Fair Equilibria in Sponsored Search Auctions: The Advertisers’ Perspective

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Abstract

In this work we introduce a new class of mechanisms composed of a traditional Generalized Second Price (GSP) auction and a fair division scheme, in order to achieve some desired level of fairness between groups of Bayesian strategic advertisers. We propose two mechanisms, β-Fair GSP and GSP-EFX, that compose GSP with, respectively, an envy-free up to one item, and an envy-free up to any item fair division scheme. The payments of GSP are adjusted in order to compensate advertisers that suffer a loss of efficiency due to the fair division stage. We investigate the strategic learning implications of the deployment of sponsored search auction mechanisms that obey to such fairness criteria. We prove that, for both mechanisms, if bidders play so as to minimize their external regret they are guaranteed to reach an equilibrium with good social welfare. We also prove that the mechanisms are budget balanced, so that the payments charged by the traditional GSP mechanism are a good proxy of the total compensation offered to the advertisers. Finally, we evaluate the quality of the allocations through experiments on real-world data.

1 Introduction

Over the last decades, online advertising has been one of the main tools for small and medium business (SMB) to grow. Online advertising allows SMBs to reach potential customers without geographical or demographic barriers. Moreover, it offers better return-on-investments than other advertising mediums thanks to its highly personalized system. Given its crucial role in the growth of businesses (and, in its turn, society), it is natural to study how the mechanisms implemented in online advertising platforms can be improved to obey to different fairness criteria for advertisers. There are indeed various settings where one may care about balancing ads allocations, even just between a majority and a minority group of advertisers. For example, one setting is when large companies and small businesses are competing for the same set of users, and the platform may want to ensure that small companies get reasonable visibility despite smaller budgets. As another example, there may be businesses based in different geographical locations but offering similar products/services. These businesses may have unbalanced budgets due to their location, but one may want to guarantee to each business some visibility in a target set of users regardless of geographical attributes. More in general, one could think of incorporating fairness in sponsored search auctions whenever two different parties are advertising to increase users’ awareness about some sensitive topic.

We study how fairness notions borrowed from the fair division literature can be used to model fairness with respect to advertisers. Notions from fair division already found applications in online advertising [Chawla and Jagadeesan, 2020; Ilvento et al., 2020]. In the fair division literature, the dominant notion of fairness aims at providing guarantees for individual agents (see, e.g., [Brandt et al., 2016; Moulin, 2003]). However, we argue that a group approach would be more practical, better aligned with societal expectations, and easier to implement. Following recent works in group fair division literature [Kypourou et al., 2020; Manurangsi and Saksompong, 2017; Manurangsi and Saksompong, 2019; Segal-Halevi and Saksompong, 2019], we propose to use envy-freeness [Foley, 1967] to study group fairness. Group envy-freeness guarantees that no group of agents envies the allocation obtained by any other group. Unfortunately, envy-freeness cannot be guaranteed for indivisible items even in simple settings with two agents and one item. Thus, we focus on two natural relaxations: envy-freeness up to one good (EF1) [Budish, 2011; Lipton et al., 2004], and envy-freeness up to any good (EFX) [Caragiannis et al., 2019; Gourvès et al., 2014].

In practice, any attempt to guarantee such properties in real advertising platforms will inevitably collide with real-world engineering constraints. Therefore, a credible solution should be a mechanism that can be easily integrated with a pre-existing auction framework, without requiring substantial changes to it. We focus on the generalised second price (GSP) auction framework [Edelman et al., 2007], which is one of the most frequently adopted mechanisms for the allocation of advertising opportunities in large Internet advertising companies. In this setting, we show the existence of simple mechanisms that guarantee some notion of group EF1 (resp., group EFX) for advertisers. Such mechanisms can be

1See the full paper for additional discussion of related work.
implemented as a post-auction layer to be run after a standard GSP mechanism. In the spirit of the work by [Dwork and Ilvento, 2019], we study the properties of such composite mechanisms.

Original contributions We focus on a Bayesian setting with incomplete information (i.e., the valuations for advertising opportunities are stochastic, and each bidder does not observe the realized valuations of the other bidders). For each auction, bidders are divided in two groups (a majority group and a minority group) based on their characteristics and competitiveness. Given the different characteristics, the users interact in different ways with the ads from the two groups. This is modeled through group specifics click-through rates and quality factors. Moreover, they can be computed efficiently with two fair division schemes. We show that group settings can model, for example, cases in which different groups of advertisers mainly resort to different media types. Finally, each group $G$ is associated with a quality factor $\gamma_w$ in $[0, 1]$, which reflects the clickability of ads from bidders belonging to group $w$. For example, ads coming from advertisers in the majority group may have higher clickability than ads from advertisers in the minority groups because of differences in the available budgets to develop the campaigns. Quality factors are private knowledge of the advertising platform, and not known by the bidders.

2 Preliminaries

Throughout the paper, we use bold case letters to denote column vectors. Given a vector $y$, its $i$-th component is denoted by $y_i$. The set $\{1, \ldots, x\}$ is denoted by $[x]$, and $\Delta_X$ is the $|X|$-dimensional simplex over the discrete set $X$.

Sponsored search framework There is a set $I$ of $n$ bidders and a set $J$ of $m$ slots. An outcome is an assignment of bidders to slots. Each bidder $i$ has a private type $v_i$, representing their valuation on the item which is being sold. The vector of types is denoted as $v = (v_1, \ldots, v_n)$. Each bidder $i$ belongs to a group from a finite set of possible groups $G$. The function $g : [n] \rightarrow G$ maps bidders to their group, that is, we write $g(i)$ to denote the group to which bidder $i$ belongs. We assume that bidders may belong to one of two groups $G = \{h, \ell\}$ (e.g., a majority group and a minority group) and the two groups may have different sizes. We denote the set of bidders belonging to the two groups by $I_h$ and $I_\ell$. We make the assumption that $|I_h| \geq m$ and $|I_\ell| \geq m$. This assumption is reasonable in the context of large Internet advertising markets. As it is customary in the literature, we use the model of separable click probabilities (see, e.g., [Edelman et al., 2007; Varian, 2007]), in which each slot $j$ is associated with a click-through rate $\alpha_j, g(i)$ for group $g(i)$. We assume that, for each group $w \in G$, $\alpha_{1,w} \geq \alpha_{2,w} \geq \ldots \geq \alpha_{m,w}$, and without loss of generality we take $n = m$. Group-specific click-through rates can model, for example, cases in which different groups of advertisers mainly resort to different media types. Finally, each group $w \in G$ is associated with a quality factor $\gamma_w \in [0, 1]$, which reflects the clickability of ads from bidders belonging to group $w$. For example, ads coming from advertisers in the majority group may have higher clickability than ads from advertisers in the minority groups because of differences in the available budgets to develop the campaigns. Quality factors are private knowledge of the advertising platform, and not known by the bidders.

GSP auction A mechanism elicits a bid $b_i \in \mathbb{R}_{\geq 0}$ for each bidder $i$, which is interpreted as a type declaration, and computes an outcome as well as a price $p_i(b, \gamma)$ for each bidder $i$. We denote by $\pi(b, \gamma, j)$ the bidder assigned to slot $j$ when the mechanism observes the bid vector $b$ and vector of quality factors $\gamma$. We also denote by $\nu(b, \gamma, i)$ the slot assigned to bidder $i$ when the mechanism observes the bid vector $b$ and vector of quality factors $\gamma$. When the vectors of bids and quality factors are clear from the context we simplify the notation by writing $\pi(j)$, $\nu(i)$, and $p_i$ in place of $\pi(b, \gamma, j)$, $\nu(b, \gamma, i)$, and $p_i(b, \gamma)$, respectively. The value perceived by bidder $i$ when they are allocated $\nu(i)$ is $\alpha_{\nu(i), g(i)} \gamma_{g(i)} v_i - p_i$. We focus on a family of mechanisms derived from the Generalized Second Price (GSP) auction [Varian, 2007]. In a GSP auction the mechanism assigns the slots in order from 1 to $m$ and sets $\pi(b, \gamma, j)$ to be the bidder with the highest effective bid $\gamma_{g(i)} \alpha_{j, g(i)} b_i$ not yet assigned (breaking ties arbitrarily).

Our results hold also in the case of advertiser-dependent clickability, because that does not alter the relative ordering of items.
For any bid profile $b$, quality factors $\gamma$ and for each $j \in [m]$, $i = \pi(j)$, the price charged to bidder $i$ is computed as
\begin{equation}
\label{eq:price}
p_i^T(b, \gamma) = \frac{\gamma_{\pi(j)}(\pi(j+1)) \Omega_{j, \pi(j+1)}(\pi(j+1))}{\gamma_{\pi(i)}(i)} b_{\pi(j+1)},
\end{equation}
where we set $b_{n+1} = 0.5$. The mechanism is Individually Rational (IR) if, for each bidder $i \in I$, $u_i(b, v, \gamma) \geq 0$, for all $b, v,$ and $\gamma$. The mechanism is Individually Rational at the Equilibrium (IRE) if it is IR at the equilibrium bid vectors.

Online Bayesian framework The $n$ bidders participate in a series of GSP auctions. At each iteration $t$, each bidder $i$ observes a valuation $v_i^t$ for the item being sold at time $t$. Let $V_i$ be the finite set of types of bidder $i$. The vector of types $v^t = (v_1^t, \ldots, v_n^t)$ is drawn at each time $t$, from a (possibly correlated) probability distribution $\mathcal{F}$ supported on a finite set of joint types $\mathcal{V}$, that is, $\mathcal{V} := \times_{i \in [n]} V_i$. Moreover, at each $t$, a vector of quality factors $\gamma_t \in [0, 1]^{|\mathcal{G}|}$ is drawn from a (possibly correlated) distribution $\mathcal{G}$. Each bidder $i$ has an arbitrary finite set of available bids $B_i \subseteq \mathbb{R}_{\geq 0}$, with $B_i := \max B_i$ and $B_i \supseteq V_i$. Moreover, we denote by $B := \times_{i \in [n]} B_i$ the set of all possible joint bid profiles. A bidding strategy $\sigma_i$ for bidder $i$ is a (possibly randomized) mapping from their types $V_i$ to their bids on $B_i$. We represent such strategies as a $|\mathcal{V}| \times |\mathcal{B}_i|$ right stochastic matrix in which each row specifies a well-defined probability distribution over bids: bidder $i$’s strategy space is $\Sigma_i := \{\sigma_i \in \mathbb{R}_{\geq 0}^{|\mathcal{V}| \times |\mathcal{B}_i|} : \sigma_i \mathbb{1} = \mathbb{1}\}$. We observe that bidders cannot condition their bids on their quality factors $\gamma$, since they are only known to the platform, and not to advertisers. Finally, we define the set of joint bidding strategies as
\[\Sigma := \left\{ \sigma \in \Delta_{\mathcal{V} \times \mathcal{B}} : \sum_{v, b} \sigma[v, b] = \mathcal{F}(v), \quad \forall v \in \mathcal{V} \right\}.
\]

Regret and equilibria Given a sequence of decisions $(b_1^t, \ldots, b_n^t)$ up to time $T$, the external regret of bidder $i$ in type $v_i$ is how much they regret not having played the best fixed action in hindsight at each iteration in which they observed type $v_i$. Formally, the regret experienced by bidder $i$ under a certain type $v_i \in V_i$ is
\begin{equation}
R^T_{v_i} := \max_{b \in B_i} \sum_{t=1}^T \mathbb{1}[v_i = v_i^t]\left(u_i(b) - u_i^*(b_i^t)\right),
\end{equation}
where we set $u_i^{n+1} = 0.5$. The mechanism is Individually Rational (IR) if, for each bidder $i \in I$, $u_i(b, v, \gamma) \geq 0$, for all $b, v,$ and $\gamma$. The mechanism is Individually Rational at the Equilibrium (IRE) if it is IR at the equilibrium bid vectors.

3 Group Fairness in GSP Auctions

In this section we present the two group fair division schemes that will be added as a post-auction layer to GSP.

Preliminary definitions Consider an arbitrary stage $t$ of the repeated auctions process (dependence on $t$ will be omitted when clear from the context). For each group $w \in G$, let $\text{ALG}_{\text{G}_w}(b) := \sum_{i \in I_w} \gamma_{w, a_{\pi(i)}, w} b_i$ be the value obtained by group $w$ via a generic mechanism with allocation rule $\nu$, on bid vector $b$. Since $G = \{h, \ell\}$, the overall value is $\text{ALG}(b) = \text{ALG}_{\text{G}_h}(b) + \text{ALG}_{\text{G}_\ell}(b)$. Moreover, given a set of slots $J' \subseteq [m]$ assigned to group $w \in G$, we define
\[\text{ALG}_{\text{G}_w}(J', b) := \sum_{j \in [|J'|]} \gamma_{w, a_{J'}[j], w} b_{I_w[j]},\]
where $J'[j]$ denotes the slot with the $j$-th click-through-rate among slots in $J'$, and $I_w[j]$ is the bidder belonging to $I_w$ with the $j$-th effective bid in decreasing order (e.g., $I_h(1)$ is the bidder of group $h$ with the highest effective bid). Intuitively, $\text{ALG}_{\text{G}_w}(J', b)$ is the maximum value attainable by group $w$ when it is allocated $J'$ and the bid vector is $b$.

$\beta$-EF1 mechanism The first mechanism that we describe employs a fair division scheme that guarantees the following notion of group fairness.

Definition 1. (Group $\beta$-EF1 fairness) Let $\beta := \xi_\ell/\xi_h$, with $\xi_\ell, \xi_h \in \mathbb{N}^+ \cup \{0\}$, $\xi_\ell \geq \xi_h$, and $\xi_h + \xi_\ell \leq m$. We say that an allocation is group $\beta$ envy-free up to one good ($\beta$-EF1 fair) for $\beta \leq 1$ and bid profile $b$ if, for each pair of groups $h, \ell \in G$, there exists one item $j_h \in J_h$ such that $\text{ALG}_{\text{G}_h}(J_h \setminus \{j_h\}, b) \geq \beta \text{ALG}_{\text{G}_h}(J_h, b)$.

We fix $\xi_\ell$ and $\xi_h$ to be the smallest integers that satisfy $\beta = \xi_\ell/\xi_h$. A group $\beta$-EF1 fair allocation can be obtained through a round robin procedure that assigns $\xi_h$ slots to group $h$ for each $\xi_\ell$ slots assigned to group $\ell$. The proof of this result is very similar to the one for the classical EF1 fair division scheme by Markakis [2017]. As an example, if we assume group $h$ to be the majority group (i.e., advertisers from group $h$ are allocated the highest slots in the ranking), then the result of the application of the group $\beta$-EF1 round robin procedure is the shift of advertisers assigned to a position $j \in J_h$ to
position at most $\lceil (1 + \beta) j \rceil - 1$. For more details on how this fair division scheme is implemented see the extended version of the paper. We observe that a round robin procedure guarantees group EFX fair even when the number of groups is $|G| > 2$. The reason is that, for any pair $w, w' \in G$, and positions $j \in [\lceil |J_w| \rceil], j' \in [\lceil |J_w'| \rceil]$, with $j \leq j'$, it holds $\alpha_{f_{j,j}|w} \geq \alpha_{f_{j',j'|w'}}$. Moreover, it is possible to show that a round robin procedure also guarantees group $\beta$-EFX with respect to valuations (proofs can be found in the extended version of the paper).

**Theorem 1.** Given a bid profile $b$, the allocation computed by the composite mechanism is $\beta$-EFX fair with respect to the valuation profile $v$. In particular, given the allocation of slots to the two groups $J_h, J_l$, for each pair of groups $h, \ell \in G$, there exists one item $j \in J_\ell$ such that $\text{ALG}_h(J_h, v) \geq \beta \text{ALG}_\ell(J_\ell \setminus \{j\}, v)$.

Intuitively, this is because the $\beta$-EFX mechanism computes $J_h, J_l$ without employing the reported bid profile, which is used only to determine the per-group ranking.

**$\beta$-EFX mechanism** The second notion of fairness which we consider is group $\beta$-EFX fairness.

**Definition 2.** (Group $\beta$-EFX (fairness) An allocation is group $\beta$-envy free up to any good ($\beta$-EFX fair) for $\beta \leq 1$ and bid profile $b$ if, for each pair of groups $h, \ell \in G$, and for each item $j_h \in J_h$, it holds $\text{ALG}_h(J_h, b) \geq \beta \text{ALG}_\ell(J_\ell \setminus \{j_h\}, b)$.

A group $\beta$-EFX fair division scheme can be obtained through a “group version” of the envy-cycle elimination algorithm by Lippton et al. [2004]. In particular, we propose the Group Envy-Cycle-Elimination algorithm (GECE). The GECE algorithm can be summarized as follows: denote by $J_h$ and $J_l$ the set of slots assigned, respectively, to group $h$ and $\ell$. We say that group $h$ envies group $\ell$ if $\text{ALG}_h(J_h, b) < \beta \text{ALG}_h(J_\ell, b)$. Initially, all the slots are not assigned, that is, $J_h = J_\ell = \emptyset$. Then, the algorithm iterates through the slots in decreasing order of click-through rate. The first slot is assigned to group $h$. For each subsequent slot $j$, the algorithm checks if groups $h$ and $\ell$ envy each other, and, if this is the case, the algorithm swaps their allocations. Otherwise, if group $h$ does not envy group $\ell$, then the next slot is assigned to group $h$, else, if $\ell$ envies $h$, the slot is assigned to group $\ell$. In the following theorem, we prove that the GECE algorithm is guaranteed to obtain a $\beta$-EFX allocation.

**Theorem 2.** The allocation computed by the group envy-cycle-elimination (GECE) algorithm is group $\beta$-EFX fair.

It is possible to show that a variation of GECE yields a group EFX fair allocation (i.e., $\beta$-EFX with $\beta = 1$) even with more than 2 groups.

**Corollary 1.** The allocation computed by the k-group envy-cycle-elimination (k-GECE) algorithm is group EFX.

4 Efficiency and Budget Balance

In this section, we study the efficiency and budget balance of the two mechanisms obtained by combining GSP with the two fair division schemes described in Section 3. Let us denote one such composite mechanism by $C$. The post-auction layer of the composite mechanism $C$ is modifying the GSP allocation of slots to bidders. Ideally, no bidder should be penalized for this re-allocation. Therefore, we need to update the payments so that bidders’ utility is not negatively affected by the composition of GSP with the fair division scheme. Interestingly, we can do so starting from the payments of GSP. In particular, denote by $p^{\ell}_{i}$ and by $p^{C}_{i}$ the payments charged to advertiser $i$ computed by GSP and by the composite mechanism, respectively. Moreover, let $\nu^{GSP}_{i}$ and $\nu^{C}_{i}$ be the slots assigned to advertiser $i$ by GSP and by the composite mechanism, respectively. Then, we define the payments charged by the composite mechanism $C$ as:

$$
p^{C}_{i} = \begin{cases} 
 p^{\ell}_{i} & \text{if } \nu^{C}_{i} \leq \nu^{GSP}_{i} \\
 p^{\ell}_{i} - 2b_i \gamma_{g(i)} \left(\alpha_{\nu^{GSP}_{i},g(i)} - \alpha_{\nu^{C}_{i},g(i)}\right) & \text{else},
\end{cases}
$$

The pricing rule shows that the composite mechanism $C$ compensates the loss of social welfare of the advertisers that obtain a worse slot by reducing their payments. In order to ensure individual rationality, the advertisers that obtain a better slot are not asked to compensate with a higher payment. Observe that the pricing rule does not exclude positive transfers to the advertisers.

Now, let us define an appropriate notion of budget balance for a composite mechanism which assigns a payment $p^{C}_{i}$ to bidder $i \in I$, with respect to the GSP mechanism. Let $p^{\ell}_{i} := \sum_{i \in I} p^{\ell}_{i}$, and $p^{C}_{i} := \sum_{i \in I} p^{C}_{i}$.

**Definition 3.** A composite mechanism is $\alpha$-budget balanced, for $\alpha \geq 0$, if $p^{\ell}_{i} - p^{C}_{i} \leq \alpha p^{\ell}_{i}$.

An $\alpha$-budget balanced mechanism is therefore able to cover via the GSP payments at least an $\alpha$ fraction of the compensations given to the bidders by the composite mechanism.

Let $\text{ALG}^{\ell}_{GSP}(b)$ and $\text{ALG}^{C}_{GSP}(b)$ be, respectively, the value of the GSP mechanism and of the fair composite mechanism on an arbitrary bid vector $b$. Moreover, we use the following two assumptions in the analysis of the composite mechanisms.

**Assumption 1.** The value of the minority group increases after the application of the composite mechanism, i.e., $\text{ALG}^{C}_{GSP}(b) \geq \text{ALG}^{\ell}_{GSP}(b)$.

**Assumption 2.** The first slot is assigned by GSP to $I_h(1)$, i.e., the first bidder of group $h$.

Assumption 1 is natural since the basic goal of the proposed mechanisms would be to make the minority group better off with respect to the GSP case. Assumption 2 only requires that the advertiser with the highest bid belongs to the majority group. This is natural, for example, in settings where groups have different economic power. First, we provide efficiency and budget balance results for the $\beta$-Fair GSP mechanism.

**Theorem 3.** The $\beta$-Fair GSP mechanism achieves a value that is at least a $1/(1 + \beta)$ fraction of the value of GSP, i.e., for all bid vectors $b \in B$, $\text{ALG}^{C}_{GSP}(b) \geq \text{ALG}^{\ell}_{GSP}(b)/(1 + \beta)$.

**Theorem 4.** The $\beta$-Fair GSP mechanism is 2-budget balanced.

This result does not rule out that the mechanism may suffer a net loss. However, such loss is bounded by a small constant (see Section 6 for an empirical evaluation of such loss).
Second, we study efficiency and budget balance of the GSP-EFX mechanism. The allocation done by the mechanism is described in Section 3 and it is obtained by the composition of the GSP mechanism with the group EFX fair division scheme. The payments of GSP-EFX are computed as in Equation (2). Then, we have the following.

Theorem 5. GSP-EFX achieves a value that is at least a fraction 1/3 of the value of GSP, i.e., for all bid vectors $b \in \mathcal{B}$, $\text{ALG}^c(b) \geq \frac{1}{3} \text{ALG}^G(b)$.

Finally, we prove that GSP-EFX is able to compensate at least 1/4 of the total welfare loss generated by the application of the EFX fair division scheme. Formally,

Theorem 6. The GSP-EFX mechanism is 4-budget balance.

Remark 1. Given a number of groups $k > 2$ and $\beta = 1$, Theorem 3 and Theorem 5 can be extended to show that the group EFX (resp., group EFX) mechanism achieves a value that is at least a 1/2k fraction (resp., a fraction 1/3k) of the value of GSP.

5 Price of Composition

Equipped with the results from Section 4 we can study the performance of the proposed mechanisms at equilibrium. We study the quality of the equilibria emerging as the results of the no-regret learning dynamics in which each bidder behaves as an external-regret minimizer. To do so, we propose the price of composition (PoC) as a natural measure to evaluate the social welfare guarantee of our mechanisms at equilibrium. For an arbitrary mechanism, the social welfare attained for bids $b$, valuations $v \in \mathcal{V}$, and quality factors $\gamma$ is

$$SW(b, v, \gamma) := \sum_{j \in [n]} \alpha_{j, b, \gamma}(\pi(j)) \gamma(\pi(j)) v(\pi(j)).$$

Given an equilibrium strategy $\sigma \in \Sigma^*$ in an incomplete-information game, its social welfare is evaluated by comparing it to the expected ex-post social welfare of the GSP mechanism, which we denote by $E_{b, v, \gamma}[SW^G(b, v, \gamma)]$. In particular, we can define the following worst-case ratio.

Definition 4. The price of composition (PoC) of a composite mechanism $C$ is defined as

$$\text{PoC} := \inf_{\sigma \in \Sigma^*, b \in \mathcal{B}, \sigma \in \Sigma^*} \frac{E_{b, v, \gamma, \sigma}[SW^C(b, v, \gamma)]}{E_{b, v, \gamma}[SW^G(b, v, \gamma)]}.$$

By the definition of the composite mechanisms, their social-welfare is at most equal to the social welfare of the GSP mechanism (see Section 4), i.e., PoC $\in [0, 1]$. Moreover, bounding the worst case PoC automatically yields a PoC/2 guarantee on the price of total anarchy [Blum and Mansour, 2007]. This is because $SW^G(b, v, \gamma) \geq SW^*\gamma(b, v, \gamma)/2$, where $SW^*$ is the optimal social welfare with valuations $v$.

In order to characterize the PoC of our mechanisms, we introduce a natural smoothness condition for composite mechanisms, which is a generalization of the notion of semi-smoothness by Lucier and Paes Leme [2011]. Let $SW^G$ be

the social welfare of the baseline mechanism (that is, in our setting, GSP), and let $SW^C$ be the social welfare provided by the composite mechanism (in our case, $\beta$-Fair GSP or GSP-EFX) given an arbitrary vector or valuations. Then, our notion of smoothness states that social welfare gaps between the social welfare at the baseline mechanism with truthful bid vector $v$, and the social welfare for an arbitrary joint strategy profile $\sigma \in \Sigma$ in the composite mechanism, can be captured by the marginal increases in the individual agents’ utilities when unilaterally switching to a deviation strategy profile $\sigma' = (\sigma'_1, \ldots, \sigma'_n)$. Formally, a composite mechanism $C$ is $(\lambda, \mu)$-semi-smooth with respect to a baseline mechanism $G$ if there exists a profile of individual bidding strategies $\sigma' = (\sigma'_1, \ldots, \sigma'_n) \in \times \Sigma^i$ such that, for any joint strategy $\sigma \in \Sigma$, $v \in \mathcal{V}$, and $\gamma$,

$$E_{b \sim \sigma, [w]} \left[ \sum_{i \in [n]} u_i^c((b'_i, b_{-i}), v_i, \gamma) \right] \geq \lambda SW^G(v, \gamma) - \mu E_{b \sim \sigma}[SW^C(b, v, \gamma)].$$

In this setting, we make the additional assumption that bidders are conservative and, therefore, they do not overbid. This is in line with previous works studying the price of anarchy of auctions with conservative bidders (see, e.g., [Leme and Tardos, 2010; Christodoulou et al., 2016; Feldman et al., 2013]). Under this assumption, the following holds.

Lemma 1. The $\beta$-Fair GSP mechanism is $(1/2, (1 + \beta))$-semi-smooth, and the GSP-EFX mechanism is $(1/2, \beta)$-semi-smooth. Both mechanisms are individually rational at the equilibrium.

By the fact that $(\lambda, \mu)$-semi smoothness implies a $\frac{\lambda}{1 + \mu}$ PoC, we characterize the PoC of the mechanisms as follows.

Theorem 7. The price of composition with uncertainty of $\beta$-Fair GSP is $\text{PoC} = (2(2 + \beta))^{-1}$. This result shows that, even if the bidders are self-interested (i.e., they take decisions so as to minimize their own individual regret), our composite mechanism can guarantee group fairness, as well as convergence to a good equilibrium point. Analogously, it is possible to prove the following for GSP-EFX:

Theorem 8. The price of composition with uncertainty of GSP-EFX is $\text{PoC} = 1/8$.

6 Experimental Evaluation

We experimentally evaluate the quality of the equilibria emerging as the results of no-regret learning dynamics in which agents interact through the $\beta$-Fair GSP mechanism.

Regret minimization for the Bayesian setting For each bidder $i$, we consider a discrete set of bids $\mathcal{B}_i$. We focus on the partial-information setting, in which, at each time instant $t$, each bidder observes only the reward $u_i^b(b_i^t)$ associated to

\footnote{Our bounds on PoC can be adapted with minor modifications to the case in which bidders are $\delta$-conservative [Bhawalkar and Roughgarden, 2011], i.e., $b_i \leq \delta v_i, \forall i$. In particular, all price of composition factors are multiplied by an additional $\delta$ factor.}
its choice $b^*_i \in B_i$. This is in line with what happens in real-world sponsored search auctions, where advertisers do not observe competing bids (i.e., they cannot compute a counterfactual utility $u^*_i(b)$ for each $b \in B_i$), but can only observe the outcome associated to their decision $b^*_i$. We instantiate an external-regret minimizer $R_{t,v}$ for each bidder $i$ and $v \in V_i$. In practice, we use the EXP3 algorithm by [Auer et al., 2002] for each external regret minimizer. Then, we build a regret minimizer for the Bayesian bidder $i$ as follows: at each $t$, bidder $i$ observes their realized type $v^t \in V_i$ and selects $b^*_i \in B_i$ according to $R_{t,v^t}$. Then, after utility $u^*_i(b^*_i)$ is observed, only $R_{t,v^t}$ is updated. This simple procedure guarantees that $\lim_{T \to \infty} \sup_{t} R^t_i / T \leq 0$, which implies that the empirical frequency of play $\tilde{\sigma}^T$ of the dynamic converges almost surely in the limit to a Bayesian coarse correlated equilibrium (see, e.g., [Hartline et al., 2015, Lemma 10]).

**Experimental setting** We construct a real-world dataset through logs of a large Internet advertising company. We test our $\beta$-Fair GSP mechanism in an artificial environment where we have 20 advertisers, equally distributed among two groups, and competing for ad opportunities over a sequence of $T = 10^4$ auctions. For each $i$, we set $V_i = \{x/100 : x \in [100] \cup \{0\}\}$. Moreover, in order to be able to create a steep unbalance between the two groups, we let, for each $v \in V_i$, $\mathcal{F}(v) = \mathcal{F}_1(v_1) \cdot \ldots \cdot \mathcal{F}_n(v_n)$, with $\mathcal{F}_i \in \Delta V_i$ for each $i$. Then, for each $i \in I_h$ (i.e., advertiser $i$ belongs to the majority group), we artificially set value distributions to be such that $\mathcal{F}_i(1) = 1$, and $\mathcal{F}_i(v) = 0$ for each $v \in V_i \setminus \{1\}$. Each bidder $i \in I_t$ has a value distribution $\mathcal{F}_i$ built by normalizing the distribution of bids observed from real-world bidding data of a large Internet advertising company. We use bids as a proxy for true valuations of advertisers. Discount curves are computed by averaging and normalizing in $[0,1]$ real-world discount factors. In particular, we estimate discount curves on ads optimizing for two distinct conversion types, one per group. This models different per-group preferences on the one hand, and figures display the resulting mean and standard deviation. In particular, Figure 1–(Top-Right) reports the per-group social welfare at equilibrium. Finally, Figure 1–(Bottom) describes the fraction of the revenue which is lost due to compensation for advertisers and revenue losses incurred by the platform where this type of mechanisms could be viable in practice.

**7 Discussion and Future Works**

We proposed a class of composite mechanisms constructed through the combination of GSP with different fair division schemes. We show that these composite mechanisms yield group fairness guarantees for advertisers, and we characterized the costs which the platform has to incur. In the future, it would be interesting to study fairness guarantees in the space of valuations, and in the space of utilities. We already provided some preliminary results going in this direction (Theorem 4). Extending this to EFX presents non-trivial additional challenges, such as showing that there exists a monotone version of GECE. We leave this as a future research direction.
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