A Logic of Intention

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Abstract

There is a lot of research on formalization of intention. The common idea of these theories is to interprete intention as an unary modal operator in Kripkean semantics. These theories suffer from the side-effect problem seriously. We introduce an alternative approach by establishing a non-classical logic of intention. This logic is based on a novel non-Kripkean semantics which embodies some cognitive features. We show that this logic does provide a formal specification and a decidable inference mechanism of intention consequences. All and only the instances of side-effects, except ones in absorbent forms, are forbidden in the logic.

1 Introduction

Formalization of intention has drawn the attention of researchers [Cohen and Levesque 1990; Rao and Georgeff 1991; Konolige and Pollack 1993; Wainer 1994; Linder *et al.,* 1995; Huang *et al.,* 1996; Singh, 1997; Schild, 1999]. The common idea is to formalize intention into a modal operator on the framework of Kripkean possible world semantics. Many varieties of the semantics have been put forward, and a lot of models of intention have been established. But all the formalizations suffer from the side-effect problem [Bratman, 1987]. The problem has two most difficult cases, the one concerning closeness of intention consequences under tautological implications and the other concerning closeness under logical equivalencies. The former is about the relation between tautological implications and intention consequences. It asks the question

(Q) Given ψ, a tautological implication of φ, whether intending that ψ is a consequence of intending that φ?

The latter is a special case of the former where "equivalence" is substituted for "implication" and "consequence".

To the question (Q), some of the previous theories answer "yes", most of the others answer "no", and the remainders give indefinite answers. We argue that both of the "yes" and the "no" answers are wrong. Moreover, the first answer causes confusion between an agent's goal and its side-effects, and the second answer results in forsaking the specification of intention consequences.

We provide an alternative approach to formalizing intention. The basic idea is to develop a novel semantics which introduce "cognitive abstraction" into interpretation rules. It turns out that the semantics produces an appropriate specification of intention consequences to the extent that all and only the instances of side-effects, except those in absorbent forms, are avoided. Besides that, the semantics supports a decision procedure for the intention consequences defined in the semantics.

In the next section, we examine the side-effect problem and previous work on it. Section 3 states our motivation and basic ideas. The semantics is developed in section 4 and discussed in depth in section 5. Finally, in section 6, we draw some conclusions and point the way toward further development of our logic of intention.

2 The Side-effect problem

The language we use in this section is a modal extension of a propositional language with operators I and B, where I and B represent intention and belief, respectively, φ and ψ are arbitrary formulas, $\supset$ and $\equiv$ represent material implication and equivalence, respectively. The so-called side-effect problem is captured by the following cases

(SEB) $\vdash \mathbf{B}(\varphi\supset\psi) \Rightarrow \vDash \mathbf{I}(\varphi)\supset\mathbf{I}(\psi)$

(SET) $\vDash \varphi\supset\psi \Rightarrow \vdash \mathbf{I}(\varphi)\supset\mathbf{I}(\psi)$

(SEL) $\vdash \varphi\equiv\psi \Rightarrow \vDash \mathbf{I}(\varphi)\equiv\mathbf{I}(\psi)$

(SEB) is the case of side-effect under belief implications. In this case, an agent's intentions are closed under his/her belief implications. (SET) is the case of side-effect under tautological implications (logical consequences), that is, an agent's intentions are closed under tautological implications. It can be regarded as a special case of the well-known problem of *logical omniscience* [Hintikka, 1962]. As pointed out in [Konolige and Pollack, 1993], this problem is more serious and harmful to intention than belief: logically consequential closure cannot be assumed for intention, even as an idealization; not all the consequences of an agent's intention are intentions of the agent, even the consequences he/she has anticipated. (SEL) is another case of the side-effect problem.

There are good solutions to (SEB), e.g. [Cohen and Levesque, 1990; Rao and Georgeff, 1991; Wainer, 1994], so we will not consider it in this paper. But, on the other hand, neither (SET) nor (SEL) has been solved satisfactorily. A major difficulty in dealing with (SET) comes from the fact that either of the following rules has exceptions,

(C1) $\forall\varphi,\psi$: $\vdash\varphi\supset\psi \Rightarrow\ \models\mathbf{I}(\varphi)\supset\mathbf{I}(\psi)$

(C2) $\forall\varphi,\psi$: $\vdash\varphi\supset\psi \Rightarrow\ \not\models\mathbf{I}(\varphi)\supset\mathbf{I}(\psi)$

That is, tautological implications cannot be transferred to intention consequences in a globally uniform manner.

Previous work concerning (SET) can be classified into three categories. The models of intention in the first category take (CI) and reject (C2) [Cohen and Levesque, 1990; Wainer, 1994]. These models employ normal modal logics to characterize cognitive states including intention. This makes them tolerate and hence suffer from (SET)(Strictly speeking, [Cohen and Levesque, 1990] takes some more constraints on (CI), which avoids (SET) in some way. But Cohen and Levesque did not consider the way satisfactory). The models in the second category take (C2) with constraint $\not\models\psi\supset\varphi$. For example, the representationalist theory of intention [Konolige and Pollack, 1993] employs the minimal model semantics [Chellas, 1980] to interpret the operator I. The only way to infer formulas containing I is by using the rule $\varphi\equiv\psi/\mathbf{I}(\varphi)\equiv\mathbf{I}(\psi)$. Hence, if any intention is derivable from another in the theory, then their content must be logical equivalence. This means that both (SET) and the intention consequence are eliminated from the model. The models in the third category restrict both (CI) and (C2) such that neither of them are valid for all q> and ψ [Linder *et al,* 1995; Huang *et al,* 1996]. Obviously, these models are more appropriate than the ones in the other two categories. But these models are not self-contained: one could not employ these models by themselves to decide whether f"7$\mathbf{I}(\varphi)\supset\mathbf{I}(\psi)$ holds for any φ and ψ. For instance, the model proposed in [Linder *et al,* 1995] uses *awareness* [Fagin and Halpern, 1988] to define preferences, and then goals (intentions). Because the *explicit preferences* cannot be characterized by a formalized system, the intention consequences in the model are not specified. Moreover, no remedial measures to overcome this shortage have been put forward in literature.

As for (SEL), it is usually considered harmless [Linder *et al,* 1995]. But from the point of view of bounded rationality and resource-boundness, (SEL) is inappropriate and harmful: logical equivalencies are not "cognitive equivalencies". For example, from

the side-effectV is introduced. Perhaps one may think the situation could be remedied by demanding that agents always take the "simplest" content of an intention. This requires the specification of "simplest content", the criterion of which has not been established definitely. It follows from the discussion above that all the models mentioned are inadequate. We believe the inadequacy results from the formal tools. We are to deal with these issues in depth.

3 Motivation

For simplicity, in this paper we only consider the formal specification of the intention consequence relation between the *content* of two intentions. Hence we assume in the rest of this paper that any formula in our formal language $\mathbf{L}_1$ represents the content of an intention. For any $\varphi,\psi\in\mathbf{L}_1$, that ψ is an intention consequence of φ means that $\mathbf{I}(\psi)$ is an agent's intention whenever $\mathbf{I}(\varphi)$ is the agent's intention. Thus, $\mathbf{L}_1$ need not contain the modal operator I. A binary operator '$\rightarrow$' is added into $\mathbf{L}_1$ and $\varphi\rightarrow\psi$ means that ψ is an intention consequence of φ.

In all existing theories, the semantic interpretation of an intention $\mathbf{I}(\varphi)$ is based on some set of the intended worlds, where each intended world is a classical possible world [Chellas 1980] satisfying φ, the content of the intention. However, classical possible worlds have following properties that are harmful to the appropriate specification of intention and intention consequences. Suppose φ is the formula representing the content of the intention being considered (and hence being satisfied by all the intended worlds).

(H1) All tautologies are satisfied by each classical possible world. Thus all tautologies are always intentions of any agent. This is a special case of (SET).

(H2) Any ψ logically equivalent to φ is satisfied by each of the intended worlds. So ψ is also an intention whenever φ is. This is (SEL).

(H3) Suppose formula ψ is stronger than φ. Then ψ may not be satisfied by an intended world, or even specified by a set of the intended worlds. This causes the failure to the specification of "strong consequences" (see below).

To overcome all the drawbacks, we introduce a new sort of possible worlds based on "cognitive abstraction". In our semantics, an intended world about φ is a "minimal model" of φ in the sense that only a possibly smallest number of propositional symbols occurring in φ are assigned classical truth values while others are assigned the same abstract value (0 or 1). Any proposition assigned the abstract values are considered to be "abstracted cognitively" (neglected cognitively). A world of this sort is a cognitively finite object, just matching the ability or nature of resource-bounded agents. Based on the set of minimal models, the intention consequence defined in the next section will draw or extract ψ from φ such that ψ is a piece of "partial content" of φ. As a result, the semantics avoids all the harmful properties listed above and fits our purpose well. Most importantly, it supports a well-defined and decidable inference scheme that can derive both "strong consequences" and "weak consequences" of an intention. If an agent intend that $\varphi\wedge\psi$, then both φ and ψ are "partial content" (or "subgoals" as usually called) of the agent's intention. Generally, ψ is called a *weak consequence* of φ, if φ is stonger than ψ in the classical logic. Sometimes an agent need derive from his/her intention φ some ψ as a "means" to the "end" φ, where ψ is stronger than φ. We call such ψ a *strong consequence* of φ. The compositions of strong and weak consequences are called hybrid consequences. For example, given intention $(x_1\vee x_2)\wedge x_3$, its weak consequences are $x_1\vee x_2$ and x_3, strong consequences are $x_1\wedge x_2$ and $x_2\wedge x_3$, and hybrid consequences are x_1 and x_2.

We will establish a logic of intention, L_{mp4c}, and employ the set of the L_{mp4c}'s valid formulas of the form $\varphi\rightarrow\psi$ to provide a formal specification for the intention consequence. Therefore, for any $\varphi,\psi\in L_1$, the problem "if ψ is an intention consequence of φ" is reduced to the problem "if $\models\varphi\rightarrow\psi$ holds in L_{mp4c}". The three kinds of intention consequence described above will be defined uniformly in L_{mp4c}, this makes it an uniform mechanism of inferring intention consequences. Moreover, we can also provide an algorithm to decide whether an intention consequence is a strong, weak, or hybrid one. If φ and ψ satisfy the single-level description assumption (SDA), i.e., all items of the primitive intention content (represented by proposition symbols in L_1) occurring in ψ also occur in φ, then that $\models\varphi\rightarrow\psi$ holds in L_{mp4c} will guarantee that ψ will *realize or elaborate φ without side-effect.*

It follows from the discussion above that the validity characterized by our logic should conform to the following principles

(P1-1) $\models x_1\wedge x_2\rightarrow x_1$
(P1-2) $\models(\neg x_1\wedge x_1)\rightarrow x_1$
(P1-3) $\models x_1\wedge(x_1\vee x_2)\rightarrow x_1$
(P1-4) $\models x_1\wedge(x_1\vee x_2)\rightarrow(x_1\vee x_2)$
(P2-1) $\models(x_1\vee x_2)\rightarrow x_1$
(P2-2) $\models x_1\rightarrow(\neg x_1\vee x_1)$
(P2-3) $\models x_1\vee(x_1\wedge x_2)\rightarrow x_1$
(P2-4) $\models x_1\vee(x_1\wedge x_2)\rightarrow(x_1\wedge x_2)$
(P3) $\models x_1\rightarrow(x_1\vee x_2)$
(P4) $\models(x_1\wedge x_2)\leftrightarrow(x_1\vee x_2)$
(P5-1) $\models x_1\wedge(x_2\wedge x_3)\leftrightarrow(x_1\wedge x_2)\wedge x_3$
(P5-2) $\models x_1\vee(x_2\vee x_3)\leftrightarrow(x_1\vee x_2)\vee x_3$
(P6-1) $\models x_1\wedge x_2\leftrightarrow x_2\wedge x_1$
(P6-2) $\models x_1\vee x_2\leftrightarrow x_2\vee x_1$
(P7-1) $\models x_1\wedge(x_2\vee x_3)\leftrightarrow(x_1\wedge x_2)\vee(x_1\wedge x_3)$
(P7-2) $\models x_1\vee(x_2\wedge x_3)\leftrightarrow(x_1\vee x_2)\wedge(x_1\vee x_3)$
(P8-1) $\models\neg(x_1\wedge x_2)\leftrightarrow(\neg x_1\vee\neg x_2)$
(P8-2) $\models\neg(x_1\vee x_2)\leftrightarrow(\neg x_1\wedge\neg x_2)$
(P9) $\models\neg\neg x_1\leftrightarrow x_1$
(P10-1) $\models x_1\leftrightarrow x_1\wedge x_1$
(P10-2) $\models x_1\leftrightarrow x_1\vee x_1$

4 Formalization

Let L_1 be the propositional language with a set of propositional symbols Atom=$\{x_1, x_2, \ldots\}$ and logical connectives $\neg$, $\wedge$, and $\vee$. The formulas of L_1 are defined as usual. Let L, the language of L_{mp4c}, be an extension of L_1 with only one additional operator $\rightarrow$. Any formula of L has the form $\varphi\rightarrow\psi$ where $\varphi,\psi\in L_1$. The semantic interpretation of L_1 is defined over the set $\mathbf{T}=\{t,f,0,1\}$, where t and f mean truth and falsehood, respectively, and 0 and 1 represent two states of "cognitive abstraction".

Definition 1 (Assignment) A **0-assignment** is a mapping
$g0$: Atom$\rightarrow\{t,f,0\}$.
A **1-assignment** is a mapping
$g1$: Atom$\rightarrow\{t,f,1\}$.
An **assignment** is either a 0-assignment or a 1-assignment. □

We will use M_0 and M_1 to denote the set of 0-assignments and the set of 1-assignments, respectively, and M to denote $M_0\cup M_1$.

The connectives $\neg$, $\wedge$ and $\vee$ are interpreted by operators -, *, and + on $\mathbf{T}$, respectively. These operators are defined in Figure 1.

x	-x
t	f
f	t
0	1
1	0

*	t	f	0	1
t	t	f	0	t
f	f	f	0	f
0	0	0	0	0
1	t	f	0	1

+	t	f	0	1
t	t	t	t	1
f	t	f	f	1
0	t	f	0	1
1	1	1	1	1

Figure 1. The definitions of -, * and +

Definition 2 (Valuation) **A valuation** on L_1 under $\pi\in M$ is a mapping V_π assigning to each formula in L_1 a value in T, such that for all x∈ Atom and $\varphi,\psi\in L_1$

(1) $V_\pi(x) = \pi(x)$;

(2) $V_\pi(\neg\varphi) = -V_\pi(\varphi)$;

(3) $V_\pi(\varphi\wedge\psi) = V_\pi(\varphi)*V_\pi(\psi)$; and

(4) $V_\pi(\varphi\vee\psi) = V_\pi(\varphi)+V_\pi(\psi)$. □

V_n is an extension of π and will be abbreviated π hereafter. For simplicity, we will use $M_0(M_1,M)$ to denote also the set of valuations that are extensions of assignments in $M_0(M_1,M)$.

Definition 3 (Model) For any $\pi\in M$ and $\varphi\in L_1$, π is a **model** of φ if and only if $\pi(\varphi)\in\{t,f\}$. □

If is a model of φ and $\pi(\varphi)$=t(f), then π is called a t-model (f-model) of φ. Hereafter $[\varphi]_t([\varphi]_f)$ denotes the set oft-models(f-models)of φ and $[\varphi]$ denotes $[\varphi]_t\cup[\varphi]_f$.

The key idea to embody the cognitive characteristics of resource bounded agents is to introduce a "cognitive abstraction" relation on M.

Definition 4 (Cognitive abstraction) For any $\pi,\pi'\in M$,

(1) π is a **O-abstraction** of π', denoted by $\pi\leq_0\pi'$, iff

(3) n is a **cognitive abstraction** of π', denoted by $\pi\leq\pi'$, if and only if $\pi\leq_0\pi'$ or $\pi\leq_1\pi'$. □

Definition 5 (Minimal model) For any $\varphi\in L_1$ and $\pi\in[\varphi]$, π is a **minimal model** of φ iff there exists no $\pi'\in[\varphi]$ such that $\pi\neq\pi'$, $\pi'\leq\pi$ and $\pi'(\varphi)=\pi(\varphi)$. □

For any $\varphi\in L_1$, $[\![\varphi]\!]$ denotes the set of minimal models of φ.

Definition 6 (Validity) For any $\varphi,\psi\in L_1$, $\pi\in M$,

(1) $\varphi\to\psi$ is **true** under π, denoted by $\pi\models\varphi\to\psi$, if $\pi\in[\![\psi]\!]\Rightarrow\exists\pi'\in[\![\varphi]\!]$: $\pi\leq\pi'$ & $\pi(\psi)=\pi'(\varphi)$; otherwise, $\varphi\to\psi$ is false under π.

(2) $\varphi\to\psi$ is **valid** in L_{mp4c}, denoted by $\models\varphi\to\psi$, if $\pi\models\varphi\to\psi$ for all $\pi\in M$. □

Theorem 7 (Reflexivity and Transitivity) For any $\varphi,\chi,\psi\in L_1$, (i) $\models\varphi\to\varphi$; (ii) if $\models\varphi\to\chi$ and $\models\chi\to\psi$, then $\models\varphi\to\psi$. □

Theorem 8 (Decidability) The intention consequence in L_{mp4c} is decidable. That is, there exists an algorithm A such that for any $\varphi,\psi\in L_1$, A returns "yes" if $\models\varphi\to\psi$ and "no" if $\not\models\varphi\to\psi$. □

5 Features of L_{mp4c}

We show the side-effect-free property of L_{mp4c} by examining (SEL) and (SET) separately.

5.1 Free from (SEL)

We describe the relative results following the thread of their proofs, but omit these proofs here.

Any $\varphi,\psi\in L_1$ are called intention equivalent in L_{mp4c}, denoted by $\models\varphi\leftrightarrow\psi$, if $\models\varphi\to\psi$ and $\models\psi\to\varphi$.

Theorem 9 L_{mp4c} has properties (P4)-(P 10-2). □

This indicates that all the methods for transferring formulas into normal forms are preserved in L_{mp4c}. So we have

Corollary 10 For any $\varphi\in L_1$, there exists $\psi\in L_1$ f normal form such that $\models\varphi\leftrightarrow\psi$. □

Any ψ is called a *normal form* of φ if ψ is in normal form and $\models\varphi\leftrightarrow\psi$. For any $\varphi,\psi\in L_1$, define $[\varphi]\equiv[\psi]$ as that $[\varphi]=[\psi]$ and $\pi(\varphi)=\pi(\psi)$ for all $\pi\in M$. And $[\![\varphi]\!]\equiv[\![\psi]\!]$ is defined similarly.

Theorem 11 For any $\varphi,\psi\in L_1$, following assertions are equivalent:

(1) $\models\varphi\leftrightarrow\psi$;

(2) $[\varphi]\equiv[\psi]$;

(3) $[\![\varphi]\!]\equiv[\![\psi]\!]$. □

There are two sorts of equivalencies closely relative to (SEL). A formula $\varphi\in L_1$ is called an *absorbent form* if φ has a sub-formula of the forms $\psi\wedge(\psi\vee\chi)$ or $\psi\vee(\psi\wedge\chi)$. ψ is called an *absorbent normal form* of φ if ψ is in absorbent form and is a normal form of φ. ψ is called a *strict normal form* of φ if ψ is a normal form of φ and Atom(φ)=Atom(ψ), i.e., for all x∈Atom, x occurs in ψ iff x occurs in φ.

Theorem 12 (Equivalence) For any $\varphi,\psi\in L_1$, where ψ is in normal form, $\models\varphi\leftrightarrow\psi$ iff

(1) ψ is a strict normal form of φ; or

(2) ψ is an absorbent normal form of φ. □

Corollary 13 (Side-effect-free in equivalence under

Atom) For any $\varphi,\psi\in L_1$ not being of absorbent form, if $\vdash\varphi\leftrightarrow\psi$ then Atom(φ)=Atom(ψ). □

This result can be strengthened further even in situations where Atom(φ)=Atom(ψ).

Theorem 14 (Side-effect-free in equivalence)

(1) $\vDash\varphi\leftrightarrow\varphi\wedge(\neg\psi\vee\psi)$;

(2) $\vDash\varphi\leftrightarrow\varphi\vee(\neg\psi\wedge\psi)$. □

The instances of this invalidity can be found next subsection, e.g., (PI-6), (PI-7) and (P2-6), etc.

Now the feature of intention equivalencies in L_{mp4c} can be summarized in two aspects. First, intention equivalencies cover all strict and absorbent normal forms. Second, all of the other forms of equivalencies in the standard logic are forbidden. This means that all instances of (SEL), except absorbent equivalencies, are avoided in L_{mp4c}.

5.2 Free from (SET)

Theorem 15 (Side-effect-free in consequence under Atom) For any $\varphi,\psi\in L_1$, if $\vdash\varphi\rightarrow\psi$ and ψ is not in absorbent forms, then Atom(ψ)$\subseteq$Atom(φ). □

At the first glance, the property is shared with Bochvar's 3-valued logic, where ψ is a semantic consequence of φ iff it is in the classical logic and Atom(ψ)$\subseteq$Atom(φ)[Mo, 1954]. However, there are remarkable differences between Bochvar's and ours, which will become clear in the following discussion.

Theorem 16 L_{mp4c} has the properties (PI-1)-(P3). □

This theorem, together with theorem 9, shows that L_{mp4c} follows all the principles previously put forward.

It seems from properties (PI-1) and (PI-3) that L_{mp4c} is similar to the standard logic in deriving weak consequences: a conjunct of a conjunctive intention is also an intention, or a "sub-intention". (PI-4) and (PI-2) reveal that it is not the case. In particular, the inference in L_{mp4c} is not "driven" only by the surface layer of the syntactic description of formulas, while at least in some extent the standard logic is. Moreover, (PI-4) also gives an instance that L_{mp4c} forbids the *unnecessary weakening* of intentions.

Properties (P2-1) and (P3) indicate other significant differences between the intention consequence in L_{mp4c} and the logical consequence in the standard logic: L_{mp4c} can derive strong consequences of a given intention, i.e., sufficient means for the end without side-effects. The difference is made clearer by (P2-3) and (P2-4), where the latter also shows that *unnecessary strengthening* is not allowed in L_{mp4c}. Further properties about strong consequences are given as follows.

Theorem 17

(P2-5) $\vDash x_1\rightarrow(\neg x_1\wedge x_1)$

(P2-6) $\vDash x_1\rightarrow x_1\vee(\neg x_2\wedge x_2)$

(P2-7) $\vDash x_1\vee(\neg x_2\wedge x_2)\rightarrow(\neg x_2\wedge x_2)$

(P2-8) $\vdash x_1\vee(\neg x_2\wedge x_2)\rightarrow x_1$

Properties (P2-5), (P2-6) and (P2-7) are three more instances of prohibiting intention consequences from deriving side-effects. These side-effects contain logical contradictions, but they may not contain new atoms. However, a "contingent" intention can be extracted from its compound with a logical contradiction, as (P2-8) shows. It is also forbidden in L_{mp4c} to infer intention consequences from contradictory intentions, an instance of this property is (PI-2).

Another kind of side-effects concerns tautological intentions. Fortunately, this kind of side-effects is also refused by L_{mp4c}.

Theorem 18

(P1-6) $\vDash x_1\rightarrow x_1\wedge(\neg x_2\vee x_2)$

(P1-7) $\vdash x_1\wedge(\neg x_2\vee x_2)\rightarrow x_1$

(P1-8) $\vdash x_1\wedge(\neg x_2\vee x_2)\rightarrow(\neg x_2\vee x_2)$

(P1-9) $\vDash (\neg x_1\vee x_1)\vee x_2\rightarrow x_2$

(P1-10) $\vDash (\neg x_1\vee x_1)\rightarrow x_1$

From above discussion we can draw the conclusion that L_{mp4c} avoids all side-effects usually appearing in intention consequences, except ones in absorbent form.

6 Conclusion

L_{mp4c} is rather simple and powerful. It does provide a formal specification and a decidable inference mechanism of intention consequences. Most strikingly, all and only the instances of side-effects we have identified, except those in absorbent forms, are forbidden in L_{mp4c}. Meanwhile, all the rules for transferring formulas into normal forms in classical logics are preserved in L_{mp4c}.This indicates that the logic has a moderate descriptive granularity, lying between that of the standard semantics and that of the syntactic approach [Eberle, 1974; Konolige, 1986]. Perhaps the traditional idea of interpreting intention as an unary modal operator in Kripke's semantics is misleading, although this idea has many advantages. In addition, L_{mp4c} is not just a subsystem of the classical proposition logic — both weak and strong consequences can be derived in it with a unified mechanism. So it could be used as a new tool of reasoning about plans.

We have concentrated on intention consequences in this paper. Some further development is also under consideration. First, $\mathbf{L_{mp4c}}$ need to be extended to languages including action terms and be able to function beyond the SDA (see section 3). Second, we have discovered that $\mathbf{L_{mp4c}}$ provides an interesting definition and mechanism of some kind of non-monotonic reasoning. Third, it is deserved to consider whether the semantics of $\mathbf{L_{mp4c}}$ can be generalized and employed to treat other kinds of cognitive state such as belief, and to model the interaction among these cognitive operators. Last, the tractability of the decision procedure for $\mathbf{L_{mp4c}}$ is an open problem.

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