

2.1 Defeasible network

A defeasible (inheritance) network Γ is defined as a finite collection of positive and negative links between nodes. If x, y are nodes then $y \leftarrow x$ (resp. $y \not\leftarrow x$) represents a positive (resp. negative) *link* from x to y where x is called the *root* and y is called the *head.* Nodes are divided into two disjoint classes: *object nodes* which are denoted as a, a_1, ..., and *property nodes* which are denoted as p, q, p_1, ... We assume that an object node can only be used as a root node. If a link's root is an object node, then this link is called an *object link.* Two links are said to be *in conflict* if they have the same head but one is positive and the other is negative. We say that $(x_1, x_2), ..., (x_{n-1}, x_n)$ (or simply $x_1...x_n$, $n \geq 2$) is a *general path* of Γ if for every $i = 1, ..., n-1$, (x_i, x_{i+1}) is a link of Γ whose root is x_i and whose head is x_{i+1}. The path above is called a *general cycle* if $x_1 = x_n$. A network is said to be *acyclic* if it has no general cycle.

A network Γ is *consistent* if there exist no two nodes x, y such that both $y \leftarrow x$ and $y \not\leftarrow x$ belong to Γ.

A *path* of Γ is either a link of Γ or a sequence of Γ's links $x_1 \rightarrow x_2, ..., x_n \rightarrow x_{n+1}$ (called a *positive path*), $n \geq 1$ (resp. $x_1 \rightarrow x_2, ..., x_{n-1} \rightarrow x_n, x_n \not\rightarrow x_{n+1}$, called a *negative path*). We simply denote the above positive path by $\pi(x_1, \sigma, x_{n+1})$ and negative path by $\overline{\pi}(x_1, \sigma, x_{n+1})$.

We now give the off-path credulous semantics of Horty [Horty, 1994].

Definition 2.1 (Path constructibility and conflict in a path set) Suppose Γ is a defeasible network, Φ is a path set of Γ. A path σ of Γ is *constructible* in a path set Φ if σ is an object link or $\sigma = \pi(x_1, \sigma_1, x_n, x_{n+1})$ (resp. $\sigma = \overline{\pi}(x_1, \sigma_1, x_n, x_{n+1})$) and $\pi(x_1, \sigma_1, x_n) \in \Phi$.

A path σ of Γ is *conflicting* in Φ if $\sigma = \pi(x_1, \sigma_1, x_n)$ (resp. $\sigma = \overline{\pi}(x_1, \sigma_1, x_n)$) and $\sigma' = \overline{\pi}(x_1, \sigma_1', x_n) \in \Phi$ (resp. $\sigma' = \pi(x_1, \sigma_1', x_n) \in \Phi$). □

Definition 2.2 (Preemption)
A positive path $\pi(x, \sigma, u, y)$ (resp. negative path $\overline{\pi}(x, \sigma, u, y)$) is *preempted* (see Net 6 of Figure 2) in Φ iff there is a node v such that (i) $v \not\rightarrow y \in \Gamma$ (resp. $v \rightarrow y \in \Gamma$) and (ii) either $v = x$ or there is a path of the form $\pi(x, \tau 1, v, \tau 2, u) \in \Phi$. □

Definition 2.3 (Defeasible inheritability)
Path σ is *defeasible inheritable* in Φ, written as $\Phi \vdash_d \sigma$, iff either σ is an object link[2] or σ is a compound path, $\sigma = \pi(x, \tau, y)$ (likewise for negative path) such that
(i) σ is constructible in Φ;
(ii) σ is not conflicting in Φ; and
(iii) σ is not preempted in Φ. □

Definition 2.4 (Credulous extension)
A set Φ of paths is a *credulous extension* of a net Γ iff $\Phi = \{\sigma \mid \Phi \vdash_d \sigma\}$. □

Example 2.5 Consider *Net 7* of Figure 2. This network allows one credulous extension: $\Phi_1 = \{1, 12, 4, 43\}$. □

2.2 Translation

For the presentation purpose, we first present a translation that is intuitive but not tractable, and then show how to modify it slightly to make it tractable.

Given a network Γ, a path $\pi(u, \sigma, x)$ is called a *simple path* if there is no path $\pi(u, \sigma', x)$ such that the set of links in $\pi(u, \sigma', x)$ is a proper subset of the links in $\pi(u, \sigma, x)$. We say that *a simple path* $\pi(u, \sigma, x)$ *causes conflict in node* r if two links from u to r and from x to r are in conflict in Γ. We say that *a simple path causes conflict* if it causes conflict in some node r. E.g. in *Net 7*, the simple path 12 causes conflict in node p, and it is the only simple path of *Net 7* that causes conflict.

We now relate the credulous semantics of a defeasible network with Reiter's extension semantics of its translated default theory. First, we transform a network to a default theory. In the following transformation, we use a predicate $in_l(a)$ to mean that *link* l *is in a path from object node* a. The role played by $in_l(a)$ is similar to

[2] In [Horty, 1994], it is any *direct link.* The difference, however, is inessential.

the normality predicate in nonmonotonic reasoning; the link l in a path to conclude a property of a is normally accepted unless there is a reason to reject it.

Definition 2.6 Let Γ be an arbitrary defeasible network. We translate it into a default theory $\Pi(\Gamma)$ as:

$d_l = \frac{:B, in_l(a)}{B \wedge in_l(a)}$, for every object link $l \in \Gamma$, if $l = p \leftarrow a$, then $B = p(a)$; if $l = p \not\leftarrow a$, then $B = \neg p(a)$;

$d_l(x) = \frac{p(x):B, in_l(x)}{B \wedge in_l(x)}$ for every non-object link $l \in \Gamma$,if $l = q \leftarrow p$, then $B = q(x)$; if $l = q \not\leftarrow p$, then $B = \neg q(x)$;

$d_{(\pi(u,\sigma,v),l)}(x) = \frac{\wedge\{in_l(x) \mid l \in \pi(u,\sigma,v)\}:}{\neg in_l(x)}$, for every simple path $\pi(u, \sigma, v)$ and link l from v to r such that it conflicts with a link from u to r. □

Here, the underlying first order language of $\Pi(\Gamma)$ contains the predicate set of the property nodes of Γ and the newly introduced predicate in_l for every link l of Γ; Its constant set is the set of the object nodes of Γ.

The first two kinds of defaults are translations directly from individual links. Let us denote the set of all these defaults by $\Pi_m(\Gamma)$. The last kind of defaults are translated from the simple paths that lead to a conflict. We denote it by $\Pi_p(\Gamma)$. Clearly, $\Pi(\Gamma) = \Pi_m(\Gamma) \cup \Pi_p(\Gamma)$.

We use the following examples to illustrate how a defeasible network is translated into a default theory and their relationships.

Example 2.7 Continue with *Net 7* of Figure 2. The translated default theory $\Pi(Net\ 7)$ is:

$\Pi_m(Net\ 7) =$
$\{\frac{:p(a), in_1(a)}{p(a) \wedge in_1(a)}, \frac{p(x):q(x), in_2(x)}{q(x) \wedge in_2(x)}, \frac{q(x):\neg p(x), in_3(x)}{\neg p(x) \wedge in_3(x)}, \frac{:q(b), in_4(b)}{q(b) \wedge in_4(b)}\}$

$\Pi_p(Net\ 7) = \{\frac{in_1(x) \wedge in_2(x):}{\neg in_3(x)}\}$

The defaults in $\Pi_m(Net\ 7)$ are self-explainable. The default in $\Pi_p(Net\ 7)$ is due to the fact that links 1 and 3 are in conflict and the simple path 12 is from link 1's root to link 3's root. This default theory has one R-extension

$$E = Th(\{p(a), q(a), q(b), \neg p(b), in_1(a), in_2(a), \neg in_3(a), in_4(b), in_3(b)\}),$$

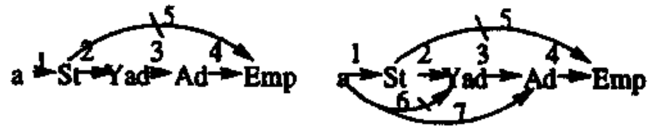
which corresponds (after removing $in_l(x)$ literals) to the only credulous extension as given in Example 2.5. □

Example 2.8 Consider the network *Net_8* adopted from [Dung and Son, 1996] in Figure 3 where $St(x)$ means *x is a student*, $Yad(x)$ *x is a young adult*, $Ad(x)$ *x is an adult* and $Emp(x)$ *x is employed*. This network has only one credulous extension $\Phi = \{1, 12, 15, 123\}$. Now we translate it into a default theory:

$\Pi_m(Net\ 8) = \{\frac{:St(a), in_1(a)}{St(a) \wedge in_1(a)}, \frac{St(x):Yad(x), in_2(x)}{Yad(x) \wedge in_2(x)}, \frac{Yad(x):Ad(x), in_3(x)}{Ad(x) \wedge in_3(x)}, \frac{Ad(x):Emp(x), in_4(x)}{Emp(x) \wedge in_4(x)}, \frac{St(x):\neg Emp(x), in_5(x)}{\neg Emp(x) \wedge in_5(x)}\}$

$\Pi_p(Net\ 8) = \{\frac{in_2(x) \wedge in_3(x):}{\neg in_4(x)}\}$

This default theory has only one R-extension, which implies $St(a), Yad(a), Ad(a), \neg Emp(a)$, along



Net 8 Net 9

Figure 3

with $in_1(a), in_2(a), in_3(a), \neg in_4(a), in_5(a)$. It corresponds to the unique credulous extension of the network.

If we add two more rules $6: Yad \not\leftarrow a$ and $7: Ad \leftarrow a$ into the network *Net_8*, the new network, *Net 9*, has two credulous extensions: $\Phi_1 = \{1, 6, 7, 15\}$ and $\Phi_2 = \{1, 6, 7, 74\}$. When we translate it into a default theory $\Pi(Net\ 9)$, we get $\Pi_m(Net\ 9)$ and $\Pi_p(Net\ 9)$ where

$\Pi_p(Net\ 9) = \{\frac{in_1(x):}{\neg in_3(x)}, \frac{in_2(x) \wedge in_3(x):}{\neg in_4(x)}\}$.

$\Pi(Net\ 9)$ has two extensions, which imply, respectively,

$\{St(a), \neg Yad(a), Ad(a), \neg Emp(a)\}$,

$\{St(a), \neg Yad(a), Ad(a), Emp(a)\}$,

They correspond to the two credulous extensions of *Net 9*. □

We now show that the relationship demonstrated above holds for all consistent networks. First we explain some notation.

Let Γ be a defeasible network, E a first older theory under the language of default theory $\Pi(\Gamma)$, and Φ a path set of the network Γ. We use E/Γ to mean the union of set $\{p(a) \mid p(a) \in E$, p is a property node and a is an object node$\}$ and set $\{\neg p(a) \mid \neg p(a) \in E$, p is a property node and a is an object node$\}$.

The path set $\mathcal{P}(E)$ of Γ is defined as follows: $\mathcal{P}(E) = \{\pi(a, \sigma, p) \mid \forall l \in \pi(a, \sigma, p), in_l(a) \in E\} \cup \{\overline{\pi}(a, \sigma, p) \mid \forall l \in \overline{\pi}(a, \sigma, p), in_l(a) \in E\}$.

The consequence set of path set Φ of the net Γ is defined by $Cons(\Phi) = \{p(a) \mid \exists \pi(a, \sigma, p) \in \Phi\} \cup \{\neg p(a) \mid \exists \overline{\pi}(a, \sigma, p) \in \Phi\}$.

Theorem 2.9 *For any consistent network Γ, Φ is its credulous extension iff $\Pi(\Gamma)$ has an R-extension E such that $\mathcal{P}(E) = \Phi$ and thus $E/\Gamma = Cons(\Phi)$.* □

2.3 A note on translation

Clearly, the number of defaults in a translated default theory depends on the number of the simple paths that lead to a conflict in the given network. In the worst case, this number is exponential to the node size. However, this still improves Dung and Son's translation [Dung and Son, 1995] from consistent and acyclic networks to argumentation frameworks, where the number of arguments could be infinite.

Selman and Levesque [Selman and Levesque, 1993] show that deciding whether a credulous extension exists for a defeasible network is NP-complete. Kautz and Selman [Kautz and Selman, 1991] show that the same decision problem for the simple default logic (restricted to literals) is also NP-complete. These results provide strong evidence that an extension preserving, polynomial transformation exists (note that these results only guarantee the existence of a polynomial transformation such that this same decision problem is preserved).

We now present such a transformation. The key idea is to use a predicate, $c_{p,q}$, to represent a positive path from node p to q where all the links on it have been *accepted*. Therefore, we modify $\Pi_p(\Gamma)$ to contain only the following defaults:

$\frac{in_{p\leftarrow a}(a):}{c_{a,p}(a)}$, where $p \leftarrow a$ is a positive link of the network.

$\frac{in_{q\leftarrow p}(x), c_{p',p}(x):}{c_{p',q}(x)}$, i.e., $c_{p,q}$ is transitive.

$\frac{c_{p,q}(x):}{\neg in_l(x)}$, where l is a link from q to r and the network has another link from p to r that conflicts with l.

Clearly, the size of $\Pi_p(\Gamma)$ is polynomial to the node size of Γ. By encoding the connection relation between any two nodes, we can avoid enumerating all the possible paths that connect the two nodes.

An interesting implication of such a polynomial translation is that any decision problem for a defeasible network based on the credulous semantics is *no harder* than the corresponding decision problem for the simple default logic (cf. [Kautz and Selman, 1991]).

3 Improved Semantics

The default interpretation of defeasible networks presented above provides a way to understand, indirectly, the possible semantics of defeasible networks. Namely, any semantics of default logic yields a semantics of defeasible networks. If a semantics is defined for all default theories, then it is defined for all defeasible networks. In this section we propose two of such semantics to address the problems of no extension and inconsistency.

As we know, not every default theory has an extension. This is because an R-extension E of a default theory *D* is a fixpoint of the following *anti-monotonic* operator:

Definition 3.1 (Anti-monotonic operator $\mathcal{R}$)
Suppose D is a default theory. For any first order theory E, we define $\mathcal{R}(E)$ to be *the smallest formula set* that satisfies the following conditions:

1. $Th(\mathcal{R}(E)) = \mathcal{R}(E)$;
2. For every $d \in D$, if $Pre(d) \in \mathcal{R}(E)$ and for any $B \in Just(d)$, $\neg B \notin E$, then $Cons(d) \in \mathcal{R}(E)$. □

$\mathcal{R}$ is anti-monotonic because for any theories E_1 and E_2, $E_1 \subseteq E_2$ implies $\mathcal{R}(E_2) \subseteq \mathcal{R}(E_1)$. Thus, $\mathcal{R}^2$, the operator that applies $\mathcal{R}$ twice, is monotonic. It thus has a least fixpoint and maximal fixpoints. Such a fixpoint is called an *alternating fixpoint* of R.

A number of researchers have proposed to use the technique of alternating fixpoints to define partial semantics fer default logic and logic programs. [Baral and Subrahmanian, 1991; Przymusinska and Przymusinski, 1991],

Definition 3.2 Let D be a default theory. A fixpoint E of $\mathcal{R}^2$ is said to be an *alternating extension* of D. It is said to be *normal* if $E \subseteq \mathcal{R}(E)$.

The *well-founded semantics* of *D* is defined by the least alternating extension (which is necessarily normal). The *regular semantics* of *D* is defined by the set of all maximal normal alternating extensions. D

We call the least normal alternating extension the *well-founded extension,* and a maximal normal alternating extension a *regular extension.*

However, this is not enough. It is well known that the well-founded extension defined this way is rather weak. Since in classic logic inconsistency is a global phenomenon, the possibility of deriving contradictory conclusion could nullify other expected conclusions. E.g. the default theory $\{\frac{:c}{c}, \frac{:\neg c}{\neg c}, \frac{:p}{p}\}$ has a well-founded extension $Th(\{\})$ and two regular extensions $Th(\{c, p\})$ and $Th(\{\neg c, p\})$. Obviously, we need to make p as a conservative conclusion in the well-founded semantics. Thus, we modify the definition of closure $Th(E)$ as follows: $Th(E) = \{p \mid \exists$ consistent $E' \subseteq E$ such that $E' \models p\}$. Under this subtle modification, we can see that the preceding default theory has the well-founded extension $Th(\{p\})$ and the same regular extensions.

Theorem 3.3 *For any network* Γ, *its translated default theory* $\Pi(\Gamma)$ *must have a unique, consistent well-founded extension* W, *and one or more consistent regular extension* E. *Conversely, the well-founded extension and any regular extension of the default theory* $\Pi(\Gamma)$ *must be consistent.* □

Based on this result, we define the well-founded extension of a defeasible network Γ by $\mathcal{P}(W)$ and a regular extension of Γ by $\mathcal{P}(E)$.

3.1 Examples

Example 3.4 Consider *Net 11* in Fig. 5. Its default translation yields
$\{\frac{:p(a), in_1(a)}{p(a) \wedge in_1(a)}, \frac{p(x):q(x), in_2(x)}{q(x) \wedge in_2(x)}, \frac{p(x):\neg q(x), in_3(x)}{\neg q(x) \wedge in_3(x)}, \frac{p(x):r(x), in_4(x)}{r(x) \wedge in_4(x)}\}$
Its well-founded extension is

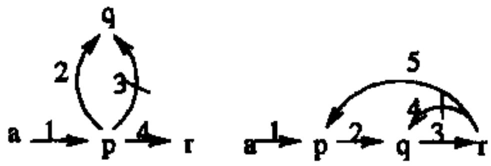
$W = Th(\{p(a), r(a), in_1(a), in_4(a)\})$,

and the two regular extensions are

$E_1 = Th(\{p(a), r(a), q(a), in_1(a), in_4(a), in_2(a)\})$ and
$E_2 = Th(\{p(a), r(a), \neg q(a), in_1(a), in_4(a), in_3(a)\})$.

Thus, *Net* 11 has the well-founded extension, $\{1, 14\}$, and two regular extensions, $\{1, 12, 14\}$ and $\{1, 13, 14\}$. □

Example 3.5 Consider the cyclic network *Net 12* of Fig. 5 (same as *Net 5* in Fig. 1). Its translated default theory is:



Net 11 Net 12

Figure 5

$$\Pi_m(Net\ 12) = \{\frac{:p(a), in_1(a)}{p(a) \wedge in_1(a)}, \frac{p(x):q(x), in_2(x)}{q(x) \wedge in_2(x)},$$
$$\frac{q(x):r(x), in_3(x)}{r(x) \wedge in_3(x)}, \frac{r(x):\neg q(x), in_4(x)}{\neg q(x) \wedge in_4(x)}, \frac{r(x):p(x), in_5(x)}{p(x) \wedge in_5(x)}\}$$
$$\Pi_p(Net\ 12) = \{\frac{in_5(x):}{\neg in_2(x)}, \frac{in_2(x) \wedge in_3(x):}{\neg in_4(x)}\}$$

The default theory $\Pi(Net\ 12)$ has the well-founded extension $Th(\{p(a), in_1(a)\})$ which is also its unique regular extension.

Thus, *Net* 12 has {1} as its well-founded extension and regular extension. □

3.2 Properties

The most important property is a soundness property for any well-founded, regular extension of a network *T.* Essentially, it says that any path constructed from such an extension cannot be preempted.

Theorem 3.6 (Soundness property) *Let T be any defeasible network, Φ is the well-founded or regular extension of T. Then,*

1. $\Phi \subseteq \{\sigma \mid \Phi \vdash_d \sigma\}$*;*

2. For any path β *of* Γ*, if* $\beta \notin \Phi$*, and* $\Phi \vdash_d \beta$*, then* $\{\sigma \mid \Phi \vdash_d \sigma\} \not\vdash_d \beta$*.* □

If a network is consistent and acyclic, the correspondence between regular extensions and Horty's credulous extensions is one-to-one.

Theorem 3.7 *Suppose* Γ *is a consistent and acyclic defeasible network.* Φ *is a regular extension of* Γ *iff* Φ *is a credulous extension of* Γ*.* □

Since the least alternating fixpoint of a simple default theory can be tractably computed, we get

Theorem 3.8 *The set of literals that are implied by a network under the well-founded semantics can be computed tractably.* D

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