Acceptability Semantics for Weighted Argumentation Frameworks

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Abstract

The paper studies semantics that evaluate arguments in argumentation graphs, where each argument has a basic strength, and may be attacked by other arguments. It starts by defining a set of principles, each of which is a property that a semantics could satisfy. It provides the first formal analysis and comparison of existing semantics. Finally, it defines three novel semantics that satisfy more principles than existing ones.

1 Introduction

In an argumentation setting, an argument may be attacked by other arguments. It may also have a *basic strength*, which may represent various issues like its probability [Hunter, 2013; Thimm, 2012; Li *et al.*, 2011], certainty degree of its premises [Benferhat *et al.*, 1993], votes provided by users [Leite and Martins, 2011], importance degree of a value it promotes [Bench-Capon, 2003], trustworthiness of its source [da Costa Pereira *et al.*, 2011]. In all these disparate cases, the basic strength may be expressed by a numerical value, leading to *weighted argumentation graphs*. The question of evaluating the *overall strength* or *overall acceptability* of an argument in such graphs raises naturally.

Two families of semantics were proposed in the literature for answering this question. The first family, studied in [Amgoud and Cayrol, 2002; Bench-Capon, 2003; Modgil, 2009; Li *et al.*, 2011; Thimm, 2012; Hunter, 2013], extends the semantics proposed by Dung [1995], which compute extensions of arguments. The second family, investigated in [da Costa Pereira *et al.*, 2011; Leite and Martins, 2011; Gabbay and Rodrigues, 2015; Baroni *et al.*, 2015; Rago *et al.*, 2016], computes the overall strengths of arguments.

The two families of semantics have never been formally compared and their foundational principles not investigated. This is mainly due to the absence of formal properties that serve as guidelines for such analysis. In case of flat (i.e. non-weighted) graphs, there are few attempts in defining such properties. Baroni and Giacomin [2007] defined some principles that extension semantics à la Dung would satisfy. Those principles are mainly properties of extensions and not of overall acceptability of arguments. Amgoud and Ben-Naim [2013] proposed another set of principles for ranking semantics. The set was used in [Bonzon *et al.*, 2016] for comparing some semantics devoted to flat graphs. It was also recently extended, and refined in [Amgoud and Ben-Naim, 2016a] by decomposing some principles into more elementary ones.

The first contribution of this paper consists of extending the recent set of principles to account for basic strengths of arguments. We also introduce four novel principles, one of them deals with basic strengths, and the others describe three strategies that a semantics may use when it faces a conflict between the quality of attackers and their quantity. The three strategies are: i) privileging quality (QP), ii) privileging cardinality (CP), or iii) simply allowing compensation.

The second contribution of the paper consists of providing the first formal analysis and thorough comparison of most of the above cited semantics. These shed light on underpinnings, strengths and weaknesses of each semantics, as well as similarities and differences between pairs of semantics. The results also reveal three limitations in the literature. First, there is no semantics satisfying (CP), which is unfortunate since (CP) is a viable choice in (multiple criteria) decision making [Dubois *et al.*, 2008]. Second, there is only one semantics satisfying (QP), however, it may return counter-intuitive results (see Section 5). Third, several semantics satisfy compensation, however, none of them satisfies all the principles that are compatible with the compensation principle.

The third contribution consists of filling the previous gaps by introducing three novel semantics, one for each strategic option. The new semantics satisfy *all* the principles that are compatible with their strategic principle. They thus enjoy more desirable properties than existing semantics.

2 Basic Concepts

A *weighted argumentation graph* is a set of arguments and an attack relation between them. Each argument has a weight in the interval $[0, 1]$ representing its basic strength (the smaller the weight, the weaker the argument).

Definition 1 (WAG) *A weighted argumentation graph (WAG) is an ordered tuple* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*, where* $\mathcal{A}$ *is a non-empty finite set of arguments,* w *is a function from* $\mathcal{A}$ *to* $[0, 1]$*, and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*.*

Intuitively, $w(a)$ is the basic strength of argument a, and $(a, b) \in \mathcal{R}$ (or $a\mathcal{R}b$) means argument a *attacks argument* b.

An isomorphism between WAGs is defined as follows.

Definition 2 (Isomorphism) *Let* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle$ *be two WAGs. An* isomorphism *from* $\mathbf{G}$ *to* $\mathbf{G}'$ *is a bijective function* f *from* $\mathcal{A}$ *to* $\mathcal{A}'$ *such that: i)* $\forall\, a \in \mathcal{A}$, $w(a) = w'(f(a))$*, ii)* $\forall\, a, b \in \mathcal{A}$, $a\mathcal{R}b$ *iff* $f(a)\mathcal{R}'f(b)$.

An acceptability semantics is a function assigning a value, called *acceptability degree*, to every argument in a weighted argumentation graph. This value represents the *overall strength* of an argument, and is issued from the *aggregation* of the basic strength of the argument and the overall strengths of its attackers. The greater this value, the more acceptable the argument. Unlike extension semantics where arguments are either accepted or rejected, we consider *graded* semantics, which may assign various acceptability degrees to arguments. Thus, a rich scale of acceptability degrees is needed. Throughout the paper, we consider the scale $[0, 1]$.

Definition 3 (Semantics) *A* semantics *is a function* $\mathbf{S}$ *transforming any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *into a vector* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}$ *in* $[0, 1]^n$*, where* $n = |\mathcal{A}|$*. For* $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a)$ *is called* acceptability degree *of* a.

We present next the list of all notations used in the paper.

Notations: Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG and $a \in \mathcal{A}$. $\mathtt{Att}_{\mathbf{G}}(a)$ denotes the set of all *attackers* of a in $\mathbf{G}$ (i.e. $\mathtt{Att}_{\mathbf{G}}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}a\}$). For $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ and $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, $\mathbf{G} \oplus \mathbf{G}'$ is the WAG $\langle \mathcal{A} \cup \mathcal{A}', w'', \mathcal{R} \cup \mathcal{R}' \rangle$ where for any $x \in \mathcal{A}$ (resp. $x \in \mathcal{A}'$), $w''(x) = w(x)$ (resp. $w''(x) = w'(x)$).

3 Foundations of Semantics

This section discusses principles, which are important for i) a better understanding of semantics, ii) the definition of reasonable semantics, iii) comparing semantics, iv) choosing suitable semantics for applications.

3.1 Principles

We propose 15 principles, which describe the role and impact of attacks and basic strengths in the evaluation of arguments, and how these two elements are aggregated. Some of the principles extend those proposed by Amgoud and Ben-Naim [2016a] for flat (i.e. non-weighted) graphs. The first principle, called *anonymity*, can be found in almost all axiomatic studies including those in cooperative games [Shapley, 1953]. In the argumentation literature, anonymity is called abstraction in [Amgoud and Ben-Naim, 2013] and language independence in [Baroni and Giacomin, 2007].

Principle 1 (Anonymity) *A semantics* $\mathbf{S}$ *satisfies* anonymity *iff, for any two WAGs* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle$*, for any isomorphism* f *from* $\mathbf{G}$ *to* $\mathbf{G}'$*, the following property holds:* $\forall\, a \in \mathcal{A}, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(f(a))$.

The second principle, called *independence*, states that the acceptability degree of an argument should be independent of any argument that is not connected to it.

Principle 2 (Independence) *A semantics* $\mathbf{S}$ *satisfies* independence *iff, for any two WAGs* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *and* $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle$ *such that* $\mathcal{A} \cap \mathcal{A}' = \emptyset$*, the following holds:* $\forall\, a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G} \oplus \mathbf{G}'}(a)$.

The next principle states that the acceptability degree of an argument a in a graph can depend on argument b only if there is a *path* from b to a, i.e., a finite non-empty sequence $\langle x_1, \ldots, x_n \rangle$ s.t. $x_1 = b$, $x_n = a$ and $\forall i < n$, $x_i \mathcal{R} x_{i+1}$. This principle is more general than the Circumscription axiom presented in [Amgoud and Ben-Naim, 2016a] even when the arguments have the same basic strengths.

Principle 3 (Directionality) *A semantics* $\mathbf{S}$ *satisfies* directionality *iff, for any two WAGs* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\mathbf{G}' = \langle \mathcal{A}, w, \mathcal{R}' \rangle$ *s.t.* $\mathcal{R}' = \mathcal{R} \cup \{(a, b)\}$*, it holds that:* $\forall x \in \mathcal{A}$*, if there is no path from* b *to* x*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(x)$.

The next principle, called *neutrality*, states that an argument, whose acceptability degree is 0, is lifeless and thus has no impact on the arguments it attacks.

Principle 4 (Neutrality) *A semantics* $\mathbf{S}$ *satisfies* neutrality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$*, if i)* $w(a) = w(b)$*, and ii)* $\mathtt{Att}_{\mathbf{G}}(b) = \mathtt{Att}_{\mathbf{G}}(a) \cup \{x\}$ *with* $x \in \mathcal{A} \setminus \mathtt{Att}_{\mathbf{G}}(a)$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = 0$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

The condition $w(a) = w(b)$ ensures that the attacks from $\mathtt{Att}_{\mathbf{G}}(a)$ have the same effect on both arguments a and b.

Equivalence principle ensures that the overall strength of an argument depends only on the basic strength of the argument and the overall strengths of its (direct) attackers.

Principle 5 (Equivalence) *A semantics* $\mathbf{S}$ *satisfies* equivalence *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$*, if i)* $w(a) = w(b)$*, and ii) there exists a bijective function* f *from* $\mathtt{Att}_{\mathbf{G}}(a)$ *to* $\mathtt{Att}_{\mathbf{G}}(b)$ *s.t.* $\forall x \in \mathtt{Att}_{\mathbf{G}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(f(x))$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Maximality principle states that an unattacked argument receives an acceptability degree equal to its basic strength.

Principle 6 (Maximality) *A semantics* $\mathbf{S}$ *satisfies* maximality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a \in \mathcal{A}$*, if* $\mathtt{Att}_{\mathbf{G}}(a) = \emptyset$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = w(a)$.

The role of attacks is *weakening* their targets. Indeed, when an argument receives an attack, its overall strength decreases whenever the attacker is "alive".

Principle 7 (Weakening) *A semantics* $\mathbf{S}$ *satisfies* weakening *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a \in \mathcal{A}$*, if i)* $w(a) > 0$*, and ii)* $\exists b \in \mathtt{Att}_{\mathbf{G}}(a)$ *s.t.* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) < w(a)$.

Weakening leads to strength loss as soon as an argument is attacked by at least one alive attacker. *Counting* principle states that *each* alive attacker has an impact on the overall strength of the argument. Thus, the more numerous the alive attackers of an argument, the weaker the argument. Even reasonable semantics may violate this principle, namely those that look for particular attackers (e.g., the strongest ones) as we will see later. It is however, very useful for formal comparisons of different semantics.

Principle 8 (Counting) *A semantics* $\mathbf{S}$ *satisfies* counting *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$*, if i)* $w(a) = w(b)$*, ii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$*, and iii)* $\mathtt{Att}_{\mathbf{G}}(b) = \mathtt{Att}_{\mathbf{G}}(a) \cup \{y\}$ *with* $y \in \mathcal{A} \setminus \mathtt{Att}_{\mathbf{G}}(a)$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > 0$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Weakening soundness principle goes further than weakening by stating that attacks are the *only* source of strength loss.

Principle 9 (Weakening Soundness) *A semantics* $\mathbf{S}$ *satisfies* weakening soundness *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a \in \mathcal{A}$ *such that* $w(a) > 0$, *if* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) < w(a)$, *then* $\exists b \in \mathtt{Att}_{\mathbf{G}}(a)$ *such that* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$.

The next two principles are about the *intensity* of an attack. Intensity depends on the strength of the source of the attack as well as that of the target. *Reinforcement* principle states that the stronger the source of an attack, the greater its intensity.

Principle 10 (Reinforcement) *A semantics* $\mathbf{S}$ *satisfies* reinforcement *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$, *if i)* $w(a) = w(b)$, *ii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$ *or* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$, *iii)* $\mathtt{Att}_{\mathbf{G}}(a) \setminus \mathtt{Att}_{\mathbf{G}}(b) = \{x\}$, *iv)* $\mathtt{Att}_{\mathbf{G}}(b) \setminus \mathtt{Att}_{\mathbf{G}}(a) = \{y\}$, *and v)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Example 1 *Consider the WAG* $\mathbf{G}_1$ *made of four arguments* a, b, c, d *whose basic strengths are respectively 0.50, 0.25, 0.90, 0.25, and* a *attacks* b *and* c *attacks* d. *Let* $\mathbf{S}$ *satisfy Maximality, so* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_1}(a) = 0.5$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_1}(c) = 0.9$. *Reinforcement ensures* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_1}(b) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_1}(d)$.

Resilience principle states that an attack cannot completely kill an argument. To motivate it, consider debates deprived of formal rules, like those on societal issues (e.g. capital punishment, abortion). In such debates, an argument cannot use a formal rule to kill another argument, which is perhaps why there is still no consensus on the above issues. This principle is thus optional, and its use depends on the application at hand. Finally, it is worth recalling that in case of flat argumentation graphs, it was shown by Amgoud and Ben-Naim [2016a] that Resilience is one of the main principles, which distinguishes Dung's semantics from those proposed in [Amgoud and Ben-Naim, 2013; Besnard and Hunter, 2001].

Principle 11 (Resilience) *A semantics* $\mathbf{S}$ *satisfies* resilience *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a \in \mathcal{A}$, *if* $w(a) > 0$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$.

Remark: If the basic strength of any argument in a given graph is equal to 1, then the previous principles coincide with those proposed in [Amgoud and Ben-Naim, 2016a] except Directionality, which is more general than Circumscription.

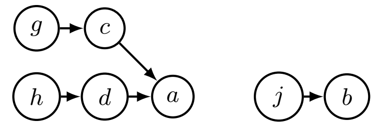
Proportionality states that the stronger the target of an attack, the weaker its intensity.

Principle 12 (Proportionality) *A semantics* $\mathbf{S}$ *satisfies* proportionality *iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$ *s.t. i)* $\mathtt{Att}_{\mathbf{G}}(a) = \mathtt{Att}_{\mathbf{G}}(b)$, *ii)* $w(a) > w(b)$, *and iii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$ *or* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Example 2 *Consider the WAG* $\mathbf{G}_2$ *made of the three arguments* a, b, c *whose basic strengths are respectively 0.30, 0.70, 0.20, and* a *attacks both* b *and* c. *From Proportionality, it follows that* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_2}(b) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}_2}(c)$.

The three last principles concern possible choices offered to a semantics when it faces a conflict between the *quality* and the *number* of attackers as shown by the following example.

Example 3 *Consider the WAG* $\mathbf{G}_3$ *depicted below, and whose arguments have all a basic strength equal to* 1.



The argument a *has two weak attackers (each attacker is attacked). The argument* b *has only one attacker, but a strong one. The question is which of* a *and* b *is more acceptable?*

The answer to the previous question depends on which of quantity and quality is more important. *Cardinality precedence* principle states that a great number of attackers has more effect on an argument than just few. For an example of application, consider a debate on the best YouTuber. An argument against YouTuber A is typically that viewer X follows B, but not A. That argument is stronger if X is a celebrity (or a strong YouTuber itself), but ignoring this fact (in a first step) and just counting the number of such arguments is a good strategy, because big numbers of followers is particularly appreciated in social networks.

Principle 13 (Cardinality Precedence) *A semantics* $\mathbf{S}$ *satisfies* cardinality precedence *(CP) iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$, *if i)* $w(a) = w(b)$, *ii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b) > 0$, *and iii)* $|\{x \in \mathtt{Att}_{\mathbf{G}}(a) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) > 0\}| > |\{y \in \mathtt{Att}_{\mathbf{G}}(b) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > 0\}|$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Quality precedence principle gives more importance to the quality. It is important, for instance, in debates requiring expertise. If a Fields medal says P, whilst three students say $\neg P$, we probably believe P. It is worth mentioning that this principle is similar to the *Pessimistic* rule in decision under uncertainty [Dubois and Prade, 1995].

Principle 14 (Quality Precedence) *A semantics* $\mathbf{S}$ *satisfies* quality precedence *(QP) iff, for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\forall a, b \in \mathcal{A}$, *if i)* $w(a) = w(b)$, *ii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$, *and iii)* $\exists y \in \mathtt{Att}_{\mathbf{G}}(b)$ *s.t.* $\forall x \in \mathtt{Att}_{\mathbf{G}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$ *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Compensation says that several weak attacks may compensate the quality. In the graph $\mathbf{G}_3$, the two attackers of a compensate the strong attacker of b, thus a is as acceptable as b.

Principle 15 (Compensation) *A semantics* $\mathbf{F}$ *satisfies* compensation *iff, there exists a WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *such that for two arguments* $a, b \in \mathcal{A}$, *i)* $w(a) = w(b)$, *ii)* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$, *iii)* $|\{x \in \mathtt{Att}_{\mathbf{G}}(a) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) > 0\}| > |\{y \in \mathtt{Att}_{\mathbf{G}}(b) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > 0\}|$, *iv)* $\exists y \in \mathtt{Att}_{\mathbf{G}}(b)$ *s.t.* $\forall x \in \mathtt{Att}_{\mathbf{G}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Two axioms, similar to CP and QP, were proposed for the first time by Amgoud and Ben-Naim [2013] for flat graphs and ranking semantics. Recall that ranking semantics do not compute acceptability degrees, but rather define a preference relation between arguments. Thus, the equivalent axiom of QP uses that preference relation while the one corresponding to CP counts simply the number of attackers even the lifeless ones. Our principles are finer since they do not consider lifeless attackers. The three previous principles (CP, QP, Compensation) were also investigated by the same authors in [2016b] for weighted argumentation graphs where arguments may only *support* each other. Our principles are much simpler since they capture with less constraints the dilemma between quality and quantity of attackers.

3.2 Properties

Some of the principles are incompatible, that is they cannot be satisfied all together by a semantics. This is particularly the case of the last three principles. Quality Precedence is also incompatible with another subset of principles.

Proposition 1 *The three following properties hold.*

1. *CP, QP and Compensation are pairwise incompatible.*
2. *Independence, Directionality, Equivalence, Resilience, Reinforcement, Maximality and QP are incompatible.*
3. *CP (respectively Compensation) is compatible with all principles 1–12.*

The following result shows some dependencies between the principles. Namely, Weakening, Weakening Soundness, and Counting follow from other principles.

Proposition 2 *Let* $\mathbf{S}$ *be a semantics which satisfies Directionality, Independence, Maximality, Neutrality. Then:*

- $\mathbf{S}$ *satisfies Weakening Soundness.*
- *If* $\mathbf{S}$ *satisfies Reinforcement, then it also satisfies Counting and Weakening.*

Arguments that are attacked only by lifeless attackers keep their basic strength in case the semantics satisfies Independence, Directionality, Neutrality and Maximality.

Proposition 3 *If a semantics* $\mathbf{S}$ *satisfies Independence, Directionality, Neutrality, and Maximality, then for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*, for any* $a \in \mathcal{A}$ *such that* $\mathtt{Att}_{\mathbf{G}}(a) \neq \emptyset$*, if for any* $x \in \mathtt{Att}_{\mathbf{G}}(a)$*,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = 0$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = w(a)$*.*

A semantics satisfying Independence, Directionality, Proportionality, Neutrality, Weakening and Maximality, assigns to each argument a degree between 0 and its basic strength.

Theorem 1 *If a semantics* $\mathbf{S}$ *satisfies Independence, Directionality, Neutrality, Proportionality, Weakening and Maximality, then for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*, for any argument* $a \in \mathcal{A}$*, it holds that* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) \in [0, w(a)]$*.*

Another property which follows from a subset of principles is Counter-Transitivity. It was introduced in [Amgoud and Ben-Naim, 2013] for ranking semantics in case of non-weighted graphs. It states that: "if the attackers of an argument b are at least as numerous and strong as those of an argument a, then a is at least as strong as b".

Theorem 2 *If a semantics* $\mathbf{S}$ *satisfies Independence, Directionality, Equivalence, Reinforcement, Maximality, and Neutrality, then for any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*,* $\forall a, b \in \mathcal{A}$*, if* $w(a) = w(b)$*, and there exists an injective function* f *from* $\mathtt{Att}_{\mathbf{G}}(a)$ *to* $\mathtt{Att}_{\mathbf{G}}(b)$ *such that* $\forall x \in \mathtt{Att}_{\mathbf{G}}(a)$*,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(f(x))$*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$*.*

4 Three Novel Semantics

This section introduces three novel semantics: one for each of the three last principles.

4.1 Weighted Max-Based Semantics

The first semantics satisfies quality precedence, thus it favors the quality of attackers over their cardinality. It is based on a scoring function which follows a multiple steps process. At each step, the function assigns a score to each argument. In the initial step, the score of an argument is its basic strength. Then, in each step, the score is recomputed on the basis of the basic strength as well as the score of the *strongest* attacker of the argument at the previous step.

Definition 4 ($\mathtt{f_m}$) *Let* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *be a WAG. We define the* weighted max-based function $\mathtt{f_m}$ *from* $\mathcal{A}$ *to* $[0, +\infty)$ *as follows: for any argument* $a \in \mathcal{A}$*, for* $i \in \{0, 1, 2, \ldots\}$*,*

$$\mathtt{f}^{\mathtt{i}}_{\mathtt{m}}(a) = \begin{cases} w(a) & \text{if } i = 0 \\ \frac{w(a)}{1 + \max_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{f}^{\mathtt{i-1}}_{\mathtt{m}}(b)} & \text{otherwise} \end{cases}$$

By convention, $\max_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{f}^{\mathtt{i}}_{\mathtt{m}}(b) = 0$ *if* $\mathtt{Att}_{\mathbf{G}}(a) = \emptyset$*.*

The value $\mathtt{f}^{\mathtt{i}}_{\mathtt{m}}(a)$ is the score of the argument a at step i. This value may change at each step, however, it converges to a unique value as shown in the next theorem.

Theorem 3 $\mathtt{f}^{\mathtt{i}}_{\mathtt{m}}$ *converges as* i *approaches infinity.*

Weighted max-based semantics is based on the previous scoring function. The acceptability degree of each argument is the *limit* reached using the scoring function $\mathtt{f_m}$.

Definition 5 ($\mathtt{Mbs}$) *The* weighted max-based semantics *is a function* $\mathtt{Mbs}$ *transforming any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *into a vector* $\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}$ *in* $[0, 1]^n$*, with* $n = |\mathcal{A}|$ *and for any* $a \in \mathcal{A}$*,* $\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}(a) = \lim_{i \to \infty} \mathtt{f}^{\mathtt{i}}_{\mathtt{m}}(a)$*.*

Example 3 (Cont) $\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}_3}(a) = 0.66$ and $\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}_3}(b) = 0.5$. $\mathtt{Mbs}$ privileges quality to quantity.

We show next that the limit scores of arguments satisfy a nice property, namely the equation of Definition 4.

Theorem 4 *For any WAG* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*, for any* $a \in \mathcal{A}$*,*

$$\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}(a) = \frac{w(a)}{1 + \max_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}(b)}. \tag{1}$$

The next result states that equation (1) is not just a property of weighted max-based semantics, but also its characterization. Indeed, it is the only function satisfying the equation. Due to this characterization, equation (1) represents an alternative definition of weighted max-based semantics.

Theorem 5 *Let* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ *be a finite WAG, and let* $D : \mathcal{A} \to [0, +\infty)$*. If* $D(a) = \frac{w(a)}{1 + \max_{b \in \mathtt{Att}(a)} D(b)}$*, for all* $a \in \mathcal{A}$*, then* $D \equiv \mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}$*.*

Weighted max-based semantics satisfies Quality Precedence as well as *all* the principles which are compatible with it. It violates, however, Counting since by definition, this semantics focuses only on the strongest attacker of an argument, and neglects the remaining attackers.

Theorem 6 *Weighted max-based semantics violates Cardinality Precedence, Compensation, Counting and Reinforcement. It satisfies all the remaining principles.*

We show next that an attacked argument cannot lose more than half of its basic strength with this semantics. This is reasonable since only one attacker has effect on the argument.

Theorem 7 *For any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, for any $a \in \mathcal{A}$,* $\mathtt{Deg}^{\mathtt{Mbs}}_{\mathbf{G}}(a) \in [\frac{w(a)}{2}, w(a)]$.

4.2 Weighted Card-Based Semantics

The second semantics, called *weighted card-based*, favors the number of attackers over their quality. It considers only arguments that have a basic strength greater than 0, called *founded*. This restriction is due to the fact that unfounded arguments are lifeless and their attacks are ineffective.

Definition 6 (Founded Argument) *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG and $a \in \mathcal{A}$. The argument a is* founded *iff $w(a) > 0$. It is* unfounded *otherwise. Let $\mathtt{AttF}_{\mathbf{G}}(a)$ denote the set of founded attackers of a.*

This semantics is based on a recursive function which assigns a score to each argument on the basis of its basic strength, the number of its founded attackers and their scores.

Definition 7 ($\mathtt{f_c}$) *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG. We define the* weighted card-based function $\mathtt{f_c}$ *from $\mathcal{A}$ to $[0, +\infty)$ as follows: for any argument $a \in \mathcal{A}$, for $i \in \{0, 1, 2, \ldots\}$,*

$$\mathtt{f}^{\mathtt{i}}_{\mathtt{c}}(a) = \begin{cases} w(a) & \textit{if } i = 0 \\ \frac{w(a)}{1 + |\mathtt{AttF}_{\mathbf{G}}(a)| + \frac{\sum_{b \in \mathtt{AttF}_{\mathbf{G}}(a)} \mathtt{f}^{\mathtt{i}-1}_{\mathtt{c}}(b)}{|\mathtt{AttF}_{\mathbf{G}}(a)|}} & \textit{otherwise} \end{cases}$$

By convention, $\frac{\sum_{b \in \mathtt{AttF}_{\mathbf{G}}(a)} \mathtt{f}^{\mathtt{i}-1}_{\mathtt{c}}(b)}{|\mathtt{AttF}_{\mathbf{G}}(a)|} = 0$ *if $\mathtt{AttF}_{\mathbf{G}}(a) = \emptyset$.*

The value $\mathtt{f}^{\mathtt{i}}_{\mathtt{c}}(a)$ is the score of the argument a at step i. This value converges to a unique value as i becomes high.

Theorem 8 $\mathtt{f}^{\mathtt{i}}_{\mathtt{c}}$ *converges as i approaches infinity.*

The acceptability degree of each argument is the limit reached using the scoring function $\mathtt{f_c}$.

Definition 8 ($\mathtt{Cbs}$) *The* weighted card-based semantics *is a function $\mathtt{Cbs}$ transforming any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ into a vector $\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}$ in $[0, 1]^n$, with $n = |\mathcal{A}|$ and for any $a \in \mathcal{A}$,* $\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}(a) = \lim_{i \to \infty} \mathtt{f}^{\mathtt{i}}_{\mathtt{c}}(a)$.

Example 3 (Cont) $\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}_4}(a) = 0.3$ while $\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}_4}(b) = 0.33$. $\mathtt{Cbs}$ clearly privileges quantity to quality.

We show next that the limit scores of arguments satisfy the equation of Definition 7.

Theorem 9 *For any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, for any $a \in \mathcal{A}$,*

$$\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}(a) = \frac{w(a)}{1 + |\mathtt{AttF}_{\mathbf{G}}(a)| + \frac{\sum_{b \in \mathtt{AttF}_{\mathbf{G}}(a)} \mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}(b)}{|\mathtt{AttF}_{\mathbf{G}}(a)|}}. \quad (2)$$

We also show that equation (2) represents an alternative definition of weighted card-base semantics, i.e., $\mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}$ is the only function which satisfies the equation.

Theorem 10 *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a finite WAG, and let $D : \mathcal{A} \to [0, +\infty)$. If $D(a) = \frac{w(a)}{1 + |\mathtt{AttF}_{\mathbf{G}}(a)| + \frac{\sum_{b \in \mathtt{AttF}_{\mathbf{G}}(a)} D(b)}{|\mathtt{AttF}_{\mathbf{G}}(a)|}}$, for all $a \in \mathcal{A}$, then $D \equiv \mathtt{Deg}^{\mathtt{Cbs}}_{\mathbf{G}}$.*

Weighted card-based semantics satisfies CP as well as all the principles that are compatible with it.

Theorem 11 *Weighted card-based semantics satisfies all the principles except Quality Precedence and Compensation.*

Corollary 1 *For any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, for any $a \in \mathcal{A}$,* $\mathtt{Deg}^{\mathtt{Rbs}}_{\mathbf{G}}(a) \in (0, w(a)]$.

4.3 Weighted h-Categorizer Semantics

This semantics extends h-categorizer, initially proposed by Besnard and Hunter [2001] for *non-weighted* and *acyclic* graphs. We extend its definition to account for varying degrees of basic strengths, and any graph structure.

Definition 9 ($\mathtt{f_h}$) *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG. We define the* weighted h-categorizer function $\mathtt{f_h}$ *from $\mathcal{A}$ to $[0, +\infty)$ as follows: for any argument $a \in \mathcal{A}$, for $i \in \{0, 1, 2, \ldots\}$,*

$$\mathtt{f}^{i}_{h}(a) = \begin{cases} w(a) & \textit{if } i = 0; \\ \frac{w(a)}{1 + \sum_{b_i \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{f}^{i-1}_{h}(b_i)} & \textit{otherwise.} \end{cases}$$

By convention, if $\mathtt{Att}_{\mathbf{G}}(a) = \emptyset$, $\sum_{b_i \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{f}^{i-1}_{h}(b_i) = 0$.

Like the two previous scoring functions, $\mathtt{f}^{i}_{h}$ converges.

Theorem 12 $\mathtt{f}^{i}_{h}$ *converges as i approaches infinity.*

The acceptability degree of each argument in a weighted graph is the limit reached using the function $\mathtt{f_h}$.

Definition 10 ($\mathtt{Hbs}$) *The weighted h-categorizer semantics is a function $\mathtt{Hbs}$ transforming any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ into a vector $\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}$ in $[0, 1]^n$, with $n = |\mathcal{A}|$ and for any $a \in \mathcal{A}$,* $\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(a) = \lim_{i \to +\infty} \mathtt{f}^{i}_{h}(a)$.

Example 3 (Cont) It can be checked that $\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}_4}(a) = \mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}_4}(b) = 0.5$. Thus, compensation is applied.

We now show that the limit scores of arguments satisfy the equation of Definition 9.

Theorem 13 *For any WAG $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, for any $a \in \mathcal{A}$,*

$$\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(a) = \frac{w(a)}{1 + \sum_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(b)}. \quad (3)$$

The following theorem states that equation (3) is a characterization of weighted h-categorizer semantics.

Theorem 14 *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG, and let $D : \mathcal{A} \to [0, +\infty)$. If $D(a) = \frac{w(a)}{1 + \sum_{b \in \mathtt{Att}_{\mathbf{G}}(a)} D(b)}$, for all $a \in \mathcal{A}$, then $D \equiv \mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}$.*

Finally, we show that this semantics satisfies compensation as well as all the principles that are compatible with it.

Theorem 15 *Weighted h-categorizer semantics satisfies all the principles except CP and QP.*

Corollary 2 *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be a WAG. For any $a \in \mathcal{A}$,* $\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(a) \in (0, w(a)]$.

We implemented the three semantics from this section. The degrees can be calculated in 1 or 2 seconds even for complex/large graphs.

5 Formal Analysis of Existing Semantics

This section analyses and compares for the first time existing semantics, thanks to the principles from Section 3. The first family of semantics for weighted argumentation graphs extends Dung's semantics with preferences between arguments [Amgoud and Cayrol, 2002; Bench-Capon, 2003; Modgil, 2009], or with weights on attacks [Cayrol *et al.*,

	Grounded	Stable	Preferred	Complete	IS	(DF-)QuAD	TB	Mbs	Cbs	Hbs
Anonymity	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Independence	$\bullet$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Directionality	$\bullet$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Neutrality	$\bullet$	$\bullet$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Equivalence	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Maximality	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Weakening	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$	$\bullet$	$\bullet$	$\bullet$
Counting	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$	$\times$	$\bullet$	$\bullet$
Weakening soundness	$\bullet$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Proportionality	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$	$\bullet$	$\bullet$	$\bullet$
Reinforcement	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$	$\times$	$\bullet$	$\bullet$
Resilience	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$
Cardinality Precedence	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$
Quality Precedence	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\times$	$\times$	$\bullet$	$\times$	$\times$
Compensation	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\times$	$\bullet$	$\bullet$	$\times$	$\times$	$\bullet$

Table 1: The symbol $\bullet$ (resp. $\times$) stands for the principle is satisfied (resp. violated) by the semantics.

2010; Dunne *et al.*, 2010; 2011]. In case of preferences, the idea is to get rid of attacks whose source is weaker than the target before computing the extensions. In a weighted graph $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, the preference relation privileges arguments with the highest basic strength. Indeed, for $a, b \in \mathcal{A}$, a is preferred to b iff $w(a) \geq w(b)$. A new *flat* graph $\langle \mathcal{A}, \mathcal{R}' \rangle$ is then generated, where $a\mathcal{R}'b$ iff $a\mathcal{R}b$ and b is not strictly preferred to a. Dung's semantics are applied to $\langle \mathcal{A}, \mathcal{R}' \rangle$. Finally, an acceptability degree is assigned to each argument as proposed by Amgoud and Ben-Naim [2016a]: An argument gets acceptability degree 1 iff it belongs to all extensions; value 0.5 if it belongs to some but not all extensions; value 0.3 if it does not belong to and is not attacked by any extension; value 0 if it does not belong to any extension and is attacked by an extension. Table 1 shows that weakening is violated by Dung's semantics in this setting. Indeed, an argument may not lose weight even when attacked by an alive attacker. Maximality is also violated since those semantics manipulate only a preference relation issued from the basic strengths.

Like preferences, weights on attacks may also come from the basic strengths of arguments. Moreover, when all attacks are assigned weight 1, the formalism coincides with Dung's one, which was already investigated in [Amgoud and Ben-Naim, 2016a]. Thus, since Dung's formalism is a particular case of the weighted formalism, every principle violated in Dung's framework is also violated by the corresponding semantics in the weighted framework.

The second family of semantics directly deals with basic strengths, and defines scoring functions assigning a numerical value to each argument in an iterative way. Da Costa et al. [2011] proposed the semantics TB, where basic strengths express degrees of trustworthiness of arguments' sources. TB violates the key principle of *weakening*. Indeed, an argument may not lose weight even if it has a strong attacker.

It was shown recently in [Amgoud *et al.*, 2017] that the social semantics of Leite and Martins [2011] may assign more than one acceptability degree to an argument, which contradicts the uniqueness conjecture.

Baroni et al. [2015] developed QuAD, which evaluates arguments in *bipolar acyclic* graphs. QuAD was extended to DF-QuAD by Rago et al. [2016] but for the same family of graphs. The two semantics coincide in case of empty support relation. To analyze them against our principles, we assume acyclic graphs and empty support relations.

Gabbay and Rodrigues [2015] developed "Iterative Schema" (IS). Basic strengths are used as initial *labels* of arguments. Value 1 corresponds to label `in`, value 0 to `out`, and any other value to `und`. If this labeling is legal, then IS returns a single extension (arguments with value 1). Otherwise, it modifies the values until reaching a legal labeling. IS and Mbs are the only semantics that satisfy QP principle. However, IS violates key principles like Maximality, and thus may return counter-intuitive results. Assume a graph made of a single argument a, which is not attacked and whose basic strength is 0 (meaning that the argument is worthless). IS returns a single extension, $\{a\}$, declaring thus a as accepted.

Table 1 shows that Cbs is the *first* semantics that satisfies CP. However, several semantics satisfy compensation, but Hbs is the only one that satisfies all the principles compatible with compensation, and for any graph structure.

6 Conclusion

For the purpose of evaluating arguments in weighted argumentation graphs, the paper proposed the *first*: i) principles that serve as guidelines for defining and comparing semantics, ii) semantics that satisfies CP, iii) reasonable semantics that satisfies QP, iv) semantics that satisfies compensation as well as all the other compatible principles, and that deals with any graph structure, and v) formal analysis and comparison of all existing semantics, except those dealing with probabilities.

Future work consists of analyzing the semantics developed in probabilistic argumentation settings [Hunter, 2013; Thimm, 2012; Li *et al.*, 2011]. Another natural line of research consists of characterizing families of semantics that satisfy all or subsets of the proposed principles.

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