A Cumulative-Model Semantics for Dynamic Preferences on Assumptions

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Abstract

Explicit preferences on assumptions as used in prioritized circumscription [McCarthy, 1986; Lifschitz, 1985; Grosof, 1991] and preferred subtheories [Brewka, 1989] provide a clear and declarative method for defining preferred models. In this paper, we show how to embed preferences in the logical theory itself. This gives a high freedom for expressing statements about preferences. Preferences can now depend on other assumptions and are thus dynamic. We elaborate a preferential semantics based on Lehmann's cumulative models, as well as a corresponding constructive characterization, which specifies how to correctly treat dynamic preferences in the default reasoning system EXCEPT [Junker, 1992].

Keywords: nonmonotonic reasoning, common sense reasoning.

1 Introduction

In the absence of complete information, it is necessary to base decisions and conclusions on assumptions. If those assumptions were arbitrary, the resulting decisions and conclusions would be arbitrary as well. Depending on the given information, *best* assumptions are chosen.

Different ways for defining best assumptions (or default rules) have been studied in nonmonotonic reasoning. A sound and declarative method is provided by preferences on assumptions. They are e.g. used in prioritized circumscription [McCarthy, 1986; Lifschitz, 1985; Grosof, 1991] and for preferred subtheories [Brewka, 1989]. Preferences decide which assumptions will be selected first in presence of conflicts between assumptions. In absence of conflicts, they don't have any effect. Furthermore, preferences enable a preferential semantics leading to clear logical properties, as well as constructive characterizations in form of inductive definitions. Finally, they allow to express the important specificity principle in inheritance systems in a clear way. All these points are difficult to achieve in alternative approaches such as default logic. A problem, however, is how to specify preferences:

1. **Static preferences** are specified outside the logical theory to which they apply. They are given in form of priorities [McCarthy, 1986; Lifschitz, 1985] or in form of a partial order on assumptions [Brewka, 1989; Grosof, 1991]. Specifying such an ordering is a minutely work. It would be preferable to write down quantified and conditional statements on preferences in the logical theory itself.
2. **Implicit preferences** are used in conditional approaches [Geffner and Pearl, 1992; Kraus *et al.*, 1990]. Default rules of the form $\alpha_i \mid\sim \beta_i$ can be (partially) ordered by exploiting specificity relations between the contexts α_i. However, other kinds of preference knowledge cannot be expressed. We refer the reader to [Brewka, 1994] who argues in favour of explicit preferences.

In order to allow a clear, explicit, and flexible specification of preferences, we embed them in the logical theory itself. As a consequence, preferences on assumptions can depend on (other) assumptions and thus become dynamic. We argue that those *dynamic preferences* are quite natural in human commonsense reasoning and illustrate this by the following example:

Jim and Jane have the following habits:

1. ***Normally, Jim and Jane go to at most one attraction each evening.***
2. ***Jim prefers the theatre to the night club.***
3. ***Jane prefers the night club to the theatre.***
4. ***If Jim invites Jane then he respects her preferences (and vice versa).***
5. ***Normally Jim invites Jane.***
6. ***An exception to 1 is Saturday.***
7. ***An exception to 5 is Jim's birthday, where Jane invites Jim.***

If no further information is given we conclude that Jim and Jane will go to the night club. When we learn that Jim has birthday we revise this and conclude that they go to the theatre. However, the day in question is a Saturday. Hence, they should go to both attractions. Finally the news tell that the theatre is closed for work. Thus we again conclude that they go to the night club.

Dynamic preferences have been examined in the scope of the TASSO-project on graphic configuration under uncertainty. The default reasoning system EXCEPT II uses dynamic preferences for determining an order in which assumptions are inspected [Junker, 1992; Junker, 1995]. Problems are provided by cyclic preferences, as well as by new preferences that contradict the already chosen part of the order. Brewka succeeded to integrate dynamic preferences into default logic [Brewka, 1994] and logic programming [Brewka, 1996]. Defaults are applied in a certain order that is chosen initially. The dynamic preferences obtained as consequences of defaults must be consistent with this order.

Both approaches do not guarantee the existence of solutions. Furthermore, they miss a clear preferential semantics as well as a constructive characterization. In this paper, we present a solution to these problems:

1. In section 2, we show how to embed assumptions and preferences in a logical language.
2. We seek a preferential semantics for static preferences in section 3. We analyse limits of existing approaches and elaborate a preferential semantics based on cumulative models [Kraus *et al.*, 1990].
3. We extend this semantics to dynamic preferences in section 4. The resulting nonmonotonic inference relation inherits all properties of Lehmann's system *C* [Kraus *et al.*, 1990].
4. This semantics then points out how to modify the constructive approach in [Brewka, 1989; Junker and Brewka, 1991] to dynamic preferences.

Finally, we discuss a simple example in section 5, *as* well as related work in section 6.

2 Preferences in a Logical Language

ID this section, we show how to express preferences on assumptions in a first-order language. For this purpose, assumptions must be named by ground terms. Similar to circumscription, we do not change the syntax of a logical language, but introduce special predicate symbols:

- a unary predicate symbol c for *chosen* assumptions[1].
- a binary predicate symbol $\leftarrow$ for preferences.

If t, t_1, t_2 are ground terms then $c(t)$ means that an assumption of name t is chosen and $t_1 \leftarrow t_2$ means that the assumption of name t_1 is preferred to the assumption of name t_2. Let $\mathcal{L}$ be a first-order language having the predicate symbols c, $\leftarrow$ and an unsatisfiable constant $\bot$.

Named default rules in the sense of [Poole, 1988] can easily be translated to this quite technical representation. A default rule $d : \alpha \supset \gamma$ of name d means that α normally implies γ. It can be written as $\alpha \wedge c(d) \supset \gamma$.

We write our example directly in the translated form. We abbreviate night club by nc and theatre by th. Furthermore, we introduce the terms one_D, $go_D(x)$, inv_D for naming default rules:

0. $\forall x.c(go_D(x)) \supset go(x)$
1. $c(one_D) \wedge go(nc) \wedge go(th) \supset \bot$
2. $invites(jim, jane) \supset go_D(nc) \leftarrow go_D(th)$
3. $invites(jane, jim) \supset go_D(th) \leftarrow go_D(nc)$
4. $c(inv_D) \supset invites(jim, jane)$
5. $saturday \supset \neg c(one_D)$
6. $hasBirthday(jim) \supset \neg c(inv_D) \wedge invites(jane, jim)$

Since the assumptions inv_D, one_D influence decisions about the assumptions $go_D(nc)$ and $go_D(th)$, but not vice versa, we put them into a level of higher priority by adding the following preferences:

7. $\forall x.one_D \leftarrow go_D(x)$
8. $\forall x.inv_D \leftarrow go_D(x)$

In order to simplify the discussion in this paper, we follow [Poole, 1988] and suppose that all possible assumptions are given explicitly. Furthermore, we suppose that the set of these assumptions is finite:

1. Let $\mathcal{N}$ be a *finite* set of ground terms of $\mathcal{L}$ that serve as names of assumptions.
2. Let $\mathcal{A} := \{c(t) \mid t \in \mathcal{N}\}$ be the corresponding set of assumptions.

Thus, only assumptions from $\mathcal{A}$ will be selected and only preferences between elements of $\mathcal{N}$ will be relevant.

Given a logical theory $\Gamma \subseteq \mathcal{L}$, we consider subsets of $\mathcal{A}$ that are consistent w.r.t. Γ. Let $\mathcal{C}_\Gamma := \{A \subseteq \mathcal{A} \mid A \cup \Gamma \not\models \bot\}$ be the set of these assumption sets. The following sections show how to define preferred elements of $\mathcal{C}_\Gamma$.

3 Static Preferences

In this section, we suppose that static preferences in form of a strict partial order $\prec \subseteq \mathcal{N} \times \mathcal{N}$ are given.

We first examine existing approaches for defining preferred assumption sets. The first one follows the idea of a preferential semantics [Shoham, 1987]. We lift the partial order $\prec$ on assumption names to a partial order $\prec_G$ on assumption sets and select the $\prec_G$-minimal elements of $\mathcal{C}_\Gamma$. We use an order proposed in [Geffner and Pearl, 1992; Grosof, 1991], where worse assumptions are exchanged by better assumptions:

Definition 3.1 *Let* $\prec_G \subseteq 2^{\mathcal{A}} \times 2^{\mathcal{A}}$ *s.t.* $A_1 \prec_G A_2$ *iff* $A_1 \neq A_2$ *and*

$$\forall c(t) \in A_2 - A_1 \exists c(t^*) \in A_1 - A_2 : t^* \prec t$$

$A \subseteq \mathcal{A}$ *is called* G-preferred assumption set *of* Γ *iff 1.* $A \in \mathcal{C}_\Gamma$ *and 2.* $A^* \prec_G A$ *implies* $A^* \notin \mathcal{C}_\Gamma$.

The order $\prec_G$ is transitive and irreflexive [Geffner and Pearl, 1992]. Since the set $\mathcal{A}$ is finite the existence of G-preferred assumption sets is thus guaranteed.

The second approach considers a partial order an incomplete specification of a strict total order $< \subseteq \mathcal{N} \times \mathcal{N}$ [Brewka, 1989]. A strict total order on a finite set $\mathcal{N}$ uniquely defines an enumeration $t_1, \ldots, t_n$ of $\mathcal{N}$ that *respects* $<$ in the sense that $t_j < t_k$ iff $j < k$ for all $j, k = 1, \ldots, n$.

[1] In fact $c(t)$ corresponds to $\neg ab(t)$.

We inspect the assumptions in increasing order and select them if this selection is consistent w.r.t. the already chosen assumptions:

Definition 3.2 *Let $< \subseteq \mathcal{N} \times \mathcal{N}$ be a strict total order and $t_1, \ldots, t_n$ be the enumeration of $\mathcal{N}$ that respects $<$. Let $A_0 := \emptyset$ and*

$$A_{i+1} := \begin{cases} A_i & \text{if } \Gamma \cup A_i \cup \{c(t_{i+1})\} \models \bot \\ A_i \cup \{c(t_{i+1})\} & \text{otherwise} \end{cases}$$

Then A_n is the selection *of $<$. A is a B-preferred assumption set iff A is the selection of a strict total order $<$ which satisfies $\prec \subseteq <$.*

This constructive definition immediately gives rise to an algorithm for computing B-preferred assumption sets for decidable sub-languages $\mathcal{L}$. [Junker and Brewka, 1991] shows that every B-preferred assumption set is a G-preferred one, but the inverse is not true. A counterexample is the partial order $t_1 \prec t_3$, $t_2 \prec t_4$ and the theory $\Gamma_0 = \{\neg c(t_1) \vee \neg c(t_2),\ \neg c(t_1) \vee \neg c(t_3),\ \neg c(t_1) \vee \neg c(t_4),\ \neg c(t_2) \vee \neg c(t_3),\ \neg c(t_2) \vee \neg c(t_4)\}$. When determining B-preferred theories, we must start an enumeration by t_1 or t_2. Thus, we obtain two B-preferred sets, namely $\{c(t_1)\}$ and $\{c(t_2)\}$. The set $\{c(t_3), c(t_4)\}$ is G-preferred, but not B-preferred. Hence, G-preferred assumption sets cannot serve as a preferential semantics for B-preferred assumption sets.

Thus, we have two approaches for treating static preferences on assumptions. G-preferred assumption sets seem to be too weak since we would not accept the worst choices $\{c(t_3), c(t_4)\}$ as long as the better choices $\{c(t_1)\}$ and $\{c(t_2)\}$ are possible.

If we vary the example we observe that there is no strict partial order $\prec'$ on assumption sets that produces B-preferred assumption sets as minimal elements of C_T. Consider the following theories and their B-preferred assumption sets:

Γ_0:	$\{c(t_1)\}, \{c(t_2)\}$
$\Gamma_1 := \Gamma_0 \cup \{\neg c(t_1)\}$:	$\{c(t_2)\}, \{c(t_3), c(t_4)\}$
$\Gamma_2 := \Gamma_0 \cup \{\neg c(t_2)\}$:	$\{c(t_1)\}, \{c(t_3), c(t_4)\}$
$\Gamma_3 := \Gamma_1 \cup \Gamma_2$:	$\{c(t_3), c(t_4)\}$

Assume that the $\prec'$-minimal elements of $\mathcal{C}_\Gamma$ coincide with the B-preferred assumption sets. From the last case, we infer that $\{c(t_3), c(t_4)\}$ is $\prec'$-smaller than the sets $\emptyset$, $\{c(t_3)\}$, and $\{c(t_4)\}$. The second and third cases show that neither $\{c(t_1)\}$, nor $\{c(t_2)\}$ are $\prec'$-smaller than $\{c(t_3), c(t_4)\}$. Hence, the set $\{c(t_3), c(t_4)\}$ must be a B-preferred set of Γ_0 which is a contradiction. We conclude that the simple semantical framework is not sufficient to give a preferential semantics to B-preferred assumption sets.

Rescue comes from Lehmann's more general framework [Kraus *et al.*, 1990], which is based on structures of the form $C := (\mathcal{S}, l, \leftarrow)$ where $\mathcal{S}$ is a set of states and $\leftarrow \subseteq \mathcal{S} \times \mathcal{S}$ is an antisymmetric relation on states. Each state $s \in \mathcal{S}$ is labelled with a set $l(s)$ of worlds (i.e. logical interpretations). A state s satisfies a theory Γ if all interpretations in $l(s)$ satisfy Γ. Let $\mathcal{S}_\Gamma$ be the set of all states satisfying Γ. A state s is a $\leftarrow$-minimal element of $\mathcal{S}_\Gamma$ iff $s \in \mathcal{S}_\Gamma$ and $s^* \leftarrow s$ implies $s^* \notin \mathcal{S}_\Gamma$. ψ can nonmonotonically be inferred from Γ (written as $\Gamma \mid\!\sim_C \psi$) iff all $\leftarrow$-minimal elements of $\mathcal{S}_\Gamma$ satisfy ψ.

The existence of $\leftarrow$-minimal states is ensured by the following property: The order $\leftarrow$ is called *smooth* on a set X iff for all $s \in X$ there exists a $\leftarrow$-minimal state s^* in X such that $s^* = s$ or $s^* \leftarrow s$. $C = (\mathcal{S}, l, \leftarrow)$ is called a *cumulative model* iff the order $\leftarrow$ is smooth on $\mathcal{S}_\Gamma$ for all theories V. Lehmann has shown that each cumulative model defines a nonmonotonic inference relation satisfying the following basic properties. Properties 4 and 5 together are called cumulativity:

1. $\alpha \mid\!\sim_C \alpha$
2. if $\models \alpha \equiv \beta$, $\alpha \mid\!\sim_C \gamma$ then $\beta \mid\!\sim_C \gamma$
3. if $\models \alpha \supset \beta$, $\gamma \mid\!\sim_C \alpha$ then $\gamma \mid\!\sim_C \beta$
4. if $\alpha \wedge \beta \mid\!\sim_C \gamma$, $\alpha \mid\!\sim_C \beta$ then $\alpha \mid\!\sim_C \gamma$
5. if $\alpha \mid\!\sim_C \beta$, $\alpha \mid\!\sim_C \gamma$ then $\alpha \wedge \beta \mid\!\sim_C \gamma$

For a given partial order $\prec$ on assumption names, we will define a particular cumulative model producing the B-preferred assumption sets as minimal states. A B-preferred set is determined by choosing a total completion of the given partial order. We make this explicit by including this order in a state. Furthermore, a state contains an assumption set A and a non-empty set of worlds that satisfy A. This set of worlds will serve as the label of a state.

Definition 3.3 *Let $A \subseteq \mathcal{A}$ be an assumption set, W be a non-empty set of worlds satisfying A, and $< \subseteq \mathcal{N} \times \mathcal{N}$ be a strict total order s.t. $\prec \subseteq <$. The triple $s := (W, A, <)$ is called a* static state. *Let $\mathcal{S}$ be the set of all static states.*

A static state $s := (W, A, <)$ is labelled with the set $l(s) := W$ of worlds. Let $\mathcal{S}_\Gamma$ be the set of all static states satisfying a theory Γ. The usage of a set of worlds instead of a single world means that static states represent incomplete information. If Γ contains a disjunction $a \vee b$ then a state satisfying Γ will contain worlds that satisfy a and worlds that satisfy b.

If a state $s := (W, A, <)$ satisfies Γ then all worlds w in W satisfy Γ, as well as A. Since W is not empty, $\Gamma \cup A$ is consistent in this case:

$$\Gamma \cup A \not\models \bot \quad \text{if} \quad (W, A, <) \in \mathcal{S}_\Gamma$$

We compare two static states $(W_1, A_1, <_1)$ and $(W_2, A_2, <_2)$ if they have the same total order $< = <_1 = <_2$. This order $<$ gives rise to the following lexicographical order.

Definition 3.4 *Let $<$ be a strict total order and $t_1, \ldots, t_n$ be the enumeration of $\mathcal{N}$ that respects $<$. Then $<^L \subseteq 2^{\mathcal{A}} \times 2^{\mathcal{A}}$ is defined as follows: $A_1 <^L A_2$ iff there exists an k s.t. $c(t_k) \in A_1 - A_2$ and*

$$A_1 \cap \{c(t_1), \ldots, c(t_{k-1})\} = A_2 \cap \{c(t_1), \ldots, c(t_{k-1})\}$$

The order $<^L$ is a strict total order and it is smooth on $\mathcal{C}_\Gamma$ for all theories Γ. B-preferred sets are obtained in the following way:

Lemma 3.1 *A is the selection of $<$ iff A is the $<^L$-minimal element of $\mathcal{C}_\Gamma$.*

We prefer a static state $(W_1, A_1, <)$ to $(W_2, A_2, <)$ if $A_1 <^L A_2$:

Definition 3.5 *Let $s_1 := (W_1, A_1, <_1)$ and $s_2 := (W_2, A_2, <_2)$ be two static states. The relation $\prec_S \subseteq \mathcal{S} \times \mathcal{S}$ is defined as follows: $s_1 \prec_S s_2$ iff 1. $<_1 = <_2$ and 2. $A_1 <_1^L A_2$.*

The resulting relation $\prec_S$ is a strict partial order which is smooth on all $\mathcal{S}_\Gamma$. Therefore, $(\mathcal{S}, l, \prec_S)$ is a cumulative model and has all the desired properties. We now establish the link to B-preferred assumption sets:

Theorem 3.2 *A is a B-preferred assumption set of Γ iff there exists a $\prec_S$-minimal state $s := (W, A, <)$ in $\mathcal{S}_\Gamma$.*

Thus, we established a preferential semantics for B-preferred assumption sets. Note that this result corrects the approach in [Roos, 1992] who gave a first trial for such a semantics.

4 Dynamic Preferences

In this section, we extend the preferential semantics and the constructive characterization to dynamic preferences. First, we modify the notion of a state. Given a triple $(W, A, <)$, we consider all preferences, i.e. ground formulas of the form $t_1 \leftarrow t_2$, that are satisfied by the state. A preference $t_1 \leftarrow t_2$ is satisfied by the state if it is satisfied by all worlds in W.

Definition 4.1 *Let $A \subseteq \mathcal{A}$ be an assumption set, W be a non-empty set of worlds satisfying A, and $< \subseteq \mathcal{N} \times \mathcal{N}$ be a strict total order such that $t_1 < t_2$ if all worlds $w \in W$ satisfy $t_1 \leftarrow t_2$ (for $t_1, t_2 \in \mathcal{N}$). The triple $s := (W, A, <)$ is called a* dynamic state. *Let $\mathcal{D}$ be the set of all dynamic states.*

A dynamic state $s := (W, A, <)$ is labelled with the set $l(s) := W$ of worlds. Let $\mathcal{D}_\Gamma$ be the set of dynamic states satisfying the theory Γ. If $s := (W, A, <)$ satisfies a theory Γ then $\Gamma \cup A$ is consistent and $<$ respects all preferences that can be derived from $\Gamma \cup A$ in the following way:

$$\leftarrow_{\Gamma,A} := \{(t_1, t_2) \in \mathcal{N} \times \mathcal{N} \mid \Gamma \cup A \models t_1 \leftarrow t_2\}$$

Let $W_{\Gamma \cup A}$ be the set of all worlds satisfying $\Gamma \cup A$. Then:

Lemma 4.1 *Let $A \subseteq \mathcal{A}$ and $< \subseteq \mathcal{N} \times \mathcal{N}$ be a strict total order. $s := (W_{\Gamma \cup A}, A, <)$ is a dynamic state iff $\Gamma \cup A \not\models \bot$ and $\leftarrow_{\Gamma,A} \subseteq <$.*

We compare two dynamic states $s_1 = (W_1, A_1, <_1)$ and $s_2 = (W_2, A_2, <_2)$ even if the orders $<_1$ and $<_2$ are different. s_1 is smaller than s_2 iff there is an $\alpha \in A_1 - A_2$ such that $<_1$ and $<_2$, as well as A_1 and A_2 agree on the elements that are $<$-smaller than α:

Definition 4.2 *Let $s_1 := (W_1, A_1, <_1)$ and $s_2 := (W_2, A_2, <_2)$ be two dynamic states. Let $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ be the two enumerations of $\mathcal{N}$ s.t. the first one respects $<_1$ and the second one respects $<_2$. Let $\prec_D \subseteq 2^{\mathcal{D}} \times 2^{\mathcal{D}}$ be defined as follows: $A_1 \prec_D A_2$ iff there exists a k s.t. $c(u_k) \in A_1 - A_2$ and*

1. $u_i = v_i$ *for* $i = 1, \ldots, k$
2. $A_1 \cap \{c(u_1), \ldots, c(u_{k-1})\} = A_2 \cap \{c(u_1), \ldots, c(u_{k-1})\}$

Lemma 4.2 *$\prec_D$ is a strict partial order.*

Thus, we obtained a simple preferential semantics for dynamic preferences. We require that the total order of a state respects the preferences that are satisfied by all worlds of the state and we use a kind of lexicographical order for comparing states with different base orders.

Before showing that $\prec_D$ is smooth and that $D := (\mathcal{D}, l, \prec_D)$ is a cumulative model, we give a constructive characterization of the $\prec_D$-minimal states of $\mathcal{D}_\Gamma$. We will, step by step, construct an assumption set A, as well as a corresponding order $<$. Since preferences are dynamic we have to avoid certain pitfalls.

1. We can obtain cyclic preferences such as $t_1 \leftarrow t_2$ and $t_2 \leftarrow t_1$. A relation $\leftarrow$ has a cycle iff its *transitive closure* $\leftarrow^+$ is not irreflexive.
2. We can obtain preferences $t_2 \leftarrow t_1$ although we have already chosen $t_1 < t_2$.

If the first case is obtained by the initial preferences which are derived from Γ then there is no dynamic state satisfying Γ:

Definition 4.3 *A theory Γ is called D-consistent iff Γ is consistent and $\leftarrow_{\Gamma,\emptyset}^+$ is irreflexive.*

Lemma 4.3 *There exists a dynamic state satisfying Γ iff Γ is D-consistent.*

The question is what to do if any of the two problematic cases above is obtained after adding an assumption α to a set A of already selected assumptions. The answer is quite simple: Don't select α because otherwise the current construction will not lead to a dynamic state. This modifies the constructive definition as follows:

Definition 4.4 *Let Γ be a consistent theory. Let $< \subseteq \mathcal{N} \times \mathcal{N}$ be a strict total order and $t_1, \ldots, t_n$ be the enumeration of $\mathcal{N}$ that respects $<$. Let $B_i := A_i \cup \{c(t_{i+1})\}$. We define $A_0 := \emptyset$ and*

$$A_{i+1} := \begin{cases} A_i & \text{if } \Gamma \cup A_i \cup \{c(t_{i+1})\} \models \bot \\ A_i & \text{if } \leftarrow_{\Gamma,B_i}^+ \text{ is not irreflexive} \\ A_i & \text{if } t_j \leftarrow_{\Gamma,B_i} t_k \text{ for a } k, j \text{ s.t.} \\ & \quad k \leq j \text{ and } k \leq i+1 \\ A_i \cup \{c(t_{i+1})\} & \text{otherwise} \end{cases}$$

Then A_i is the dynamic selection *of $t_1, \ldots, t_i$ and A_n is the* dynamic selection *of $<$. The sequence $t_1, \ldots, t_i$ is* correct *iff*

$$t_k \leftarrow_{\Gamma,A_j} t_{j+1} \quad \text{implies} \quad k < j + 1$$

for $j = 0, \ldots, i-1$. *The order* $<$ *is* correct *iff* $t_1, \ldots, t_i$ *is* correct *for all* $i = 1, \ldots, n$.

A is a D-preferred **assumption set** *of* Γ *iff A is the dynamic selection of a correct strict total order* $<$.

We now explore the properties of these definitions. First of all, dynamic selections are consistent and correct orders respect the dynamic preferences they produce:

Lemma 4.4 *Let* Γ *be a consistent theory and* $< \subseteq \mathcal{N} \times \mathcal{N}$ *be a strict total order. Let A be the dynamic selection of* $<$. *Then* $\Gamma \cup A \not\models \bot$. *If* $<$ *is correct then* $\leftarrow_{\Gamma,A} \subseteq <$.

Correct sequences $t_1, \ldots, t_i$ can be constructed incrementally. In each step, we pick a best element t_{i+1} among the non-enumerated assumptions and add it after t_i. t_{i+1} must be a best element w.r.t. the preferences $\leftarrow_{\Gamma,A_i}$ in order to guarantee that $t_1, \ldots, t_i, t_{i+1}$ is correct. Let $R_i := \mathcal{N} - \{t_1, \ldots, t_i\}$:

$$t_{i+1} \in \{x \in R_i \mid \nexists y \in R_i : y \leftarrow_{\Gamma,A_i} x\}$$

The existence of such best elements is insured since cyclic preferences are avoided:

Lemma 4.5 *Let* Γ *be a D-consistent theory. Let* $t_1, \ldots, t_i$ *be a correct sequence of elements of* $\mathcal{N}$ *and* A_i *its dynamic selection. Then there exists an enumeration* $t_{i+1}, \ldots, t_n$ *of* $\mathcal{N} - \{t_1, \ldots, t_i\}$ *s.t.* $t_1, \ldots, t_j$ *is correct for all* $j = 1, \ldots, n$.

Consider a dynamic state $s := (W, A, <)$ in $\mathcal{D}_\Gamma$. If A is the dynamic selection of $<$ then $<$ is correct and s is a $\prec_D$-minimal state in $\mathcal{D}_\Gamma$. Otherwise, there exists a state that is $\prec_D$-smaller:

Lemma 4.6 *Let* Γ *be a D-consistent theory and* $s := (W, A, <)$ *be a dynamic state in* $\mathcal{D}_\Gamma$. *Let* A^* *be the dynamic selection of* $<$.

1. *If* $A = A^*$ *then* $<$ *is correct.*
2. *If* $A = A^*$ *and* $s^* \prec_D s$ *then* $s^* \notin \mathcal{D}_\Gamma$.
3. *If* $A \neq A^*$ *then* $(W_{\Gamma \cup A^*}, A^*, <) \prec_D s$.

These lemmas allow to establish the two main theorems of the paper. First, $\prec_D$-minimal states have a constructive characterization:

Theorem 4.7 *Let* $\Gamma \subseteq \mathcal{L}$ *be a theory. A is a D-preferred assumption set of* Γ *iff there exists a dynamic state* $(W, A, <)$ *that is a* $\prec_D$*-minimal element of* $\mathcal{D}_\Gamma$.

Second, we can now demonstrate the smoothness of $\prec_D$:

Lemma 4.8 $\prec_D$ *is smooth on* $\mathcal{D}_\Gamma$ *for* $\Gamma \subseteq \mathcal{L}$.

Theorem 4.9 $D := (\mathcal{D}, l, \prec_D)$ *is a* cumulative model.

Therefore, the nonmonotonic inference relation $\mid\!\sim_D$ has the five basic properties of system $\mathcal{C}$.

It is straightforward to adapt our approach to Lehmann's preferential models. A *preferential model* is a triple $C := (S, l, \prec)$ where S is a set of states, l a function mapping each state to a *single* world, and $\prec$ is a strict partial order on states that is smooth on all S_Γ, i.e. the set of states satisfying a theory Γ. Preferential models additionally support reasoning by cases:

6. $\alpha \mid\!\sim_C \gamma$, $\beta \mid\!\sim_C \gamma$ implies $\alpha \vee \beta \mid\!\sim_C \gamma$

We obtain a preferential model by restricting dynamic states *(W, A,* $<$) to those where the set *W* of worlds is a singleton, i.e. contains only one world. Further work is needed to adapt the constructive approach to this preferential-model semantics.

5 Example

We determine the D-preferred assumption sets of our initial example. Let Γ_0 be the set of formulas 0.-8 and

$$\Gamma_1 := \Gamma_0 \cup \{birthday(jim)\}$$
$$\Gamma_2 := \Gamma_1 \cup \{saturday\}$$
$$\Gamma_3 := \Gamma_2 \cup \{\neg go(th)\}$$

We consider two correct strict total orders $<_1$ and $<_2$ where

$$go_D(nc) <_1 go_D(th)$$
$$go_D(th) <_2 go_D(nc)$$

Due to formulas 7. and 8., the assumptions one_D, inv_D are smaller than the assumptions $go_D(nc)$ and $go_D(th)$. These formulas have been included to give the assumptions one_D, inv_D a higher priority. Now we consider the dynamic selections $A_{i,j}$ of Γ_i and $<_j$:

$$A_{0,1} = \{c(one_D),\ c(inv_D),\ c(go_D(nc))\} \quad *$$
$$A_{0,2} = \{c(one_D),\ c(inv_D),\ c(go_D(th))\}$$
$$A_{1,1} = \{c(one_D),\ c(go_D(nc))\}$$
$$A_{1,2} = \{c(one_D),\ c(go_D(th))\} \quad *$$
$$A_{2,1} = \{c(go_D(nc)),\ c(go_D(th))\}$$
$$A_{2,2} = \{c(go_D(th)),\ c(go_D(nc))\} \quad *$$
$$A_{3,1} = \{c(go_D(nc))\}$$
$$A_{3,2} = \{c(go_D(nc))\} \quad *$$

Since a normal invitation $c(inv_D))$ implies $go_D(nc) \longleftarrow go_D(th)$ the order $<_2$ is not correct w.r.t. Γ_0 Since the theory T_1 implies $go_D(th) \longleftarrow go_D(nc)$ the order $<_1$ is not correct w.r.t. Γ_1, Γ_2, and Γ_3. As a consequence, each Γ_i has a unique D-preferred assumption set (marked with a *) and we obtain the following inferences:

$$\Gamma_0 \mid\!\sim_D go(nc) \qquad \Gamma_2 \mid\!\sim_D go(nc) \wedge go(th)$$
$$\Gamma_1 \mid\!\sim_D go(th) \qquad \Gamma_3 \mid\!\sim_D go(nc)$$

The conclusions change from Γ_0 to Γ_1 since the preferences change. The change from Γ_1 to Γ_2 is due to the removal of a conflict. The final change is due to a new inconsistency.

6 Related Work

Brewka has extended Reiter's default logic by dynamic preferences on defaults [Brewka, 1994). As in our approach, defaults are named by constants and preferences between defaults are expressed by a binary predicate symbol. The additional expressiveness of default logic, however, makes it difficult to establish a preferential semantics. Even normal defaults as considered in [Brewka, 1994] do not have a cumulative-model semantics as shown by Makinson. In order to compare both approaches, we restrict our attention to normal defaults without prerequisites, which correspond to assumptions.

Brewka requires that a theory Γ contains axioms stating that the predicate symbol $\leftarrow$ represents a strict partial order. These axioms ensure that no cyclic preferences are obtained. In our approach, we did not want to change the original theory Γ and therefore required a corresponding property on the meta-level. Brewka determines preferred assumption sets as follows. An assumption set $A \subseteq \mathcal{A}$ is a BD-preferred iff 1. it is the (static !) selection of a strict total order $< \subseteq \mathcal{N} \times \mathcal{N}$ and 2. $\Gamma \cup A \cup \{t_1 \leftarrow t_2 \mid t_1 < t_2\}$ is consistent. Here, a total order on assumptions is chosen initially and verified in the end by comparing the chosen preferences with those that are implied by $\Gamma \cup A$. Unfortunately, there are examples that don't have BD-preferred assumptions:

$$c(t_1) \supset t_2 \leftarrow t_1,\ c(t_2) \supset t_1 \leftarrow t_2$$

The order $t_1 < t_2$ is not compatible with its (static) selection $\{c(t_1)\}$, but correct w.r.t. its dynamic selection $\{c(t_2)\}$. The selection of $c(t_1)$ fails in the second case since the preference $t_2 \leftarrow t_1$ contradicts the order $t_1 < t_2$. An analogue argument holds for $t_2 < t_1$.

The example shows that cyclic dependencies between preferences and assumptions make the search for preferred assumption sets quite difficult. Therefore, we interleave the construction of an order and an assumption set and we do not choose assumptions that have drawbacks on the already chosen part of the order.

7 Conclusion

We showed how preferences on assumptions can directly be expressed in a logical theory. The resulting system offers a high degree of freedom for "programming" preference rules: Preferences can be used in implications, in quantified statements, and can themselves depend on other assumptions.

Finding a clear mathematical treatment of dynamic preferences turned out to be a non-trivial task. We developed a preferential semantics based on Lehmanivs cumulative models and an equivalent constructive characterization. The resulting nonmonotonic logic

1. allows to program preference rules,
2. satisfies all properties of Lehmann's system *C,*
3. can be implemented for decidable sublanguages.

In order to keep the presentation simple and intuitive, we considered only finite assumption sets in this paper. In a long version of the paper, we will generalize the results to infinite assumption sets and well-founded orders on assumptions.

Thus, an important milestone in the design of an applicable and powerful nonmonotonic logic has been achieved. It can be applied to default reasoning in inheritance system, to diagnostic reasoning, and to decision making. Future work will concentrate on algorithms and applications. Furthermore, we will elaborate a variant of our approach in Lehmann's system *V* which additionally supports reasoning by cases.

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References

[Brewka, 1989] G. Brewka. Preferred subtheories: An extended logical framework for default reasoning. In *IJCAI-89,* pages 1043-1048, Detroit, MI, 1989. Morgan Kaufmann.

[Brewka, 1994] G. Brewka. Reasoning about priorities in default logic. In *AAAI-94,* 1994.

[Brewka, 1996] G. Brewka. Well-founded semantics for extended logic programs with dynamic preferences. *Journal of Artificial Intelligence Research,* 4, 1996.

[GefTner and Pearl, 1992] H. Geffner and J. Pearl. Conditional entailment: Bridging two approaches to default reasoning. *Artificial Intelligence,* 53:209-244, 1992.

[Grosof, 1991] B. Grosof. Generalizing prioritization. In *KR'91,* pages 289-300, Cambridge, MA, 1991. Morgan Kaufmann.

[Junker and Brewka, 1991] U. Junker and G. Brewka. Handling partially ordered defaults in TMS. In R. Kruse and P. Siegel, editors, *Symbolic and Quantitative Aspects for Uncertainty. Proceedings of the European Conference ECSQAU,* pages 211-218. Springer, LNCS 548, Berlin, 1991.

[Junker, 1992] U. Junker. *Relationships between Assumptions.* Doctoral thesis, University of Kaiserslautern, Kaiserslautern, 1992.

[Junker, 1995] U. Junker. Buroeinrichtung mit EXCEPT II. In F. di Primio, editor, *Methoden der Kunsthchen Intelligenz fur Graphikanwendungen.* Addison-Wesley, 1995.

[Kraus *et* a/., 1990] S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence,* 44:167-208, 1990.

[Lifschitz, 1985] V. Lifschitz. Computing circumscription. In *IJCAI-85,* pages 121-127, Los Angelos, CA, 1985. Morgan Kaufmann.

[McCarthy, 1986] J. McCarthy. Applications of circumscription to formalizing common-sense knowledge. *Artificial Intelligence,* 28:89-116, 1986.

[Poole, 1988] D. Poole. A logical framework for default reasoning. *Artificial Intelligence,* 36:27-47, 1988.

[Roos, 1992] N. Roos. A logic for reasoning with inconsistent knowledge. *Artificial Intelligence,* 1992.

[Shoham, 1987] Y. Shoham. A semantical approach to nonmonotonic logics. In *Logics in Computer Science,* 1987.