Reasoning over Argument-Incomplete AAFs in the Presence of Correlations

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Abstract

We introduce *argument-incomplete Abstract Argumentation Frameworks with dependencies*, that extend the traditional abstract argumentation reasoning to the case where some arguments are uncertain and correlated through logical dependencies (such as mutual exclusion, implication, etc.). We characterize the complexities of the problems DSAT of deciding the satisfiability of the dependencies and PDVER$^\sigma(S)$ of verifying extensions under the possible perspective. We show how they depend on the forms of dependencies and, for PDVER$^\sigma(S)$, also on the semantics of the extensions.

1 Introduction

Incomplete Abstract Argumentation Frameworks (iAAF) are an extension of Dung's *Abstract Argumentation Frameworks* (AAFs) enabling a "qualitative" representation of the uncertainty involving arguments and attacks. Basically, an iAAF is an AAF where the set of arguments (resp., attacks) is partitioned into the sets of *certain* and *uncertain* arguments (resp., attacks): *certain* arguments/attacks are those whose presence in the argumentation is sure, while uncertain arguments/attacks may not occur in the argumentation. As observed in [Baumeister *et al.*, 2018; Coste-Marquis *et al.*, 2007; Cayrol *et al.*, 2007], iAAFs are well-suited for modeling a number of situations. For instance, when representing a dispute that may be participated by several agents, it is natural to model the arguments claimed by agents whose participation is not guaranteed as uncertain. Analogously, arguments encoding alternative interpretations of the same claim should be considered as uncertain, when it is not known which of them catches the intended meaning of the claim.

An iAAF compactly encodes the set of the alternative combinations of arguments and attacks that can actually occur in the argumentation. Each combination is called "completion" and is an AAF containing all the certain arguments/attacks of the iAAF plus a subset of its uncertain arguments/attacks. In order to take into account the possibility of different scenarios for the argumentation (as represented by the completions), the traditional notion of extension for an AAF has been re-formulated in terms of *i*-extension* [Fazzinga *et al.*, 2020]: *A set of arguments S is a possible (resp., necessary) i*-extension of the iAAF IF if, for some (resp., every) completion F of IF, the set S is an extension of F.*

Example 1 *Consider the iAAF IF over the arguments a, b, c, d and the attacks (a, b), (c, d), (d, c) depicted in Fig. 1 (disregard the dotted edges for now). Arguments a, b, c are the only uncertain terms of the argumentation, so IF has 8 completions (denoted as pairs $\langle$ arguments, attacks $\rangle$):*
$F_1 = \langle\{d\}, \emptyset\rangle$; $F_2 = \langle\{a, d\}, \emptyset\rangle$; $F_3 = \langle\{a, b, d\}, \{(a, b)\}\rangle$;
$F_4 = \langle\{a, c, d\}, \{(c, d), (d, c)\}\rangle$;
$F_5 = \langle\{a, b, c, d\}, \{(a, b), (c, d), (d, c)\}\rangle$; $F_6 = \langle\{b, d\}, \emptyset\rangle$;
$F_7 = \langle\{b, c, d\}, \{(c, d), (d, c)\}\rangle$;
$F_8 = \langle\{c, d\}, \{(c, d), (d, c)\}\rangle$.
The set $\{a, c\}$ is a possible i-extension (under the admissible semantics), since it is an admissible extension in at least one completion of IF (such as F_4 and F_5). It is easy to see that, among others, also $\{b, d\}$ and $\{b, c\}$ are possible admissible i*-extensions. Under the necessary perspective, the only admissible i*-extensions are $\emptyset$ and $\{d\}$.*

A limit of the reasoning paradigm over iAAFs is that it does not take into account possible correlations between arguments/attacks. In fact, there can be dependencies between the arguments/attacks implying that some completions represent scenarios that cannot actually occur, and this may deeply affect the reasoning process, as shown in Example 2.

Example 2 *Assume that the iAAF IF of Example 1 models the arguments that can be introduced during a trial, and that the uncertain arguments a, b, c are not independent. In particular, b and c are alternative interpretations of the same fact given by the two experts X_b and X_c, and the defendant has to choose who between X_b and X_c will testify: thus, exactly one argument in $\{b, c\}$ will occur in the argumentation. Moreover, the analyst knows that if b occurs in the argumentation, also a will occur, since a is claimed by an expert X_a that the prosecutor always uses to disqualify what said by X_b. The analyst thinks that the prosecutor may put X_a on the stand even if X_b is not called by the defendant, thus the implication between b and a in one way only. These dependencies can be formally written as* CHOICE(b, c) *and $b \Rightarrow a$, and are depicted in Fig. 1 as suitably labeled dotted edges.*

It is easy to see that some of the completions enumerated in Example 1 encode scenarios that cannot occur. Specifi-

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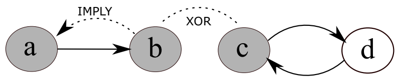

Figure 1: Grey nodes are uncertain arguments. IMPLY- and CHOICE-labeled edges represent dependencies, the other edges are attacks

cally, F_1, F_5, F_7 are not possible, since they violate the fact that exactly one expert will be called by the defendant (i.e. CHOICE(b, c)), and F_6 is not possible as well, since it violates the constraint that if b occurs, also a occurs (i.e. $b \Rightarrow a$). Hence, the only "valid" completions are F_3, F_4 and F_8.

This entails that some of the conclusions drawn in Example 1 must be revised: when verifying if a set is an i*-extension, one should focus only on the valid completions. Hence, under the possible perspective, $\{b, d\}$ is not an admissible i*-extension, since in all the valid completions where b and d occur together, the attack from a to b is not counterattacked. Analogously, $\{b, c\}$ is not an admissible i*-extension, since there is no valid completion containing b and c.

Example 2 shows that disregarding the correlations between the terms of the argumentation can yield wrong assessments: for instance, under the possible perspective, it may happen that the completions witnessing that a set S is an i*-extension are not scenarios that can occur in practice. Hence, disregarding the correlations may lead the analyst to wrongly consider S as a reasonably "robust" set of arguments (as happens in examples 1 and 2 for $\{b, d\}$ and $\{b, c\}$).

The main contribution of this paper is a study of fundamental problems supporting the reasoning over iAAFs in the presence of correlations. In particular, we focus on the case of *argument-incomplete AAFs* (aiAAFs) [Baumeister *et al.*, 2015], i.e. iAAFs where the uncertainty involves only the arguments, as in examples 1, 2. Starting from them, we introduce "*aiAAFs with dependencies*" (daiAAFs), where the analyst is allowed to specify correlations involving the uncertain arguments. We consider a practical setting, where correlations can be expressed in a user-friendly manner: the analyst can specify a set of logical dependencies, where each dependency is the application of one n-ary logical connective (namely, OR, CHOICE, NAND, or $\Rightarrow$) over a set of arguments. This way of specifying the correlations is prone to be implemented in an intuitive visual interface: binary dependencies (like the CHOICE and $\Rightarrow$ of Example 2) can be naturally depicted as (possibly oriented) labeled edges between the involved arguments, while more general n-ary correlations can be depicted by circling the involved sets of arguments (see Fig. 2).

Given this, we characterize the complexity of two problems:

- DSAT: is there a "valid" completion (i.e. where no dependency is violated) of a given daiAAF? This means deciding if the specified correlations "make sense";
- PDVER$^\sigma(S)$: the verification problem for *possible* i*-extensions for a daiAAF under a semantics σ.

In particular, we study the sensitivity of the complexity of the two problems to the form of dependency used to specify the correlations, and, for the case of PDVER$^\sigma(S)$, also to the semantics of extensions (we consider *admissible*, *complete*, *grounded*, *stable* and *preferred* semantics). Table 1 summarizes our results. Interestingly, after observing that DSAT is itself a source of complexity of PDVER$^\sigma(S)$, we show that when DSAT is trivial or polynomial-time solvable, the complexity of PDVER$^\sigma(S)$ depends on the combination $\langle$ *dependency, semantics* $\rangle$. For some combinations, the complexity is P, the same as the verification problem over iAAFs in the absence of correlations, while for others the complexity moves to NP-complete.

2 Preliminaries

An *abstract argumentation framework* (*AAF*) is a pair $\langle A, D\rangle$, where A is a finite set, whose elements are called *arguments*, and $D \subseteq A \times A$ is a binary relation over A, whose elements are called *defeats* or *attacks*. Given a set of arguments S and an argument a, we say that "*S attacks a*" if there is an argument b in S such that b attacks a, and that "*a attacks S*" if there is an argument $b \in S$ such that a attacks b. Moreover, we say that *a is acceptable w.r.t. S* if every argument attacking a is attacked by S, and say that S is *conflict-free* if there is no attack between its arguments.

Several semantics for AAFs have been proposed to identify "reasonable" sets of arguments, called *extensions* [Dung, 1995]. A set $S \subseteq A$ is: an *admissible extension* (`ad`) iff S is conflict-free and all its arguments are acceptable w.r.t. S; a *stable extension* (`st`) iff S is conflict-free and S attacks each argument in $A \setminus S$; a *complete extension* (`co`) iff S is admissible and S contains all the arguments that are acceptable w.r.t. S; a *grounded extension* (`gr`) iff S is a minimal (w.r.t. $\subseteq$) complete set of arguments; a *preferred extension* (`pr`) iff S is a maximal (w.r.t. $\subseteq$) complete set of arguments.

We recall the notion of *argument-incomplete Abstract Argumentation Framework* (aiAAF) [Baumeister *et al.*, 2015].

Definition 1 (aiAAF) *An* argument-incomplete Abstract Argumentation Framework *is a tuple $\langle A, A^?, D\rangle$, where A and $A^?$ are disjoint sets of arguments and D is a set of attacks between arguments in $A \cup A^?$. The arguments in A (resp., $A^?$) are said to be* certain *(resp.,* uncertain*), i.e. they are guaranteed (resp., not guaranteed) to occur in the argumentation.*

An aiAAF compactly represents the alternative scenarios for the argumentation, i.e. all the possible combinations of arguments and attacks that can occur according to what is certain and uncertain. Each scenario is called *completion*.

Definition 2 (Completion) *Given an aiAAF $IF = \langle A, A^?, D\rangle$, a completion for IF is an AAF $F = \langle A', D'\rangle$ where $A \subseteq A' \subseteq (A \cup A^?)$ and $D' = D \cap (A' \times A')$.*

In [Fazzinga *et al.*, 2020], *i*-extensions* were introduced to adapt the notion of extension to the case of iAAFs (and, consequently, to aiAAFs). Specifically, since an iAAF encodes several alternative scenarios, i*-extensions were defined under both the possible and the necessary perspective, where the condition of being extension is required to be true in *at least one* and *every* scenario, respectively. Example 1 contains examples of possible and necessary i*-extensions over aiAAFs.

Definition 3 (i*-extension) *Given an aiAAF IF and a semantics σ, a set S is a possible (resp., necessary) i*-extension* *for IF (under σ) if, for at least one (resp., for every) completion F of IF, the set S is an extension of F under σ.*

3 Specifying Correlations over aiAAFs

The reasoning paradigm based on i*-extensions considers the uncertain arguments as "independent": the presence/absence of an argument is not supposed to influence the presence of other arguments. In fact, when addressing the verification problem, every completion is a potential witness (resp., counter-witness) of the fact that S is an i*-extension under the possible (resp., necessary) perspective. However, the independence assumption may not be reasonable, since some correlation is known to exist between uncertain arguments, and this has important consequences, as discussed below.

Impact of the correlations on the reasoning. As shown in Example 2, introducing dependencies may have the effect of discarding some completions, as they turn out to describe scenarios that cannot occur. This has a strong impact on the reasoning. In fact, under the possible perspective, a set that is an i*-extension when dependencies are not considered may be no longer an i*-extension when dependencies are taken into account (Example 2 shows that this happens with $\{b, c\}$ and $\{b, d\}$, since the completions witnessing that they are extensions are impossible scenarios, according to the CHOICE- and IMPLY- dependencies). Under the necessary perspective, a set that, with no dependency, is not an i*-extension may become an i*-extension when the dependencies are considered. For instance, consider an aiAAF consisting in a conflict-free set S of certain arguments and two uncertain arguments a and b attacking each other. If no dependency is considered, S is not a necessary complete i*-extension, as there are two completions F_1 and F_2 where S is not complete (i.e. the completions where either b or a is missing). Consider now the two IMPLY-dependencies $a \Rightarrow b$ and $b \Rightarrow a$. It is easy to see that, considering these dependencies, F_1 and F_2 cannot be considered as possible scenarios, and S becomes a necessary complete i*-extension.

Embedding correlations into aiAAFs. We now introduce the forms of dependency that we allow to use for encoding the known correlations. In order to make the task of specifying them easy and intuitive, we consider dependencies expressed by commonly used logical connectives.

Definition 4 (Dependency) *A* dependency *δ over an aiAAF $IF = \langle A, A^?, D\rangle$ is an expression $X \Rightarrow Y$ (*IMPLY$^\vee$*-dependency) or* OP(X) *(*OP*-dependency), where* OP $\in \{$OR, NAND, CHOICE$\}$ *and X, Y are non-empty subsets of $A^?$.*

Imposing dependencies allows the user to distinguish "valid" completions from "invalid" completions (i.e. scenarios that can actually occur from impossible scenarios).

Definition 5 (Valid completion) *A completion $F = \langle A', D'\rangle$ is* valid w.r.t *a dependency δ (written $F \models \delta$) iff*
- *δ is* OR(X) *and $X \cap A' \neq \emptyset$,*
- *δ is* NAND(X) *and $X \cap A' \subset X$,*
- *δ is* CHOICE(X) *and $|X \cap A'| = 1$,*
- *δ is $X \Rightarrow Y$ and, if $X \subseteq A'$, then $Y \cap A' \neq \emptyset$.*

F is valid *w.r.t. a set of dependencies Δ if $\forall \delta \in \Delta$ $F \models \delta$.*

Thus, an OR- (resp., CHOICE-) dependency imposes that at least (resp., exactly) one of the specified arguments is in the completion; a NAND-dependency imposes that the specified arguments cannot occur all together; IMPLY$^\vee$ means that if a completion contains all the arguments on the left-hand side, then it must contain at least one of the arguments of the right-hand side. We did not consider AND and NOR as here they make no sense: an AND- (resp., NOR-) dependency requires that each (resp., none) of the specified arguments is in the completion, but this can be done by putting these arguments in A (resp., removing these arguments from $A^?$).

Definition 6 (daiAAF) *An argument-incomplete Abstract Argumentation Framework with dependencies (daiAAF) is a pair $DIF = \langle IF, \Delta\rangle$, where IF is an aiAAF and Δ a set of dependencies over IF.*

The completions of a daiAAF $DIF = \langle IF, \Delta\rangle$ are the completions of IF, and the valid completions of DIF are the completions valid w.r.t. Δ. The notion of i*-extension is naturally adapted to daiAAFs to take into account in the reasoning only the valid completions.

Definition 7 (i*-extensions over daiAAFs) *Given a daiAAF DIF and a semantics σ, a set of arguments S is a possible (resp., necessary) i*-extension for DIF (under σ) if, for at least one (resp., for every) valid completion F of DIF, the set S is an extension of F under σ.*

Motivation of the forms of the dependencies and their expressiveness. In the context of abstract argumentation, since the structure of arguments is not modeled, it is reasonable to model correlations as propositional formulae over sets of variables representing the presence of arguments. Now, any propositional formula Φ of this kind can be expressed as a set of dependencies of our form. In fact, every clause C of a CNF Φ can be translated into a single dependency, reasoning by cases on the form of C:
- $C = x_1 \vee \cdots \vee x_n$ (i.e. C contains only positive literals): C is equivalent to OR$(x_1, \cdots, x_n)$,
- $C = \neg x_1 \vee \cdots \vee \neg x_n$ (i.e. C contains only negative literals): C is equivalent to NAND$(x_1, \ldots, x_n)$,
- $C = \neg x_1 \vee \cdots \vee \neg x_m \vee x_{m+1} \vee \cdots \vee x_n$ (i.e. C contains both positive and negative literals): C is equivalent to $x_1, \ldots, x_m \Rightarrow x_{m+1}, \ldots, x_n$

Thus, a first motivation for considering these forms of dependency is that they are "maximally" expressive (w.r.t. what is reasonably supposed to be expressible).

Observe that what said above means that CHOICE-dependencies are not strictly necessary, as they could be translated into sets of dependencies of the other forms. However, our choice to include CHOICE as a form of dependency is aligned with the rationale of expressing the correlations as a set of dependencies rather than as a propositional formula with $\wedge$, $\vee$, $\neg$ and brackets: providing the analyst with a set of "intuitive" primitives, whose name explicitly describes the semantics of the correlations. Furthermore, in practical cases, encoding the correlations into the set Δ gives the possibility to distinguish the various forms of correlations imposed by the analyst, which could be otherwise "hidden" if a general propositional formula were used. This allows us to provide the contribution presented in the next section: a fine-grained analysis of the impact of correlations on the computational complexity of the fundamental reasoning problems over aiAAFs. This contribution is still of interest if a propositional formula is used to encode correlations instead of the set of dependencies: our study can be viewed as a sensitivity analysis of the complexity of the reasoning tasks w.rt. some syntactic restrictions of practical interest.

4 Reasoning over daiAAFs

We first introduce two fundamental problems that support the reasoning over daiAAFs and that are the object of our study.

Definition 8 (DSAT) DSAT *is the problem of verifying the existence of a valid completion of a given daiAAF DIF.*

Definition 9 (PDVER$^\sigma(S)$) *Let DIF be a daiAAF, S a set of arguments, and σ a semantics.* PDVER$^\sigma(S)$ *is the problem of verifying if S is a possible i*-extension for DIF under σ.*

Basically, DSAT is the problem of deciding if the set of dependencies specified in a daiAAF is not contradictory, and is somehow preliminary to the reasoning task encoded by the verification problem PDVER$^\sigma(S)$. As for the latter, observe that we focus on the possible perspective, and defer the study of the verification of necessary i*-extensions to future work.

We thoroughly analyze the complexity of DSAT and PDVER$^\sigma(S)$: we investigate its sensitivity to the form of dependencies appearing in the daiAAF and, for PDVER$^\sigma(S)$, also to the semantics σ. In order to obtain fine-grained insights on the sources of complexity, we include in our analysis two restrictions of the dependencies of Definition 4, namely CHOICE^2-dependency (i.e. an CHOICE over a pair of arguments) and IMPLY-dependency (i.e. a $\Rightarrow$ with a singleton on the right-hand side, thus no disjunction in the head).

The results are summarized in Table 1. Here, EMPTY means $\Delta = \emptyset$, and the corresponding row is a result from [Fazzinga *et al.*, 2020], where the verification problem over iAAFs (with no dependencies) was shown to be in P for $\sigma \in \{\texttt{ad}, \texttt{st}, \texttt{co}, \texttt{gr}\}$ and Σ_2^p-complete for $\sigma = \texttt{pr}$. ANY OTHER stands for "*any combination of* 2, 3, *or more forms of dependencies different from the combinations in the other rows*". The combinations summarized in ANY OTHER are those whose complexity is implied by the other rows. This does not happen, for instance, for OR+NAND, that is in a distinguished row: in this case, the NP-completeness of DSAT and of PDVER$^\sigma(S)$ under $\sigma \in \{\texttt{ad}, \texttt{st}\}$ is not implied by the polynomiality of these problems with only OR or only NAND.

The relevance of our sensitivity analysis is that, besides giving an insight on the sources of complexity of DSAT and PDVER$^\sigma(S)$, it highlights the presence of restrictions (to the set of logical connectives) that are of practical interest (as they still allow the analyst to model several scenarios) and that make reasoning over daiAAFs an efficient task.

Complexity of DSAT. The correctness of the results in Table 1 on DSAT is stated in the following theorem. Observe that, in Table 1, "*Trivial*" means that DSAT is always true.

	DSAT	PDVER$^\sigma(S)$			
		ad, st	co	gr	pr
1 EMPTY	Trivial	P	P	P	Σ_2^p
2 OR	Trivial	P	P	NP	Σ_2^p
3 NAND	Trivial	P	NP	NP	Σ_2^p
4 IMPLY$^\vee$	Trivial	NP	NP	NP	Σ_2^p
5 IMPLY	Trivial	P	NP	NP	Σ_2^p
6 CHOICE	NP	NP	NP	NP	Σ_2^p
7 CHOICE^2	P	P	NP	NP	Σ_2^p
8 OR+NAND	NP	NP	NP	NP	Σ_2^p
9 OR+CHOICE^2	NP	NP	NP	NP	Σ_2^p
10 NAND+CHOICE^2	NP	NP	NP	NP	Σ_2^p
11 IMPLY+CHOICE^2	NP	NP	NP	NP	Σ_2^p
12 IMPLY$^\vee$+OR	Trivial	NP	NP	NP	Σ_2^p
13 IMPLY+OR	Trivial	NP	NP	NP	Σ_2^p
14 IMPLY$^\vee$+NAND	Trivial	NP	NP	NP	Σ_2^p
15 IMPLY+NAND	Trivial	P	NP	NP	Σ_2^p
16 ANY OTHER	NP	NP	NP	NP	Σ_2^p

Table 1: Complexity of DSAT and PDVER$^\sigma(S)$, where NP means NP-complete and Σ_2^p means Σ_2^p-complete

This is obvious for the case where $\Delta = \emptyset$, and can be straightforwardly seen in the other cases: when at most IMPLY$^\vee$ and OR are allowed, the completion containing all the uncertain arguments is always valid; when at most IMPLY$^\vee$ and NAND are allowed, the completion where all the uncertain arguments are discarded is valid.

Theorem 1 *The complexity of* DSAT*, for the various restrictions on the set of allowed forms of dependencies, is that reported on the second column of Table 1.*

Complexity of PDVER$^\sigma(S)$. We first introduce general upper bounds for PDVER$^\sigma(S)$ that are independent from the allowed forms of dependencies.

Theorem 2 PDVER$^\sigma(S)$ *is in NP for $\sigma \in \{ad, st, co, gr\}$ and in Σ_2^p for $\sigma = pr$.*

The following two theorems distinguish two cases where this upper bound is not strict, as PDVER$^\sigma(S)$ is in P.

Theorem 3 PDVER$^\sigma(S)$ *is in P if $\sigma \in \{ad, st\}$ and only* IMPLY+NAND *or only one among* OR, NAND, IMPLY, CHOICE^2 *is allowed.*

Theorem 4 PDVER$^\sigma(S)$ *is in P when $\sigma = co$ and only* OR *dependencies are allowed.*

We now move to the cases not covered by Theorems 3, 4, showing that the upper bounds of Theorem 2 become strict. For some rows of Table 1 (i.e. 6, 8, 9, 10, 11, 16), this is implied by the following proposition, entailing that if DSAT is NP- hard, also PDVER$^\sigma(S)$ is NP-hard.

Proposition 1 *For any $\sigma \in \{ad, st, co, gr, pr\}$, there is a Karp-reduction from* DSAT *to* PDVER$^\sigma(S)$*, where the dependencies allowed in* PDVER$^\sigma(S)$ *and* DSAT *are the same.*

Proof. Given an instance I^{sat} of DSAT consisting in the daiAAF $\langle IF, \Delta\rangle$, consider the instance I^{ver} of PDVER$^\sigma(S)$ consisting in the pair $\langle\langle IF', \Delta\rangle, S\rangle$, where IF' is IF augmented with a new argument x attacking all the arguments in IF, and $S = \{x\}$. It is easy to see that the answer of I^{sat} coincides with the answer of I^{ver} for any σ. □

We now consider the cases where DSAT is not NP-hard, thus the NP lower bound for PDVER$^\sigma(S)$ must be proved.

Theorem 5 *If only* OR*-dependencies are allowed,* PDVER$^\sigma(S)$ *is NP-hard under* $\sigma = \mathtt{gr}$.

Proof. We show a reduction from SAT. Let Φ be a 3CNF and $DIF(\Phi) = \left\langle\langle A, A^?, D\rangle, \Delta\right\rangle$ be the daiAAF constructed as follows. For each variable x_i in Φ, A contains the pair of arguments i, i' and $A^?$ the pair of arguments x_i, $\neg x_i$, while D contains the four attacks $(x_i, i), (i, \neg x_i), (\neg x_i, i'), (i', x_i)$, and Δ contains the dependency OR$(x_i, \neg x_i)$. Moreover, for each clause $C_j = l_1^j \vee l_2^j \vee l_3^j$ in Φ (where every l_i^j is a literal), Δ contains OR(l_1^j, l_2^j, l_3^j). See the left-hand side of Fig. 2 for an example of construction. We prove the equivalence: "Φ is satisfiable" $\Leftrightarrow$ "$S = \{1, 1', 2, 2', \ldots, n, n'\}$ is a possible i*-extension of $DIF(\Phi)$ under $\sigma = \mathtt{gr}$".
$\Rightarrow$: Given a truth assignment ta making Φ true, let F be the completion of $IF(\Phi)$ containing, for each $i \in [1..n]$, the argument x_i if $ta(x_i) =$ *true*, and $\neg x_i$ otherwise. Obviously, F is a valid completion. Moreover, S is the grounded extension for F since, for each $i \in [1..n]$, one of the arguments i, i' is attacked by no argument, and defends the other one.
$\Leftarrow$: Let F be a valid completion whose grounded extension is S. Since S contains, for each $i \in [1..n]$, both i and i', at least one of the arguments $x_i, \neg x_i$ must not belong to F. Combining this with what imposed by dependency OR$(x_i, \neg x_i)$, we have that F contains exactly one of these arguments, for each $i \in [1..n]$. Hence, F encodes a truth assignment ta for $x_1, \ldots, x_n$, where $ta(x_i) =$ true iff x_i belongs to F. As F is valid, the OR-dependencies encoding the clauses of Φ are satisfied, thus ta is a truth assignment making Φ satisfied. □

Theorem 6 *If only* IMPLY*-dependencies are allowed,* PDVER$^\sigma(S)$ *is* NP*-hard under* $\sigma \in \{\mathtt{co}, \mathtt{gr}\}$.

Proof. We prove the case $\sigma = \mathtt{co}$ (the same reasoning works

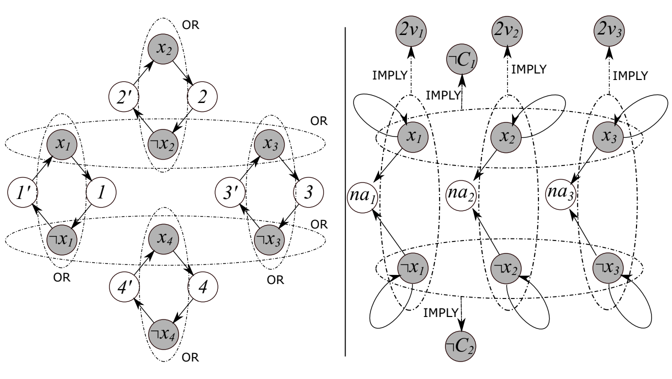

Figure 2: Left: construction of Theorem 5 for $\Phi = (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_3 \vee x_4)$; Right: construction of Theorem 6 for $\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$

for $\sigma = \mathtt{gr}$). We show a reduction from SAT. Let Φ be a 3CNF and $DIF(\Phi) = \left\langle\langle A, A^?, D\rangle, \Delta\right\rangle$ the daiAAF constructed as follows. For each variable x_i, $A^?$ contains the two arguments x_i, $\neg x_i$, and the argument $2v_i$ (meaning that *two truth values* have been assigned to x_i), while A contains the argument na_i (meaning that x_i has not been assigned a truth value). In turn, D contains the attacks (x_i, x_i), $(\neg x_i, \neg x_i)$, as well as (x_i, na_i) and $(\neg x_i, na_i)$. Moreover, for each clause $C_j = l_1^j \vee l_2^j \vee l_3^j$, $A^?$ contains the argument $\neg C_j$, and Δ the dependency $\overline{l_1^j}, \overline{l_2^j}, \overline{l_3^j} \Rightarrow \neg C_j$, where $\overline{l_k^j}$ is the argument representing the negation of l_k^j, for each $k \in [1..3]$. Finally, Δ contains $x_i, \neg x_i \Rightarrow 2v_i$, for each $i \in [1..n]$. See the right-hand side of Fig. 2 for an example of construction. We prove the equivalence: "Φ is satisfiable" $\Leftrightarrow$ "$\emptyset$ is a complete i*-extension for $DIF(\Phi)$".
$\Rightarrow$: Given a truth assignment ta for $x_1, \ldots, x_m$ making Φ true, let F be the completion of $DIF(\Phi)$ containing, for each $i \in [1..n]$, the argument x_i if $ta(x_i) =$ true, and $\neg x_i$ otherwise, but no other argument from $A^?$. Obviously, F is valid, since: 1) putting in F exactly one between x_i and $\neg x_i$ does not trigger any implication $x_i, \neg x_i \Rightarrow 2v_i$ (which would have required the presence of $2v_i$ in F); 2) as ta makes Φ true, no implication $\overline{l_1^j}, \overline{l_2^j}, \overline{l_3^j} \Rightarrow \neg C_j$ is triggered in F. Moreover, F admits no admissible extension other than $\emptyset$: an admissible extension S can contain no argument of the form x_i or $\neg x_i$ (since they are self-attacking arguments), and no argument na_i (since these arguments cannot be defended from the attacks from x_i or $\neg x_i$).
$\Leftarrow$: Let F be a valid completion whose complete extension is $S = \emptyset$. The validity of F and the fact that $S = \emptyset$ imply that, for each $i \in [1..n]$, F contains at most one between x_i and $\neg x_i$ (otherwise, the dependency $x_i, \neg x_i \Rightarrow 2v_i$ would have implied the presence of $2v_i$ in F, and since this argument is not attacked, it should belong to S). Moreover, since $\emptyset$ is complete, there can be no unattacked na_i. Since the only attacks towards na_i in D are from x_i and $\neg x_i$, this means that F, for each $i \in [1..n]$, contains exactly one between x_i and $\neg x_i$. Hence, F encodes a truth assignment ta for $x_1, \ldots, x_n$, where $ta(x_i) =$ true iff x_i belongs to F. Since $\emptyset$ is a complete extension, it means that F contains no argument $\neg C_j$ (if there were some $\neg C_j$ in F, it would be not attacked and thus present in S). This means that, for every clause C_j, no IMPLY-dependency with $\neg C_j$ on its right-hand side is triggered in F, thus ta makes all the clauses true. □

The remaining NP-hard cases are stated in the Theorem 7.

Theorem 7 *If only* NAND- *or only* CHOICE^2- *dependencies are allowed,* PDVER$^\sigma(S)$ *is NP-hard under* $\sigma \in \{co, gr\}$. *If only* IMPLY$^\vee$- *or only* IMPLY+OR- *dependencies are allowed,* PDVER$^\sigma(S)$ *is NP-hard under* $\sigma \in \{ad, st, co, gr\}$.

Finally, we consider the preferred semantics. Here, the lower bound is implied by the literature of aiAAFs without dependencies [Baumeister *et al.*, 2018].

Theorem 8 PDVER$^\sigma(S)$ *is* Σ_2^p*-hard under* $\sigma = pr$.

4.1 Summary and Discussion of the Results

As for DSAT, the satisfiability for some forms of dependencies (see rows 1-5, 12-15 of Table 1) is always guaranteed, for others (CHOICE^2) it can be checked in polynomial time, and for all the other cases DSAT is NP-complete. Observe that the expressiveness of the (combined) forms of dependencies for which DSAT is NP-complete is not necessarily the same: for instance, it is easy to see that our CHOICE-dependencies (for which DSAT is NP-complete) are not sufficient to express some correlations encoded by a propositional formula.

As for PDVER$^\sigma(S)$, our analysis highlights three sources of complexity: 1) the forms of dependencies, 2) the semantics of extensions, and 3) the combination of 1) and 2). In fact, when DSAT is NP-hard, also PDVER$^\sigma(S)$ is NP-hard. However, even for the (combined) forms of dependencies making DSAT trivial or in P, PDVER$^\sigma(S)$ may be hard. Specifically, whatever the form of dependency is, PDVER$^\sigma(S)$ is Σ_2^p-complete under $\sigma = \texttt{pr}$, and NP-complete under $\sigma = \texttt{gr}$. However, the Σ_2^p-hardness under $\sigma = \texttt{pr}$ is independent from the presence of dependencies (since it holds even over "traditional" aiAAFs, where $\Delta = \emptyset$), while under $\sigma = \texttt{gr}$ PDVER$^\sigma(S)$ is in P if $\Delta = \emptyset$. The remaining combinations $\langle$ *form of dependency, semantics* $\rangle$ are more intricate. Specifically, under $\sigma = \texttt{co}$, PDVER$^\sigma(S)$ is in P only if we restrict Δ to contain only OR-dependencies. Under $\sigma \in \{\texttt{ad}, \texttt{st}\}$, PDVER$^\sigma(S)$ is in P only if we restrict Δ to contain OR-dependencies, or CHOICE^2-dependencies, or combinations of IMPLY- and NAND- dependencies. Allowing combinations of dependencies behind these polynomial cases makes the complexity explode.

5 Related Work

i*-extensions have been introduced in [Fazzinga *et al.*, 2020] as a revision of the i-extensions defined in [Baumeister *et al.*, 2018]. The difference between i*- and i- extension is in the set used to decide if S is an extension: in the latter, the projection of S over the completions is used, rather than S. This projection can cause counter-intuitive side-effects, e.g., even a conflicting set can be an i-extension. In fact, the notion of accepted arguments used in the literature of iAAFs [Baumeister *et al.*, 2021] corresponds to argument belonging to some (or every) i*-extension (and not i-extension). Our sensitivity analysis does not apply to i-extensions as PDVER$^\sigma(S)$ in that case is already NP-complete for $\sigma = \texttt{ad}$.

Constraints in argumentation have been little investigated. In [Coste-Marquis *et al.*, 2006], AAFs are augmented with a propositional formula with the aim of refining the set of extensions. They do not consider uncertain terms. In the dynamic scenario, [Wallner, 2020] considers constraints to limit the admitted structural modifications when the sets of arguments and attacks are updated on abstract dialectical frameworks (ADFs) [Brewka *et al.*, 2017]. In this regard, the general relationship between argument-incomplete AAFs and ADFs needs some further investigation as the latter may be capable of encoding uncertain arguments and correlations.

Beside the already mentioned, other works dealing with uncertainty in AAFs are the following. The *Partial Argumentation Framework* (PAF) [Cayrol *et al.*, 2007] mainly differs from iAAFs since the semantics of extensions is not based on completions, but on a revised notion of admissibility (where, depending on the desired level of cautiousness, only certain attacks or also uncertain attacks must be defended). Reasoning over iAAFs' extensions is also related to revising AAFs to enforce the existence of an extension [Baumann and Ulbricht, 2019], or to make a set an extension [Coste-Marquis *et al.*, 2015] (where, however, only minimal sets of changes are considered), and to the credulous/skeptical conclusion problems in *Control Argumentation Frameworks (CAFs)* [Dimopoulos *et al.*, 2018]. In this regard, embedding correlations in CAFs is an interesting research direction.

Several variants of AAFs where the uncertainty is quantitatively specified have been proposed. Some of them allow the specification of preferences and/or weights [Bench-Capon, 2003; Amgoud and Vesic, 2011; Modgil, 2009; Dunne *et al.*, 2011; Coste-Marquis *et al.*, 2012; Brewka *et al.*, 2014]. In other approaches, uncertainty is specified via probabilities, according to the "epistemic" [Thimm, 2012; Hunter and Thimm, 2014]), or the "constellation" paradigm (prAAFs) [Hunter, 2014; Dung and Thang, 2010; Doder and Woltran, 2014; Dondio, 2014; Hunter, 2012; Li *et al.*, 2011; Fazzinga *et al.*, 2015]. The last ones are the most related to our framework, as prAAFs can be seen as iAAFs where a probability distribution (pdf) is defined over the completions. Typically, the correlations are hidden in the specified pdf, that is defined by enumerating the completions and explicitly assigning a probability to each of them. Up to our knowledge, the only form of prAAF allowing the explicit specification of correlations is IND-D [Fazzinga *et al.*, 2019], but only mutual exclusion and co-existence can be expressed.

6 Conclusions

An extension of argument-incomplete AAFs has been investigated, where the analyst is allowed to specify correlations among the uncertain arguments by imposing different forms of dependencies. The impact of the presence of correlations on the complexity of the satisfiability and the verification problems (under the possible perspective and for Dung's semantics) has been thoroughly investigated, by studying the sensitivity of the complexity to the form of dependencies used to define the correlations. Future directions of research stemming from this work are extending our study to characterize the complexity of the verification problem under the necessary perspective, and the search for islands of tractability for the NP-hard cases related to syntactic restrictions of the logical connectives that are still of practical interest.

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