A New Framework for Reasoning about Points, Intervals and Durations

Arun K Pujari
Artificial Intelligence Lab,
Dept. of Computer & Information Sciences
University of Hyderabad,
HYDERABAD, 500 046, INDIA
akpcs@uohyd.ernet.in

Abdul Sattar
Knowledge Representation fc Reasoning Unit,
School of Computing k Information Technology,
Oriffit h University,
NATHAN, QLD 4111, AUSTRALIA
A.Sattar@cit.gu.edu.au

Abstract

We present a new framework for reasoning about points, intervals and durations--Point Interval Duration Network (PIDN). The PIDN adequately handles both qualitative and quantitaive temporal information. We show that Interval Algebra, Point Algebra, TCSP, PDN and APDN become special cases of PIDN. The underlying algebraic structure of PIDN is closed under composition and intersection. Determinig consistency of PIDN is NP-llard. However, we identify some tractable subclasses of PIDN. We show that path consistency is not sufficient to ensure global consistency o[the tractable subclasses of PIDN. We identify a subclass for which enforcing 4-consistency suffices to ensure the global consistency, and prove that this subclass is maximal for qualitative constraints. Our approach is based on the geometric interpretation of the domains of temporal objects. Interestingly, the classical Helly\s Theorem of 1923 is used to prove the complexity for the tractable subclass.

1 Introduction

Several formalisms for expressing and reasoning with temporal constraints have been proposed [l8J, [17], [3J, [7]. Several classes of Temporal Constraint Satisfaction Problems (TCSP) are defined depending on the time entity that the variables can represent, namely time points, time intervals, durations and the class of constraints namely qualitative, metric or both. Qualitative approaches such as Allen's [l] Interval Algebra and Villain and Kautz's [17] Point Algebra have difficulties in representing metric, numerical information. On the other hand, the quantitative approach of Dechter *el. ai* [3] has shortcomings in representing qualitative information. Meiri [it] proposes a combined network-based computational model for temporal reasoning that is capable of handling both qualitative and quantitative information. There have been also efforts to combine point and interval objects in a single constraint network [l lj. In order to encode duration information, a separate network is used which is orthogonal to the interval relationship network (e.g., IA [l], PDN [12], APDN [19], metric time-point and duration model [2]) but these two networks do not function independent of each other. Barber [2] presents a duration based temporal model with metric constraint but this model cannot handle disjunctive qualitative constraint. Meiri's attempt [11] to unify the qualitative and quantitative constraint can handle both point and interval variables but cannot handle duration information.

(liven a TCSP ,S\ one important reasoning problem is to determine whether *S* is consistent. Consistency checking is NP-hard problem for the general TCSP [3], [15], while they are polynomial for PA [.16] and for some important special cases of IA and TCSP. These include a sub-algebra of Interval Algebra namely, ORD-Horn class [13] and *simple* TCSP [3]. In a landmark paper [13] Nebel and Biirckert. show that ORD-Horn class form *the* maximal tractable class. It is also known that PDN and APDN admit polynomial algorithms for the consistency checking problem only when the constraints are *simple.*

We present a new framework for representation and reasoning about points, intervals and durations-Point Interval Duration Network (PIDN). The PIDN adequately handles both qualitative and quantitative temporal information. We show that existing frameworks such as Interval Algebra, Point Algebra, TCSP framework, PDN and APDN become special cases of PIDN. The underlying algebraic structure of PIDN is closed under composition and intersection. Determining consistency of the PIDN is NP-IIard. However, we identify some tractable subclasses of PIDN. In the spirit of ORD-Horn class of Nebel and Biirkert [13], we characterize the maximal tractable subclass of this algebra. Further, we show that path consistency is not. sufficient to ensure global consistency for the tractable class of PIDN. We identify a subclass for which enforcing 4-consistency suffices to ensure the global consistency, and prove that this subclass is maximal for qualitative constraints. Our approach is based on the geometric interpretation of the domain of the temporal objects and hence, provides a better insight to some of the earlier problems. It also gives a simpler analysis of ORD-Horn classes as the inaxirnal tractable classes of the Interval Algebra.

2 Geometric Interpretation

We refer here to two types of temporal objects namely, points and intervals. Intervals correspond to time period during which events occur and points represent the beginning and end point of some event as well as the neutral points of time. A useful representation of interval relations is in terms of regions in the Euclidean plane [9; 14]. Namely. an interval is defined by a pair of real numbers (X,Y) such that $\leq$:Y . (The special case of $X = Y$ refers to an interval as a time point). Hence, the set of all intervals in that sense can be identified with the half plane H defined by the in-equation $X \leq Y$ in the (X,Y)-plane. Let (a, 6) be a fixed interval. If (a, 6) imposes a constraint on an interval (X,Y) then the admissible domain of (X,Y) is a region (not necessarily a connected region) in this half plane.

Thus a more general way of defining a Temporal Constraint Satisfaction Problem is to specify the admissible region that one temporal object imposes on another. The variables are the 2-dimensional points in this half plane representing general temporal objects. A temporal object can be a time point or a time interval. The domain of the temporal object is H. The constraints can be viewed as admissible regions and a disjunction of constraints is the union of the regions. For a given temporal object, we denote $d = Y - X$ as the duration of the object. The time points correspond to the temporal objects of 0 duration. We can define the dimension of a relation as the dimension of the associated region. Let us consider the well-known example here [11].

Example 1: *John and Fred work for a company that has local and main offices in LA. They usually work at the local office, in which case it takes John less than 20 minutes and Fred 15-20 minutes to get to work. Twice a week John works at the main office, in which case his commute to work takes at least 60 minutes. Today John left home between 7:05-7:Warn, and Fred arrived at work between 7:50-7:55am. We also know that Fred and John met at a traffic light on their way to work.*

There are four temporal objects O_1 *(John was going to work),* O_2 *(Fred was going to work),* O_3 *(John left home)* and O_4 *(Fred arrived at work).* O_1 and O_2 are time intervals and O_3 and O_4 are time points with the unary constraints *(X3* = Y3) and *(X4* = Y4). O_3 and O_4 also have other metric unary constraints which restricts their domains to regions K3, corresponding to interval [7:05,7:10] and *R4* corresponding to [7:50, 7:55], respectively (see Figure 1). Similarly, any instantiation of O_3 imposes a restriction (qualitative) on O_1 that both have same abscissa value and O_4 imposes a restriction on O_2 that they have same ordinate value. There are unary restrictions of the duration on O_1 and O_2. The admissible regions are $R_{11} \cup R_{21}$, R_2, R_3 and R_4. *0*i* imposes a binary qualitative constraint on O_1. For every instantiation of O_2 in R_2 this constraint defines the domain of O_1. The composition of the two constraints yield *ABCD* as the admissible region of O_1. The admissible domain of O_1 based on admissible instantiation of *0$* and the unary constraint on duration is $R_{11} \cup R_{21}$. Taking the intersection of the constraints, the admissible region of O_1 is only R_{21}. This illustrates the process of intersection and composition operations when we represent the domain as regions in the half plane. It may be noted that R_3 and R_4 are 1-dimensional whereas R_x and R_2 are 2-dimensional,

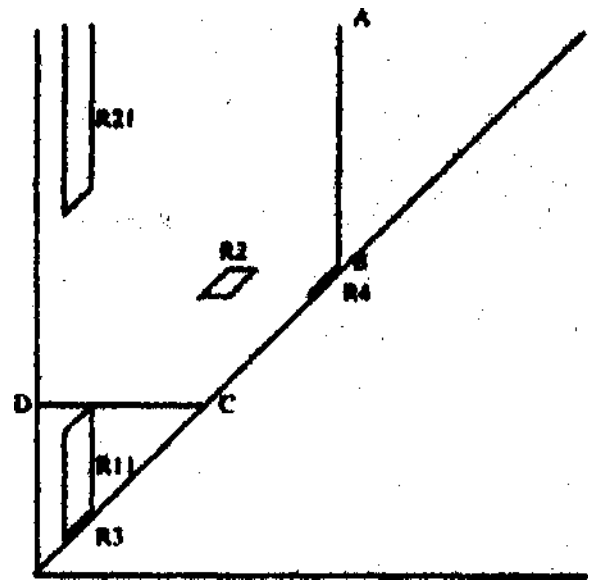

Figure 1: The admissible regions in H for different temporal constraints in Example 1

3 Point-Interval-Duration Network (PIDN)

This study is inspired by the geometric interpretation of temporal objects, constraints and their operations. There can be different ways of encoding regions. In the present study, we consider only the TCSP's having constrained regions as convex hexagon with sides parallel to three fixed lines namely, -Y-axis, y-axis, and X = *Y* line. Such regions can be described in terms of three intervals.

3.1 Definitions

A Point-Interval-Duration Network (PIDN) consists of a set of variables $O_1, O_2, \ldots, O_n$, having H as the domain, each representing a temporal object (interval or point). An *instantiation* of O_i means mapping of O_i to a point in H. A constraint is a disjunction of the form $C = (r_1) \vee (r_2) \vee \ldots \vee (r_l)$, where each r is an ordered set of three intervals (S, E, D). The intervals 5, *E* and *D* are closed or open and bounded or unbounded in either side. Intuitively, the intervals S, *E* and *D* represent the range of the domain of the start point, end point and the duration, respectively of the temporal object. A temporal object $O = (X,Y)$ is said to satisfy the constraint *C,* if it satisfies at least one of the disjuncts r< and it satisfies a r_i, if $X \in S_i$, $Y \in E_i$ and $(Y - X) \in D_i$. Following the standard notation, [a,b], (a,b], [a,b),and (a,b) represent closed, semi-closed, open intervals, respectively. ? represents a unrestricted interval or universal constraint.

A *unary constraint* C for a temporal object O_i = (X_i, Y_i) specifies a set of $r_p = (S_p, E_p, D_p)$ which constraints the domain of O_i. We assume that a unary constraint is always a quantitative constraint and the endpoints of S, *E* and *D* are real numbers.

A *binary constraint k* of the form $O_i(C)O_j$, where each disjunct r_p of C is defined i ll terms of X_i, Y_i and <*k* and it constrains the domain of *Qj*. The end points of S, E and *D* are one of the following types:

1. A nonnegative real number, say δ for metric informatiom.
2. A variable X_i, Y_i and d_i, say W_i for qualitative information.

A binary constraint is said to be a *qualitative* constraint if the endpoints of 5, *E* and *D* are of the restriction that *6* takes on values only 0 or, ∞, A network is called a Qualitative PIDN, if all its constraints are qualitative.

A *solution* is an instantiation of all 0,'s satisfying all the constraints. The network is *consistent* iff at least one solution exists.

Example 2

1. *While returning from office John took longer than the time he took to reach office.*

   If O_J *(going to office)* and O_2 is *(returning from office)* then $O_J(C)O_2$ with C as $((Y_J, \infty), ?, (d_J, \infty))$, where $d_J = Y_J - X_J$. We assume here that John returned from the office after he reached the office.

2. We can encode a qualitative constraint involving point and duration information. *Fred and John take same time to reach office but they start at different time.* Let $O_F = (X_F, Y_F)$ *(Fred going to office)* and O_J as above and $d_F = (Y_F - X_F)$.

   We write $O_F(C)O_J$, where

   $$C = ([0, X_F), ?, [d_F, d_F]) \vee ((X_F, \infty), ?, [d_F, d_F])$$

3. We can encode a disjunction of constraints on point and duration. It may not be possible to encode directly this information in PDN formalism. *Bob reaches office in less than 20 minutes if he start before 7.00am but takes at least 40 minutes due to traffic if he starts later.*

   The unary relation on O_B *(Bob was going to office)* is

   $$C = ((0, 7], ?, [0, 20)) \vee ((7, \infty), ?, [40, \infty))$$

3.2 Operations on PIDN

In order to carry out the reasoning task on PIDN, it is necessary to define 3 fundamental operations on PIDN. These are (i) Composition, (ii) Intersection and (iii) Converse.

Intersection of Constraints: Let $C^1 = \bigvee_i(r_i)$ and $C^2 = \bigvee_i(r'_i)$ be two constraints representing two sets of triplets of intervals. $C^1 \cap C^2$ is the disjunction of the pairwise intersection of r_i's and r'_j's such that if $r_i = (S, E, D)$ and $r'_j = (S', E', D')$ then $r_i \cap r'_j = (S \cap S', E \cap E', D \cap D')$. Intuitively, it is the intersection of the admissible regions defined by both the constraints. If the end points of S, (S', resp.) are a and b (c and d, respectively), then $S \cap S'$ has end points as $sup(a, c)$ and $inf(b, d)$. In case $sup(a, c) > inf(c, d)$, then S and S' are disjoint. However for all non-disjoint cases, it is not trivial to compute $S \cap S'$. If the constraints are unary constraints, the computation of the extremes are straight forward. If the constraints are qualitative constraints then using the fact that for any i, $0 \leq X_i \leq Y_i \leq \infty$, it is possible to determine the intersection of $S \cap S'$. On the other hand, when the constraints are quantitative binary constraints, the difficulty arises in computing intersection when the extremes of S and S' involve different variables. For example, if $a = Y_i + \delta_1$ and $c = X_i - \delta_2$ then it will be hard to determine $sup(a, c)$. This situation can be handled in two different ways, (i) the intersection involves two different terms with disjunction or, "(ii) approximate the intersection with any one of the term.

Converse: Since every inequality is of the form $W_i \pm \delta_3 \leq W_j \leq W_i \pm \delta_4$, it can be equivalently written as $W_j \mp \delta_4 \leq W_i \leq W_j \mp \delta_3$.

Composition in PIDN: Let C^1 and C^2 be two binary constraints such that $O_i(C^1)O_j$ and $O_j(C^2)O_k$. In other words, C^1 (C^2, respectively) defines the admissible domain of O_j(O_k) in terms of O_i (O_j). The composition of C^1 and C^2, denoted by $C^1 \otimes C^2$ defines the domain of O_k with respect to O_i. $C^1 \otimes C^2$ is obtained as the disjunctions of pairwise compositions of the disjuncts r_i of C^1 and r'_j of C^2. Thus, each interval in a disjunct of C^2 is of the form $W_j \pm \delta_1 \leq W_k \leq W_j \pm \delta_2$ and similarly, that of C^1 is of the form $W_i \pm \delta_3 \leq W_j \leq W_i \pm \delta_4$. Using the composition of inequalities, we can express W_k in terms of W_i. Intuitively, composition is the Minkowski addition of sets in two-dimensional plane.

Example 3: Let $O_1 C^1 O_2$ where $C^1 = ((Y_1, \infty), ?, (d_1, \infty))$ and $O_2 C^2 O_3$ where $C^2 = ([0, X_2), ?, ?)O_3$ then using composition we get $C^1 \otimes C^2 = ([0, Y_1), ?, ?)$.

Proposition 1: *PIDN is closed under composition, intersection and converse.*

4 Reasoning with PIDN

The most important tasks for the any TCSP (and hence for PIDN) is to decide consistency; and if the network is consistent then to find a consistent instantiation of the temporal objects.

Proposition 2: *The problem of deciding consistency of the PIDN is NP-hard.*

Proof: *(sketch)* The qualitative temporal consistency problem based Interval Algebra becomes a special instance of PIDN. We show this in Section 5 and deciding consistency of Interval TCSP is NP-hard [18]. D

4.1 Tractable cases of PIDN

Processing of disjunction: The constraint $r_1 \vee r_2$ where $r_1 = (S_1, E_1, D_1)$ and $r_2 = (S_2, E_2, D_2)$ can be **joined along** S to be equivalent to r if (i) $E_1 = E_2$, $D_1 = D_2$ and (ii) $S_1 \cup S_2$ is a single interval. In this case, $r = (S_1 \cup S_2, E_1, D_1)$. Clearly, r_1 and r_2 define two convex polygonal regions such that the union is also convex.

The constraint $r_1 \vee r_2$ can be **loosely joined along** S to be equivalent to r if $r'_1 = (closure(S_1), E_1, D_1)$ and $r'_2 = (closure(S_2), E_2, D_2)$ can be joined along S. If $S = (a, b)$ then $closure(S) = [a, b]$. It is easy to see that if S_1 and S_2 are open intervals it is not necessary that $S_1 \cup S_2$ is a single interval. Hence, though *r%* and r_2 define two convex polygonal regions in H, by loose join we only ensure that the union of these regions are connected but for a line or a point.

In the similar line, we can also join along *S* and *E* if both the disjunct have same *D* and the union results in single convex region. Thus for a given C, it may be possible to combine a set of disjunctions along 5, *E* and/or *D* to get a constraint having less number of disjunctions.

A constraint is said to be a *convex constraint* if it can be reduced to a disjunction-free constraint by the process joining along S, E and/or D. An PIDN is said to be a convex PIDN if all its constraints are convex constraints.

Example 4: If $C^1 = ((5,7),[17,20),[11,14]) \vee ((7,9),[17,20),[11,14])$ it can be loosely joined along S to yield $((5,7) \cup (7,9),[17,20),[11,14])$. The region defined by C^1 in $\mathbf{H}$ is a convex polygonal region except for a linear discontinuity of $X = 7$ as defined by the foil wing.

$$5 < X < 9, X \neq 7, 17 \leq Y < 20, 11 \leq Y - X \leq 14.$$

However, on the other hand, if $C^2 = ((5,8],[17,20),[11,14]) \vee ((8,9),[17,20),[11,14])$ then its loose join along 5 results in a convex polygonal region defined by $((5,9),[17,20),[11,14])$. If $C^3 = ((5,7),[17,20],[11,14]) \vee ((6,7.5],(17.5,20.5],[11,14])$ then also we can join these disjunct along *S* and *E* to obtain a single constraint as $((5,7.5],[17,20.5],[11,14])$.

A constraint is said to be a *preconvex constraint* if it can be written as a disjunction-free constraint by the process of loose join. A PIDN is called as pre-convex PIDN if all its constraints are pre-convex. The pre-convex constraints correspond to the polygonal admissible regions with finite number of linear discontinuities.

Proposition 3: *The Pre-convex PIDN ts closed under composition, intersection and converse.*

Proof: *(sketch)* The admissible regions defined by a convex constraint is a convex polygonal region in H. The convexity property is preserved during intersection, composition and converse operations. It is easy to see that the result holds true for pre-convex constraints too which define admissible regions as convex regions with finite number of linear discontinuities and these do not affect the three basic operations. □

4.2 ^-Consistency in PIDN

The single most important technique of determining consistency of a temporal network is the idea of enforcing some degree of local consistency to eliminate non-feasible instantiations. The PIDN is *3-consistent* if it is arc-consistent and for each triplets (i,j,k), for any consistent instantiation of O_i and O_j (i.e., satisfying binary constraint between them) there exists an instantiation of O_k which is consistent with the constraints $O_i(C_{ik})O_k$ and $O_j(C_{jk})O_k$. Similarly, we can generalized the notion to define *k-consistent.*

Proposition 4: For a Pre-convex PIDN, enforcing 4-consistency is sufficient to ensure consistency of the network.

Proof ***(sketch)*** **We prove the result initially for the Con vex PIDN and then generalize it to Pre-Convex PIDN Let us assume that the convex PIDN with temporal ob jects O_i, $1 \leq i \leq n$ is 4-consistent. Let us also assum that the admissible regions of O_n in $\mathbf{H}$, as defined by th consistent instantiation of each of O_i, $1 \leq i \leq (n-1$ are R_{in}, $1 \leq i \leq (n-1)$.Convex PIDN define the admis sible regions which are convex. Since it is 4-consistent $R_{jn} \cap R_{kn} \cap R_{ln} \neq \emptyset$. From Helly's Theorem [4], we hav $\bigcap_{1 \leq i \leq (n-1)} R_{in} \neq \emptyset$. This property also holds for pre** convex relations due to the fact that the instantiation can be an interior point of the admissible regions of the pre-convex relations [9]. □

It is well known that 3-consistency is enough to guarantee global consistency for all the tractable temporal network problems. The tractable class include Simple TCSP(or, STP) [3], STP$^{\neq}$ [5] Simple PDN [12], Simple APDN [19], Point Algebra [17], SIA subclass of IA and ORD-Horn class of IA. However, in PIDN, for its tractable subclass 3-consistency is not enough to guarantee global consistency. The following counterexample justifies the claim.

Example 5: *Fred, John, Bob and Mary work for the company. Today, Mary started before Fred. Bob left his home after Fred left. Mary reached after Bob. Mary takes less time than John to reach office. John started after Fred and reached before Bob.*

There are four temporal objects O_B, O_F, O_J and O_M. The binary relations are the following

$$C_{BM} = (?,(Y_B,\infty),?), \quad C_{FM} = ([0,X_F),?,?)$$
$$C_{JM} = (?,?,[0,Y_J - X_J)), \quad C_{FB} = ((X_F,\infty),?,?)$$
$$C_{FJ} = ((X_F,\infty),?,?), \quad C_{BJ} = (?,[0,Y_B),?)$$

One can see that there exist consistent instantiations of O_M for the consistent instantiations of any pair of *OB, Op* and O_J. The admissible r e $(C_{BM}, C_{FM}, C_{JM}$ v e pairwise non-empty intersection, but $C_{BM} \cup C_{FM} \cup C_{JM} = \emptyset$. The PIDN is inconsistent.

Determining consistency of PIDN A network can be converted to equivalent path consistent form by applying any path consistency algorithm to the network. Path consistency algorithm such as PC-2 [10]. This algorithm takes $O(n^3)$, when n is the number of nodes in the network. However, each intersection and composition operation takes 6 times more operations than the TCSP. Nevertheless, the path consistency algorithm requires polynomial time.

The 4-consistency requires $O(n^4)$ in terms of the number of temporal objects. Thus consistency checking of Pre-convex PIDN can be done in polynomial time. We now prove the following result.

Proposition 5: *For consistency checking problem of a Qualitative PIDN, the maximal tractable subclass is Pre-convex PIDN.*

Proof *(sketch)* The proof is based on following observations. If the PIDN is not pre-convex then some of its constraints define the admissible regions fall into one of the following two cases;
(i) It is the union of pre-convex regions which are separated by a dimensional region, or, (ii) it is the union

of connected set of pre~coijjtex regions. Another interesting property of the Qualitative PIDN is that the constraints which correspond to case(i) above eventually yield case(ii) in the process of transitive closure with composition and intersection. Thus, if the PIDN is not pre-convex then its transitive closure must contain the constraints of type (ii). Since the sides of these polygonal regions are parallel to three fixed lines, there can be only 12 different non-convex corners possible and out of which 6 of them are the converse of the other six. One can make use of Nebel *k* Burckert's results here to show that for these 6 cases then consistency finding is NP-complete. These 6 cases when translated into equivalent Clause algebra form include constraints of the type

$X_i \leq X_j \vee d_i \geq d_j$ or, $X_i \geq X_j \vee d_i \leq d_j$
or, $X_i \leq X_j \vee Y_i \geq Y_j$ or, $X_i \geq X_j \vee Y_i \leq Y_j$
or, $Y_i \leq Y_j \vee d_i \leq d_j$ or, $Y_i \geq Y_j \vee d_i \geq d_j$ □

It is interesting to note that this result need not be true for Pre-convex PIDN which is not qualitative. The major difference is that the transitive closure need not always contain case (ii). Since, in general PIDN is intractable, any complete algorithm must perform some sort of search over all possible combinations of disjunction-free labeling.

5 Expressive power of PIDN

We show that all the major temporal constraint satisfaction problems are special cases of PIDN.

IA as a special case of PIDN: Interesting to note that Interval algebra is a special case of Qualitative PIDN where there is no constraint on the duration and we have only one type of temporal objects namely, time intervals. Hence, without loss of generality we represent a constraint as a pair of intervals rather than a triplets of intervals. For example, $O_i \; p \; O_j \equiv O_i((Y_i, \infty), [X_j, \infty))O_j$. We call such constraints as Rectangular Constraint as the admissible regions are rectangles. The 13 basic relations can be equivalently written as Rectangular constraints.

The intuitive notion of *convex* and *pir,-convex* relations can be interpreted in this context too and these are similar to the terms introduced in [9]. It is interesting to note that the relations of IA which are equivalent to single rectangular relations are *convex relations.* For example, relation like $\{p, m, o\}$ can be combined to yield a single rectangular constraint and is a convex relation. For the IA relation {p,o} the equivalent PIDN constraints is $([0, X_i), [0, X_i)) \vee ([0, X_i), (X_i, Y_i))$. The disjunction here can be loosely joined along *E* and hence is a pre-convex relation. The preconvex relations correspond to the Nebel and Biirckert's ORD-Hom clause class [9], The following observation justifies that for IA 3-consistency is sufficient for global consistency of the tractable subclasses.

Proposition 6: *For a PIDN with rectangular constraint, 3-consistency implies global consistency.*

Proof *(sketch)* The argument of proof follows the same line and is due to the fact that if the polygonal regions have sides parallel to two fixed lines then their pairwise non-empty intersection ensures that they have a common non-empty intersection. □

TCSP as a special case of PIDN: The TCSP proposed by Dechter *et al* [3] can be viewed as a special case of PIDN. The temporal objects are time points. A constraint is a disjunction of r's where each r is a $(S, ?, ?)$. The *Simple* TCSP correspond to the convex subclass and the $\mathbf{STP^{\neq}s}$ [8], [5], class correspond to pre-convex class. However, PIDN is more expressive than these TCSP's including extensions suggested by Meiri [II].

PDN and PIDN: Navarrete and Marin [12] propose a temporal reasoning system, PDN that takes both points and durations as primitive objects and allow relative and indefinite information. PDN consists of two PA networks separately and are connected by a set of ternary constraints. One PA network represents the set of time points and the other represents duration between pairs of time points. One can visualise both networks as special cases of the PIDN, the first having just the r n 5 and the second one as a PIDN with r = *D.* However, in the proposed formalism it is possible to club the information into one network avoiding the ternary constraints linking these two networks. Since the PIDN also handles quantitative information like APDN which is an extension of the concept of PDN. On the other hand, PIDN can handle disjunctions between point and duration which is not easily possible in PDN. (see Table 1)

Other Related Work: Recently Jonsson and Backstrom [6] have proposed a unifying framework called Disjunctive Linear Relations (DLRs). The DLRs is based on the *linear programming* paradigm. Jonsson and Brackstrom use Karmarker's and Khachiyan's algorithrn to prove that Horn DLRSat (a tractable subclass of DLRs) can be solved in polynomial time.[1] Though these algorithms are polynomial in time, their optimal solutions are not exact as they are numerical methods. They generate solutions which have the limiting point that is optimal. Therefore, the numerical stability of these methods may be questionable while applying them to the satisfiability problem. On the other hand, Simplex method may be suitable in this context, but may not be polynomial. On the whole, linear programming approach, though expressive, does not take advantage of the underlying structures (e.g., domain constraints) of temporal constraints.

In comparison to DLRs, the PIDN is committed to the standard approach based on a temporal constraint network having binary constraints among time objects. Hence, PIDN has computational advantage over DLRs. For example, if there are 100 time intervals, DLR forms the constraints of 200 variables. At any given iteration it solves a LP of 200 variables. However, if we handle Binary constraint network of 100 nodes then the path consistency algorithm will carry out the composition and intersection of 4 nodes at time in one iteration. Hence PIDN handles only 12 variables. It does this for a poly-

[1] However, it is not clear whether it is the maximal tractable subclass of DLRs.

nomial time in 100. But in any iteration it handle* 12 variables. DLR also does polynomial number of iteration in 100 and in each iteration it handles 200 variables.

0 Summary and Future Scope

In the present work we propose an unified formalism which treats points, intervals and durations as temporal objects and handles quantitative and quantitative constraints (Table 1). In general, checking consistency of PIDN is NP-Hard and the maximal tractable class for qualitative case is identified. Unlike the other formalisms, for the tractable classes, the level of local consistency for guaranteeing global consistency is 4. It is shown that the all other major temporal constraint formalisms are special cases of the proposed network.

The aim of the present study is to establish the fact that a specific geometrical interpretation of the time domain helps in unifying the earlier diverse concepts. There are many unresolved questions remain to be answered. We outline some of the* possible future work: (i) To devise a specialized algorithm to compute intersection, composition for qualitative or for the general network; (ii) Determining the maximal tractable class for the general PIDN. It may be noted that the the fact that non-convex disconnected admissible regions yield connected regions by the process of composition do not hold true for the quantitative constraints. It may be possible to identify the class for which this is true; (iii) Developing an efficient search algorithm for the general network, and comparing empirical performance of PIDN and DLRs

	IA	PA	TCSP	PDN	PIDN
Qual. info.	√	√	√	x	√
Metric info.	x	x	√	√	√
Pt. info.	x	√	√	√	√
Interval info.	√	x	√	x	√
Duration info.	x	x	x	√	√
Disjunction of duration and int.	x	x	x	x	√
Consistency	NP	P	NP	NP	NP
Tractable class	ORD-Horn	All	TSP≠	Simple	Pre-Convex
Local consistency	3	3	3	3	4

Acknowledgement

The authors thank Peter van Beek for his valuable comments on an early draft of this paper. Also, the first author gratefully acknowledges CSIR, India (Research Project No.25/96/81/EMR-II) and Griffith University for supporting the research.

References

[1] J. Allen. Maintaining knowledge about temporal intervals. *Readings in Knowledge Representation,* pages 510-521, 1983.

[2] F. Barber. A metric time-point and duration-based temporal model. *SIGART Bui let me,* 4(3):30~49, 1993.

[3] R Dechter, I. Meiri, and JL Pearl. Temporal constraint networks. *Artificial Intelligence^*49:61-95, 1991.

[4] H. Edelsbrunner. *Algorithms in Combinatorial Geometry.* Springer, Berlin, 1987.

[5] A. Gerevini and M, Cristani. On finding a solution in temporal constraint satisfaction problem. *Proc. of IJCAI'97,* pages 1460-1465, 1997.

[6] P. Jonsson and C. Backstrom. A unifying approach to temporal constraint reasoning. *Artificial Intelligence,* 102(1):143~155, 1998.

[7] H. Kautz and P. Ladkin. Integrating metric and qualitative temporal reasoning. *Proc. of AAAI-91,* pages 241-246, 1991.

[8] M. Koubarakis. Dense time and temporal constraints with =. *Proceedings of the KR*92^* pages 24-35, 1992.

[9] G. Ligozat. "corner" relations in alien's algebra. *Constraints,* 3(2 and 3): 165-177, 1998.

[10] A. K. Mackworth. Consistency in network relations. *Artificial Intelligence,* 8:99-118, 1977.

[11] I. Meiri. Combining qualitative and quantitative constraints in temporal reasoning. *Artificial Intelligence,* 87:343-385, 1996.

[12] I. Navarrette and R. Marin. Qualitative temporal reasoning with points and durations. *Proc. IJCAV97,* pages 1454-1459, 1997.

[13] B. Nebel and H. Biirckert. Reasoning about temporal relations: A maximal tractable subclass of alien's interval algebra. *Journal of ACM,* 42(1):43-66, 1995.

[14] Arun K Pujari. Neighbourhood logic and interval algebra. *Technical Report No. 116, United Nations University, International Institute of Software Technology, Macau,* 1997.

[15] E. Schwalb and L. Vila. Temporal constraints: A survey. *Constraints,* 3(2 and 3):129-150, 1998.

[16] P. van Beek. Reasoning about qualitative temporal information. *The 8th AAAI-90,* pages 728-734, 1990.

[17] M. Vilain and H. Kautz. Constraint propagation algorithms for temporal reasoning. *Proceedings of AAAI-86,* pages 377-382, 1986.

[18] M. Vilain, H. Kautz, and P. van Beek. Constraint propagation algorithms for temporal reasoning: A revised report. *Readings on Qualitative Reasoning about Physical Systems,* pages 373-381, 1989.

[19] R. Wetprasit and A. Sattar. Temporal reasoning with qualitative and quantitative information about points and durations. *Proc. AAAI'98,* pages 656-663, 1998.