Tractable (Simple) Contests

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Abstract

Much of the work on multi-agent contests is focused on determining the equilibrium behavior of contestants. This capability is essential for the principal for choosing the optimal parameters for the contest (e.g. prize amount). As it turns out, many contests exhibit not one, but many possible equilibria, hence precluding contest design optimization and contestants’ behavior prediction. In this paper we examine a variation of the classic contest that alleviates this problem by having contestants make the decisions sequentially rather than in parallel. We study this model in the setting of a simple contest, wherein contestants only choose whether or not to participate, while their performance level is exogenously set. We show that by switching to the revised mechanism the principal can not only force her most desired pure-strategies based equilibrium to emerge, but also, at times, end up with an equilibrium offering a greater expected profit. Further, we show that in the modified contest the optimal prize can be effectively computed. The theoretical analysis is complemented by comprehensive experiments with people over Amazon Mechanical Turk. Here, we find that the modified mechanism offers great benefit for the principal, both in terms of an increased over-participation in the contest (compared to theoretical expectations) and increased average profit.

1 Introduction

Contests are important mechanisms for eliciting effort in multi-agent systems [Siegel, 2009; Fu and Lu, 2012]. As such, the analysis of contests, and equilibrium behavior therein, has been a major line of study [Moldovanu and Sela, 2006; DiPalantino and Vojnovic, 2009]. Naturally, the ultimate goal of such analysis is to enable the design of effective contests. Specifically, the equilibrium of a contest is taken to be the presumed (or expected) behavior of the players. With this presumption, determining the equilibrium of a contest allows the contest organizer to predict its outcome (or distribution on outcomes). This, in turn, aids the organizer in designing the contest, by choosing the contest with the best expected outcome. In particular, a major design variable is the prize amount.

Unfortunately, many contests exhibit not one, but many possible equilibria. In such cases, the core equilibrium analysis does not provide a way to determine which of the equilibria will be adopted by the players. So, the organizer cannot optimize the contest design, and in particular has no way to determine the optimal prize, since the effectiveness of a prize will depend on the exact equilibrium chosen by the players.

In this paper we focus on simple contests [Ghosh and Kleinberg, 2016; Levy et al., 2017; Sarne and Lepioshkin, 2017]. In simple contests, players only strategize on whether or not to participate in the contest; a cost is associated with participation, but once a player decides to participate, its performance level is exogenously determined, based on some predefined distribution - over which it has no control. For example, consider sport events such as the Boston Marathon. Their fame attracts world’s top-runners who have to consider whether the cost of participation (e.g., flight and accommodation) on the one hand, and potential opponents and awards offered on the other hand make participation worthy. Naturally, once deciding to participate they will do their best to win. Similar characteristics can be found in various Olympic sports, beauty pageants, photo contests and many more.

The study of simple contests has established that these are commonly characterized by multi-equilibria [Levy et al., 2017]. This, as discussed above, poses a substantial obstacle to the design of such contests. In this paper we show that a modification to the basic contest structure allows maintaining all the pure-strategies Nash equilibria of the basic mechanism, yet reliably single out one of these as the equilibrium that will be played. This is obtained by shifting from a parallel structure to a sequential one. The sequential nature of the revised mechanism dictates the use of a sub-game perfect Bayesian Nash equilibrium, which necessarily leads in our case to a single equilibrium, hence disambiguating the choice of the equilibrium to be used. By properly ordering the entrants, the organizer can not only force her most desired pure-strategies based equilibrium to emerge as the sub-game perfect equilibrium, but also, at times, end up with an equilibrium offering a greater expected profit, that could not have held in the parallel case. Despite the continuous nature of the prize that can be awarded, we show that the optimal prize belongs to a finite set of discrete values. We provide an algo-
rithm to effectively compute these values, and thus determine
the optimal prize.

Finally, we report the results of applying the new mechanism
with people, using a set of experiments carried out in
Amazon Mechanical Turk (AMT). The analysis of the results
reveals an over-participation phenomenon (compared to theo-
retical predictions) at least as great as in the parallel model.
Furthermore, we show that for the settings tested, the ac-
tual performance achieved with the new mechanism is greater
than with the original one.

2 The Model

The contest model used in this paper is standard and can
be found in several other works [Ghosh and Hummel, 2012;
Levy et al., 2017; Same and Lepioshkin, 2017]. It considers
a contest organizer and a set $A = \{A_1, \ldots, A_k\}$ of $k > 1$
heterogeneous potential contestants (denoted “players” on-
wards). All players and the organizer are fully-rational and self-interested. Each player $A_i \in A$ can either participate in
the contest, incurring some cost $c_i > 0$, or opt to avoid particip-
ing in the contest. The actual quality of contributions of
player $A_i$ (i.e., the performance in the contest if participating)
is determined only at the time of participation, based on some
probability distribution function $f_i(x)$ (where $F_i(x)$ is the
 corresponding cumulative distribution function). The contest is “simple” in the sense that contestants do not know the ac-
tual quality of their contributions (i.e., their performance in
the contest) ahead of time, hence only need to strategize on
participation in the contest.

In order to encourage participation in the contest the orga-
nizer offers a prize $M$ to the player ranked first (performance-
wise) in the contest. The model assumes performance and
prizes are additive hence the goal of the organizer is to max-
imize the expected best performance obtained by players in
the contest minus the prize awarded. In case none of the
players choose to participate in the contest, the prize is not
awarded and the performance as perceived by the organizer is
zero. The goal of each player is to maximize its own ex-
pected profit, defined as the expected prize awarded to it mi-
nus the cost incurred if participating in the contest. There-
fore in order to have at least one player participate we require
$M \geq \min(c_i | A_i \in A)$. It is assumed that $f_i(x)$, $c_i$ and $M$
are all common knowledge in the sense that they are known to
all players and to the organizer [Moldovanu and Sela, 2001;
2006; Luo et al., 2014; Levy et al., 2017].

While the above archetypal model has been used in prior
work in its fully parallel form, i.e., all players make their deci-
sions simultaneously, we propose the following modification,
which offers many advantages and yet does not change the
essence of the resulting equilibrium. The change dictates that
the contest organizer first determines an ordering of the play-
ers, and then instead of having all players decide on partic-
ipation simultaneously, each player makes its decision in its
turn after realizing the decision made by all preceding play-
ers. Meaning that the player is still unaware of the perfor-
ance obtained by others, however has information related to
the participation decision of a subset of the others.

3 Analysis

We begin with the equilibrium analysis for the case where
the prize $M$ is given. This will be used in the following sec-

| tion for determining the optimal (organizer’s expected-profit
maximizing) prize.

3.1 Notation

The parallel mechanism is denoted PAR, and the sequential
mechanism is denoted SQ. In the sequential mechanism, the
players may be ordered in different ways; $\text{SQ}(\pi)$ denotes the sequential mechanism wherein $\pi$ is the order of the players.

Strategies

In PAR a pure strategy of a player is simply a binary decision -
whether to participate or not; that is, $S_i \in \{0, 1\}$, where 1
 denotes participation and 0 nonparticipation. In $\text{SQ}(\pi)$, the strategy of player $A_i$ is a function, determining whether to
participate or not given the participation pattern of the previ-
ous players. Formally, strategy $S_i$ for player $A_i$ is a function
$S_i : \{0, 1\}^{\pi^{-1}(i)-1} \rightarrow \{0, 1\}$ (where $\pi^{-1}(i)$ is the location
of $i$ in the sequence $\pi$). As usual, a strategy profile is a tuple
of strategies $S = (S_1, \ldots, S_k) =$, with $S_i$ the strategy of player $A_i$.

Induced Participants

For each strategy profile $S$ (of either PAR or SQ) there is a set
of players $A'$ such that if all players play by $S$, then the
players of $A'$ are exactly those players that end up participat-
ing in the contest. We call this the induced set of participants
of $S$, and denote their set of indexes by $\mathcal{P}(S)$. Given $S$, $\mathcal{P}(S)$
can be readily computed.

3.2 Payoff

Consider a strategy profile $S$. Given that all players follow
$S$, the expected payoff of player $A_i$ given that the prize is $M$,
denoted $B_i(S, M)$, is zero if $i \notin \mathcal{P}(S)$, and otherwise:

$$-c_i + M \int_{y=-\infty}^{\infty} f_i(y) \left( \prod_{A_j \notin \mathcal{P}(S)} F_j(y) \right) dy$$

i.e., player $A_i$ is awarded the prize $M$ whenever its perfor-
mance $y$ is the maximum among all players that participate.
So, for any strategy profile, the payoff of each player can be
effectively computed.

Note that, given that all players indeed follow $S$, the ex-
pected payoff of any player, as well as the organizer, is fully
determined by $\mathcal{P}(S)$. Hence, the following definition:

Definition 1. Strategy profiles $S$ and $S'$ are equivalent if
$\mathcal{P}(S) = \mathcal{P}(S')$.

In this definition, $S$ and $S'$ may be of different mechanisms.

The expected payoff of the organizer can also be effectively
computed as

$$B^{\sigma_0}(S, M) = \int_{y=-\infty}^{\infty} \frac{d(\bar{F}(S, y))}{dy} dy - M$$

where $\bar{F}(S, y) = \prod_{j \in \mathcal{P}(S)} F_j(y)$ is the probability that the
maximum performance obtained in a contest involving players
from a set $\mathcal{P}(S)$ is less than $y$. Notice that if the organi-
zer is interested in maximizing a different function (e.g.,
sum of performance rather than the best performance) then only $\bar{F}(S, y)$ needs to be modified and the remaining of the analysis holds.

### 3.3 Nash Equilibria

We now show that we retain all pure strategy Nash equilibria (NE), when moving to SQ.

**Proposition 1.** For any pure-strategies Nash equilibrium $S$ of PAR and any $\pi$, there exists an equivalent pure strategies Nash equilibrium $S'$ of $SQ(\pi)$. If $S$ is strict then $S'$ is strict.

**Proof.** Define $S'$ as follows: $S'_i(x) = 1$ if $i \in P(S)$ and $S'_i(x) = 0$ otherwise. That is, the players in $P(S)$ always participate (in $S'$), regardless of what the other players do, while those not in $P(S)$ never participate. This exactly mimics the behavior of $A_i$ in $S$ - in the PAR. So, $S'$ is equilibrium, and $P(S') = P(S)$.

The reverse of Proposition 1 does not hold. There may exist NE in SQ that have no equivalent NE in PAR. This is illustrated in a numerical example provided at the end of this section. As for mixed strategy equilibria that may hold for PAR, these cannot have an equivalent equilibrium in SQ contest, simply because in every instance of the game any of the players (other than the first in the sequence) will face a different history.

**Finding the NEs**

We have already shown that, for any given strategy profile $S$, the payoff of each player can be effectively computed. This, in turn, provides an effective way to check if a strategy profile is an NE, by considering the payoffs for all possible deviations. This, in turn, provides an effective means to compute all NEs, by considering all possible strategy profiles.

### 3.4 Sub-Game Perfect Equilibrium

We have seen that all pure Nash equilibria of PAR are also Nash equilibria of SQ (up to equivalence). So, just as in PAR, there may be many Nash equilibria for SQ, no one of which necessarily dominates the others. However, SQ is a dynamic game. As such, the proper, or natural, solution concept for this model is the sub-game perfect Nash equilibrium (SPNE). We note that since $SQ(\pi)$ is a game of complete information, the SPNE can be readily computed using backward induction. Note that except for ties, the sub-game perfect Nash equilibrium is unique. Meaning that if $S$ is a strict sub-game perfect equilibrium of $SQ(\pi)$ then it is the only SPNE of $SQ(\pi)$. The unique SPNE of $SQ(\pi)$ depends on the order $\pi$. We now show that by properly ordering the players, any NE of PAR can be obtained as a SPNE of SQ.

**Proposition 2.** For any $S$, NE for PAR, there exists a $\pi$ and $S'$, such that $S'$ is an SPNE of $SQ(\pi)$, and $S'$ and $S$ are equivalent. If $S$ is strict NE then $S'$ is strict SPNE.

**Proof.** Let $\pi$ be the order in which all players in $P(S)$ appear before those not in $P(S)$. Using backward induction construct a SPNE for $SQ(\pi)$. If at any node in the induction the player, $A_i$, is indifferent between participating and not, then have her participate if $i \notin P(S)$ and not participate if $i \notin P(S)$. By construction, this is a SPNE. By induction on the game tree it is easy to see that $P(S) = P(S')$.

From Proposition 2 we conclude that by switching to the sequential mechanism we can force the best (strict) pure-strategy Nash equilibrium of PAR. Interestingly, at times we can do even better. We now give an example of a SPNE of SQ, which has a better expected benefit (for the organizer) than any NE of PAR. The example is based on three players, $A_1$, $A_2$ and $A_3$, with corresponding participation costs $c_1 = 0.1$, $c_2 = 0.33$ and $c_3 = 0.19$. The performance of $A_1$ is always 56 if $A_2$ is 60. If $A_2$ participates then its performance is 55 with probability 0.4 and 70 otherwise. The prize offered is $M = 0.4$. The only pure-strategies NE that holds with PAR in this setting is to have only $A_3$ participate. In this case, the expected profit of the organizer is 59.6. The mixed-strategies NE in this case is $(0, \frac{2}{5}, \frac{1}{5})$ which results in an expected profit of 59.675. With $SQ(1,2,3)$ the SPNE is to have $A_3$ participate only if $A_2$ does not participate; $A_2$ participate only if $A_1$ does not participate; and $A_1$ not participate (see Figure 1). This results in an expected profit of 63.6, i.e., a substantial improvement compared to PAR.

![Figure 1: SPNE for the numerical example.](attachment:image.png)
Definition 2. A prize value $M$, for $SQ(\pi)$ is critical, if for all $\epsilon > 0$,
\[
SPNE(SQ(\pi), M - \epsilon) \neq SPNE(SQ(\pi), M + \epsilon)
\]
That is, $M$ is a critical value if the SPNE of $SQ(\pi)$ changes at $M$.

We now show that while $M$ can take a continuum of values, the number of critical values is finite.

Proposition 3. For any $SQ(\pi)$, over $n$ players, the number of critical prize values is bounded by $4^n - 1$. These critical values can be effectively computed.

Proof. Consider the game tree $G$ associated with $SQ(\pi)$. Each node $v$ defines a sub-game, with a game tree to which we denote $G(v)$. For brevity we identify a sub-game with its sub-tree. Let $A(G(v))$ be the set of players of $G(v)$, and $A_v$ the player at $v$. The critical values of the sub-game, denoted $C(G(v))$ are defined analogously to those of the entire game. That is, $M$ is a critical value of $G(v)$ if the SPNE of $G(v)$ changes when the prize hits $M$.

We prove by induction (on the structure of the tree), that $|C(G(v))| \leq 4^d - 1$, where $d$ is the height (distance from leaf) of node $v$, and that $C(G(v))$ can be effectively computed.

For $v$ a leaf, $d = 0$, there is no player in the game, and hence no critical value. Consider an internal node $v$, of height $d$. Let $v_0, v_1$ be the children of $v$, where $v_1$ is the sub-game induced when the player at $v$ decides to participate, and $v_0$ induced when she decides not to participate. By the inductive hypothesis, $|C(G(v_0))| \leq 4^{d-1} - 1, b = 0, 1$. Consider $C = C(G(v_0)) \cup C(G(v_1))$. So, $|C| \leq 2 \cdot 4^{d-1} - 1$. The set $C$ divides the real axis into at most $2 \cdot 4^{d-1} - 1$ intervals. Denote this set of intervals by $I$.

Consider $I \in I$, and let $I^c$ be the interior of $I$. By construction, $SPNE(G(v_1), M)$ remains the same as long as $M \in I^c$, and the same for $SPNE(G(v_0), M)$. So, as long as the prize is within $I^c$, in the SPNE of $G(v)$, the behavior of the players of $A(G(v))$ remains the same, except for possibly $A_v$. We now show that the acts of $A_v$ can add at most one critical value within each such interval.

Recall that $v$ is a node in the game tree. Thus, the path from the root to $v$ defines the choices of the players before $A_v$. Let $A^I_{1-}$ be the set of players that decide to participate along this path. Set $A^I_{1+} = P(SPNE(G(v_1), M)) \cdot \text{the induced set of participants of the SPNE at } v_1, \text{when } M \in I^c$. Set $A_1 = A_{1-}^I \cup A^I_{1+}$. The set $A_1$ is the overall set of players that would participate alongside $A_v$, if it chooses to participate at $v$. The expected payoff of $A_1$, in this case is given in equation (1). So, given that the players of $A_1$ participate, player $A_v$ should participate whenever $M \geq M^*$, where
\[
M^* = c_v \left( \int_{y=-\infty}^{\infty} f_v(y) \left( \prod_{A_j \neq A_v, j \in A_1} F_j(y) \right) dy \right)^{-1}
\]
So, if this $M^* \in I^c$, then this adds one extra critical point to $C(G(v))$.

In all, the number of critical values for $G(v)$ is bounded by
\[
|C| + |I| = 2 \cdot 4^{d-1} - 2 + 2 \cdot 4^{d-1} - 1 < 4^d - 1
\]

The process of extracting the critical values for $SQ(\pi)$ is summarized by Algorithm 1. The algorithm traverses the graph $G$ starting with the last player’s node up the tree. For each node $v$ it maintains the set of critical values, stored in $C_v$, and the identity of players that will participate in the sub-game that starts at $v$ for each $M \in C_v$, stored in $A^I_{1+}$.

For each $M \in C_v$ the algorithm calculates the value $M^*$, i.e., the minimal prize making it worthwhile to participate given $A^I_{1+}$. If $M^*$ is below $M$, then participation is preferred throughout the interval from $M$ to its subsequent member in $C_v$. Similarly, if $M^*$ is above the subsequent member in $C_v$, then participation is not preferred throughout the interval from $M$ to the subsequent member. Finally, if it is in between, then $M^*$ is added as a new critical value. In this case participation is not preferred in $(M, M^*)$ and is preferred in the following interval. Finally, all additional critical values (stored in $M_{add}$) are added to $C_v$. The algorithm returns the list of critical values corresponding to the root node, $C^\text{root}$. Based on the output of the algorithm one can calculate $B^\text{org}(S, M)$ for each $M \in C^\text{root}$, taking $P(S) = A^I_{1+}$.
5 Experimental Evaluation

With people, the performance to be obtained with the mechanisms is likely to be different from the theoretical expectations, as people are known to be boundedly rational [Kahneman, 2000; Elmalem, 2015; Levy and Sarne, 2016]. In this section we present the results of a comparative empirical evaluation of SQ and PAR when contestants are people.

5.1 Experimental Framework

In our experimental framework, contestants were first awarded an initial bonus of $c$ cents and had to decide whether they want to participate in the contest and lose that bonus or not participate and keep the bonus. For each contestant choosing to participate the system draws a percentile within the range 0-100, representing the contestant’s performance. The contestant ranked at the highest percentile wins a bonus $M$. The fact that from the contestant’s point of view, the performance measure merely indicates the percentile her contribution falls in rather than its actual value to the organizer en-ables assigning any distribution $f(x)$ representing the value to the organizer of the contributions obtained. In PAR con-
testants do not know of the participation decisions of others whereas in SQ each contestant knows about the participation decisions of those sequenced before her. From the beginning contestants become aware of the parameters $c$, $M$, the overall number of contestants $k$, and the way contestants’ performance is determined (as a random percentile). The framework was developed as a java-script web-based game.

5.2 Experimental Design

We used six different experimental treatments, differing in the type of contest used (PAR and SQ) and the prize $M$ offered (three different prizes). The number of contestants in each was $k = 3$. The participation cost was set to be constant (arbitrarily set to $c = 5\%$) to provide a similar baseline to all treatments. When setting the prize $M$ we aimed for a varied set of values within the interval of interest, $5\% - 15\%$ (where the lower end is the point where the prize offered covers only one contestant’s participation cost, and the upper value is where even if all three participate the prize fully covers their participation costs), hence choosing $M = 6\%, M = 8\%, M = 11\%$.

Subjects were recruited and interacted through Amazon Mechanical Turk, assigning each to one treatment only, i.e., using a “between-subjects” design to prevent any carryover effect. PAR contests were run simultaneously, whereas with SQ contests we stored the state of the contest on each stage and used it as an input for the next contestant in the sequence. Overall, we had 1200 subjects taking part in our experiments, according to the following division: $300$ overall for the three PAR contests; and (b) $300$ for each of the SQ contests (to allow the completion of 100 contests for each). Subjects ranged in age (21-70), gender (50% men and 50% women) and education, with a fairly balanced division between treatments.

5.3 Experimental Results

We begin with analyzing the participation decisions made by participants in our experiments. One common finding in prior literature is that people tend to exert more effort in contests compared to the theoretical equilibrium-based (as well as alternative model-based) predictions, leading to better expected contributions overall [Sheremeta, 2013]. In particular, this has been shown in our prior work for simple parallel contests [Levy and Sarne, 2018]. This phenomenon was explained primarily by people’s non-monetary utility from winning in a contest [Sheremeta, 2010] along with a plethora of secondary explanations such as mistakes and systematic biases [Chowdhury et al., 2014], lack of experience in contests [Sheremeta, 2011] and caring about relative payoffs maximization [Mago et al., 2016]. The proposed empirical analysis thus aims to investigate both whether a similar over-participation phenomenon occurs with SQ, and if so whether to a greater or a lesser extent compared to the PAR case.

Figure 2 compares the distribution of number of contestants who chose to participate in each contest mechanism with the theoretical predictions, for different prize values. The top graph depicts the results for SQ. Here, according to equilibrium predictions only the first player should choose to participate in the contest for $M = 6$ and $M = 8$, and only the first two players should participate for $M = 11$ (in both contest mechanisms). The graph however, reflects a substantially greater participation rate compared to theory – only in $32\%$ and $22\%$ of the contests there was a single participating contestant, for $M = 6$ and $M = 8$, respectively. In fact, in a non-negligible portion of these contests all three contestants chose to participate. Similarly, for $M = 11$ in almost $30\%$ of the contests there were three contestants choosing to participate. The average number of participants in the contest is $1.77$, $1.83$ and $2.21$ for $M = 6$, $M = 8$ and $M = 11$ (where the theoretical predictions are 1, 1 and 2), respectively.

A similar over-participation phenomenon is observed in the case of PAR (bottom graph). Here, for completeness we also include the theoretical participation distribution according to the mixed-strategies equilibrium (in which participa-
tation probability of each contestant is $p = 0.177$, $p = 0.43$ and $p = 0.71$ for $M = 6$, $M = 8$ and $M = 11$ respectively. The over-participation phenomenon holds for all three treatments, both compared to the theoretical pure-strategies and mixed-strategies based equilibria. Still, with respect to both, the over-participation is to a smaller extent compared to the findings in the SQ case – the average number of participants in the contest is 1.17, 1.65 and 2.22 for $M = 6$, $M = 8$ and $M = 11$ (the theoretical predictions are 1, 1 and 2 in the pure-strategies based equilibrium and 0.51, 1.29 and 2.13 in the mixed-strategies based equilibrium), respectively. Since the theoretical participation predictions with the two mechanisms are similar, we can reason about the difference in the extent of the over-participation reflected with the two mechanisms by comparing the actual number of contestants that participated in the different contests carried out with each mechanism. Here we use both the Mann-Whitney Wilcoxon test [Mann and Whitney, 1947] and t-test. The first is a non-parametric test, checking if one population tends to have larger values than the other (hence it is the most suitable for our case), while the second is a parametric test comparing the means, and its use is legitimate in our case due to the large number of observations. With both tests we find that the difference in participation between SQ and PAR is statistically significant for $M = 6$ and non-statistically significant for $M = 8$ and $M = 11$ (for $\alpha = 0.05$). Meaning that the over-participation found with SQ is at least as great as with PAR.

Next, we compare the average profit obtained with SQ compared to with PAR, taking the expected-profit maximizing prize in each case. Naturally, given the over-participation results reported above, we expect a greater average profit for the organizer with SQ. This is indeed reflected in Figure 3 which depicts the difference between the profit (picking the treatment resulting in the maximum profit for each method) when using SQ and PAR, for different ranges over which $f(x)$ is defined (taking it to be uniform, starting at zero), both as absolute difference and in percentages of the expected profit in PAR. As can be observed from the figure, the differences are substantial and decrease as the range of values from which performance derives increases. This latter decrease is because for most ranges ($30 - 90$) the optimal prize is $M = 6$ for the SQ mechanism and $M = 11$ for PAR and the increase in profit resulting from any increase in range with $M = 6$ is smaller than the corresponding increase with $M = 11$.

6 Related Work

Contests have been widely studied in literature as a means for soliciting productive effort from agents, hence their importance for mechanism design [Lazear and Rosen, 1981; Gradstein and Konrad, 1999]. As such, various contest model variants can be found in related work, differing primarily in the number of contestants and their level of heterogeneity and tendency towards risk, structure of incentives used (e.g., rewards-allocation) and the organizer’s objective functions (maximizing best effort versus the sum of efforts) [Rosen, 1986; Glazer and Hassin, 1988; Clark and Riis, 1996; 1998; Moldovanu and Sela, 2001; 2006; Archak and Sundararajan, 2009; DiPalantino and Vojnovic, 2009; Chawla et al., 2012]. Alongside game-theoretic work in this domain, there is much empirical research analyzing people’s behavior in contests under different settings, and in particular people’s over-expenditure of effort compared to theoretical predictions [Lazear, 2000; Sheremeta, 2010; Boudreau et al., 2011; Sheremeta, 2011; 2013; Chowdhury et al., 2014; Liu et al., 2014; Levy and Sarne, 2018]. This latter phenomenon, which is also being demonstrated with the proposed SQ mechanism, received diverse explanations as discussed in the former section [Dechenaux et al., 2015].

The underlying contest model used in this paper is of a simple contest, i.e., one where contestants only strategize on participation itself. While the majority of contests body of literature has focused in “effort-based” contests, where the probability of winning is determined by the efforts each contestant exerts, simple contests have gained interest in recent years, primarily due to their applicability as contests under different settings, and in particular people’s over-expenditure of effort compared to theoretical predictions [Lazear, 2000; Sheremeta, 2010; Boudreau et al., 2011; Sheremeta, 2011; 2013; Chowdhury et al., 2014; Liu et al., 2014; Levy and Sarne, 2018]. This latter phenomenon, which is also being demonstrated with the proposed SQ mechanism, received diverse explanations as discussed in the former section [Dechenaux et al., 2015].

Alas, the study of simple contests typically assumes the contest is carried out such that all contestants make their participation decision in parallel. One exception is our prior work [Levy et al., 2017] where a strictly sequential simple contest is analyzed. That model, however, assumes complete information not only in the sense that contestants become aware of the participation decisions of others, but rather also of the performance achieved by others, prior to making their own participation decision. This latter assumption results in a critical difference in terms of the equilibrium structure and the resulting expected profit to the organizer, precluding any of the benefits discussed above. In particular, the equilibrium in that model variant relies on threshold-based contestants’ strategies and no mapping holds to the parallel model. In a more general sense, the aspect of executing a contest sequentially can be found in tournaments literature [Rosen, 1986; Clark and Riis, 1996; 1998; Gradstein and Konrad, 1999]. These, however, are very different from the model analyzed in this paper, in the sense that contestants iteratively compete against others and only those that survive a round move on to subsequent rounds.

7 Discussion and Conclusions

As proved and demonstrated throughout the paper, the choice of using SQ over the traditional parallel simple contest encompasses many advantages, among which a tight control
over the outcome of the contest, a guarantee for the maximum possible achievable (and at times even greater) pure-strategy Nash equilibria profit of PAR, and at least the same (and at times an even greater) extent of over-participation compared to theoretical prediction when contestants are people rather than self-interested agents. The expected-profit-maximizing prize determination with SQ is tractable, despite the continuous nature of the prize parameter. To the best of our knowledge this is the first attempt to deal with the continuous nature of the prize in simple contests, as far as prize-determination is concerned.

In this paper we focused on simple contests. A major future research avenue is the question of whether and how these ideas can be extended to general contests.

Acknowledgments

This research was partially supported by the ISRAEL SCIENCE FOUNDATION (grants No. 1162/17 and 1083/13) and the ISF-NSFC joint research program (grant No. 2240/15).

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