Can Abnormality be Detected by Graph Neural Networks?

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Abstract

Anomaly detection in graphs has attracted considerable interests in both academia and industry due to its wide applications in numerous domains ranging from finance to biology. Meanwhile, graph neural networks (GNNs) is emerging as a powerful tool for modeling graph data. A natural and fundamental question that arises here is: can abnormality be detected by graph neural networks? In this paper, we aim to answer this question, which is nontrivial. As many existing works have explored, graph neural networks can be seen as filters for graph signals, with the favor of low frequency in graphs. In other words, GNN will smooth the signals of adjacent nodes. However, abnormality in a graph intuitively has the characteristic that it tends to be dissimilar to its neighbors, which are mostly normal samples. It thereby conflicts with the general assumption with traditional GNNs. To solve this, we propose a novel Adaptive Multi-frequency Graph Neural Network (AMNet)[1], aiming to capture both low-frequency and high-frequency signals, and adaptively combine signals of different frequencies. Experimental results on real-world datasets demonstrate that our model achieves a significant improvement comparing with several state-of-the-art baseline methods.

1 Introduction

Detecting anomalies has attracted great research interests, with applications of great impact in numerous domains, such as telecommunication fraud detection [Yang *et al.*, 2021] and theft behavior detection [Hu *et al.*, 2020]. The nature of anomalies could exhibit themselves as inter-dependent, such as mining fake reviews in user-rating-product relations, recognizing the fraudsters on telecommunications network, and detecting money-laundering rings in trading networks. Graph data becomes ubiquitous as a powerful machinery to represent the inter-dependencies by the edges between the related instances. However, the unique characteristics of graph-based data bring additional challenges. The complex correlation in real-world datasets makes it challenging to identify the anomalies from graph objects. Detecting anomalies in graph data is a substantially more complex problem than anomaly detection in a non-relational feature space.

Recently, the advance of graph nerual networks (GNNs) has prompted various attempts to adopt GNNs for graph-based anomaly detection [Wang *et al.*, 2019; Liu *et al.*, 2019; Dou *et al.*, 2020; Liu *et al.*, 2020]. The general intuition of GNN-based anomaly detection is to leverage the expressive power of GNNs to learn node representations, aiming at distinguishing anomalous nodes from normal ones in the embedding space. Some recent studies [Wu *et al.*, 2019; Balcilar *et al.*, 2021], however, show that the expressive power of most GNN models is limited to only low-pass filters, which intensify low-frequency signals (more smooth signals) and suppress high-frequency signals (more oscillating signals). The nature of GNNs as low-pass filters succeed as most real-world networks in the real world follow the *homophily* assumption, where nodes with similar features tend to connect with each other [McPherson *et al.*, 2001]. However, this assumption may be weakened in networks containing anomalies: normal nodes still tend to share common features with their normal neighbors (low-frequency signals), whereas anomalies tend to have different features from the neighbors (high-frequency signals). Thus networks containing anomalies tend to mix both high-frequency and low-frequency local patterns (Figure 1).

We *claim* that for GNN-based anomaly detection, the direct adoption of most GNNs might not be optimal, because of the following reasons: 1) The low-pass property of GNNs essentially misaligns with the nature of networks containing anomalies. GNNs potentially smooth the difference between the representations of normal nodes and anomalous nodes by filtering out high frequency signals. As a result, the representation of anomalous nodes learned by GNNs could be indistinguishable and thus inevitably leading to sub-optimal performance for graph anomaly detection problem. 2) Most GNN-based methods apply GNNs with global filtering characteristic (low-/high-/band-pass) for all nodes of the network. However, the anomalous nodes and the normal ones could ex-

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[1]The code is available at https://github.com/zjunet/AMNet

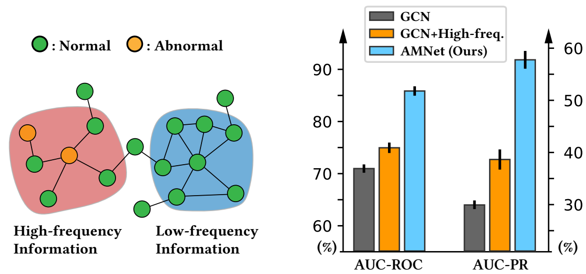

Figure 1: **Left:** An illustration of networks in graph anomaly detection. Anomalies tend to have different features from the neighbors (high-frequency information). Normal nodes tend to share common features with their normal neighbors (low-frequency information), **Right:** The performance of GCN, GCN with top 30% high-frequency graph signals and our proposed AMNet on graph anomaly detection the Yelp dataset.

ploit signals of different frequency bands, respectively. The lack of adaptivity for exploiting information of different frequencies for normal/anomalous nodes, pose a major obstacle in obtaining a more distinguishable representation.

To address the two aforementioned issues on GNN-based graph anomaly detection, one could expect a GNN model beyond a low-pass filter, which could exploit low-frequency information for normal nodes to retain the commonality, and focus on high-frequency information for anomalous ones to emphasize the difference. However, the correlation between information of different frequencies and anomaly detection task is usually very complex and agnostic. Thus we reason that for GNNs to achieve good performance on graph anomaly detection, one has to provide sufficient inductive bias that lets the model adaptively choose either low frequency, high frequency or both for distinguishing anomalous nodes. To achieve this goal, this paper proposes a novel Adaptive Multi-frequency filtering graph neural network for graph anomaly detection (AMNet). The core idea is that we fuse both low and high-frequency information adaptively to learn the node embedding for distinguishing the anomalous nodes. More specifically, instead of applying a global low-pass filter, AMNet develops a novel learnable multi-frequency filter group to effectively capture graph signals of both low frequency and high frequency simultaneously. The output signals of the filter group then convey information of multiple frequencies. In addition, we adopt a node-level attention mechanism to empower the model with ability of fusing information of different frequencies for each node substantially.

Our main contributions are summarized as follows:

- To the best of our knowledge, we are the first to identify and integrate the valuable high-frequency information from a spectral perspective in GNN-based anomaly detection. Leveraging signals beyond the low-frequency alleviates the problem that GNNs could produce confused representations as a low-pass filter for graph anomaly detection.
- We propose a novel adaptive multi-frequency GNN framework, AMNet, for graph anomaly detection, which captures information of different frequencies by our designed combinable graph filters. With the favor of the attention mechanism, different information can be adequately fused.
- Our extensive experiments on a series of datasets show that AMNet outperforms the state-of-the-art graph anomaly detection methods by an average improvement of 4.81% in AUC-ROC and 10.2% in AUC-PR.

2 Related Work

Graph-based anomaly detection. Graph structured data has been ubiquitous due to its superior capacity to model a wide range of real-world complex systems. Therefore, detecting anomaly in graphs has drawn increased interests in the community. Recently, with the advance that GNNs demonstrate its superior modeling power for graphs, various methods using GNNs have been proposed to solve the attributed network anomaly detection problem. For example, DOMINANT [Ding *et al.*, 2019] computes anomaly ranking scores using a deep GCN-based auto-encoder. GAS [Li *et al.*, 2019] also applies a GCN-based model to spam detection problems. Semi-GNN [Wang *et al.*, 2019] is a semi-supervised GNN model which leverages the hierarchical attention mechanism for fraud detection. Geniepath [Liu *et al.*, 2019] designs an novel aggregate method of GNNs to filter graph signals from neighbors of different hops away for detecting financial fraud. However, none of the aforementioned methods are aware of the limitations caused by adopting GNNs as a low-pass filter for graph anomaly detection. To the best of our knowledge, we are the first to identify the problem from a spectral perspective and attempt to alleviate the problem by our novel approach.

Graph neural networks. Graph neural network (GNN) models have achieved enormous success. Originally inspired by graph spectral theory, [Bruna *et al.*, 2014] first design learnable graph convolution operation in Fourier domain. [Defferrard *et al.*, 2016] further improve the efficiency by leveraging the Chebyshev approximation. The model proposed by [Kipf and Welling, 2017] simplifies the convolution operation by using a linear filter and becomes the most prevailing one. In addition, previous studies have shown that graph neural networks are vulnerable to abnormality [Xu *et al.*, 2022a; Xu *et al.*, 2022b]. Recently, spectral analysis on GNNs has attracted wide interests due to its valuable insight into the interpretablity and expressive power of GNNs. [Balcilar *et al.*, 2021] have attempted to show the majority of GNNs are limited only low-pass filter and argue the necessity of high and/or band-pass filters. However, the aforementioned methods do not take the unique nature of anomaly network mentioned in the introduction into consideration and we are the first to study how to adaptively integrate different signals in anomaly network with mixed frequency pattern.

3 Our Approach

3.1 Preliminaries

Problem definition. We focus on the semi-supervised graph anomaly detection on attributed graphs. Let $\mathcal{G} = (\mathcal{V}, \mathbf{A}, \mathbf{X})$ be an undirected graph, where $\mathcal{V}$ is the set of nodes. Each node $v_i \in \mathcal{V}$ has a corresponding feature vector $x_i \in \mathbb{R}^{d\times 1}$. $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_n]^T \in \mathbb{R}^{N\times d}$ denotes the feature matrix, and $\mathbf{A} \in \mathbb{R}^{N \times N}$ represents the adjacency matrix, where $\mathbf{A}_{ij} = 1$ denotes there is an edge between v_i and v_j else $\mathbf{A}_{ij} = 0$. $\mathbf{Y} \in \mathbb{R}^N$ is an indicator vector representing whether node v_i is anomalous or not. Given $\mathcal{G}$ and partial node labels, our goal is to learn a estimator to determine whether a given node is anomalous or normal.

Graph spectral filtering. According to theory of graph signal processing [Shuman *et al.*, 2013], one can define the graph filtering operation based on graph Fourier transformation. More specifically, let $\mathbf{L}$ be the symmetrically normalized Laplacian, with eigendecomposition $\mathbf{L} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$, where $\mathbf{\Lambda} = diag[\lambda_1, \cdots, \lambda_n]$ is the diagonal matrix of eigenvalues, a signal $x \in \mathbb{R}^n$ is filtered by a filter g as

$$g \star x = \mathbf{U} g(\mathbf{\Lambda}) \mathbf{U}^T x \tag{1}$$

Generally, a graph filter g is expressed by some spatial-localization parametrization methods such as cubic B-spline [Bruna *et al.*, 2014] and Chebyshev polynomial [Defferrard *et al.*, 2016], enjoying the advantanges of localization and linear complexity in the number of edges. Meanwhile, most existing GNNs adopt graph filters with a single frequency band.

3.2 Model Description

To empower GNNs with the capability of identifying abnormalities, we propose a novel framework Adaptive Multi-frequency Graph Neural Networks (AMNet). The general idea is to adaptively leverage both low- and high-frequency information.

To this end, we first design a *group* of K graph filters, each of which captures graph signals with different frequencies. Every node in the graph then obtains K signals, whose frequencies are controlled by learnable parameters of graph filters. As we have mentioned before, different nodes favor different frequency signals: normal nodes are more likely to be correlated with low frequency information, while high frequency signals manifest in anomalies who behave differently from the rest. To model the difference among signal preferences, we further propose to use a node-level attention mechanism for fusing the signals adaptively. Finally, the fused embeddings are taken for the classification task. Figure 2 illustrates how AMNet works. We next introduce the details of two major components of our model: *multi-frequency filter group* and *adaptive combination module*.

Capturing multiple frequency signals. We design the *multi-frequency filter group* in order to capture graph signals of different frequencies simultaneously. More specifically, the group consists of multiple trainable graph filters run in parallel, each of which is trained independently in an end-to-end manner. Formally, the multi-frequency filter group of K filters is denoted as $\{g_i\}_{i=1,\cdots,K}$. The graph signal $\mathbf{Z}_k$ filtered by the k-th filter can be generally defined as

$$\mathbf{Z}_k = \mathbf{U} g_k(\mathbf{\Lambda}) \mathbf{U}^T \mathbf{X} = \mathbf{U}\,\mathrm{diag}\left[g_k(\lambda_1), \ldots, g_k(\lambda_n)\right] \mathbf{U}^T \mathbf{X} \tag{2}$$

The graph filters can be implemented in several different ways. We will give one particular method based on the Bernstein polynomial parametrization in Section 3.3 and provide a theoretical analysis to explain why we choose it. Before that, we introduce how to fuse K signals produced by the multi-frequency filter group.

Combing signals adaptively. Through the graph filtering, now we have K specific signals $\{\mathbf{Z}_k\}$ with diverse frequency properties. Considering each node can focus on distinct frequency bands, we use the attention mechanism $att(\mathbf{Z}_1, ..., \mathbf{Z}_k)$ to learn the corresponding importance $(\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_k)$ as follows:

$$(\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_k) = att(\mathbf{Z}_1, ..., \mathbf{Z}_k) \tag{3}$$

where $\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_k \in \mathbb{R}^{n \times 1}$ are the attention values of n nodes with $\mathbf{Z}_1, ..., \mathbf{Z}_k$, respectively. More specifically, considering the node v_i with filtered signals $\mathbf{z}_k^i \in \mathbb{R}^{1 \times h}$(i.e., the i-th row of $\mathbf{Z_k}$), we have its attention scores as

$$\omega_k^i = \boldsymbol{q}^T \cdot \tanh\left(\mathbf{W}^Z {\mathbf{z}_k^i}^T + \mathbf{W}^X \mathbf{x}_i\right) \tag{4}$$

where $\mathbf{W}^Z \in \mathbb{R}^{h' \times h}$ and $\mathbf{W}^X \in \mathbb{R}^{h' \times d}$ indicate the weight matrices and $\boldsymbol{q} \in \mathbb{R}^{h' \times 1}$ is the shared attention vector. The final attention weights of node v_i are obtained by normalizing the attention values ω_k^i with softmax function as

$$\alpha_k^i = \mathrm{softmax}\left(\omega_k^i\right) = \frac{\exp\left(\omega_k^i\right)}{\sum_k \exp\left(\omega_k^i\right)} \tag{5}$$

Larger α_k^i implies that the node v_i favors the k-th filter's frequency band. Defining $\boldsymbol{\alpha}_k = diag\left([\alpha_k^i]\right)$, we have the final embedding $\mathbf{Z}$ by combining the filtered signals:

$$\mathbf{Z} = \sum_k \boldsymbol{\alpha}_k \mathbf{Z}_k \tag{6}$$

From another aspect, we find that AMNet actually applies a personalized graph filter $\hat{g}^i$ for each node v_i. In particular, the final embedding $\mathbf{z}^i$ of v_i can be equivalently expressed as

$$\begin{aligned} \mathbf{z}^i &= \sum_k \alpha_k^i \mathbf{z}_k^i = \mathbf{U} \sum_k \alpha_k^i g_k(\mathbf{\Lambda}) \mathbf{U}^T \mathbf{x}_i \\ &= \left(\sum_k \alpha_k^i g_k\right) \star \mathbf{x}_i \end{aligned} \tag{7}$$

where $\hat{g}^i = \sum_k \alpha_k^i g_k$ is the linear combination of filter group with attention weights. Thus AMNet provides the adaptivity for each node to learn its own graph filter.

3.3 The Choice of Graph Filter

Combinable graph filter parametrization. In this section we discuss the implementation of graph filters in the multi-frequency filter group of AMNet. From Equation 7, we see that AMNet adaptively learns a filter for each node by combining the filters in the multi-frequency filter group. However, designing a graph filter suitable for combining is non-trivial because most existing graph filters, such as cubic B-spline or Chebyshev polynomial, face following two challenges: 1) Most existing graph filters may derive negative spectral functions, which leads to complex combination result according to graph signal processing theory. 2) The frequency characteristic of a filter is scale-invariant. It means that the filters

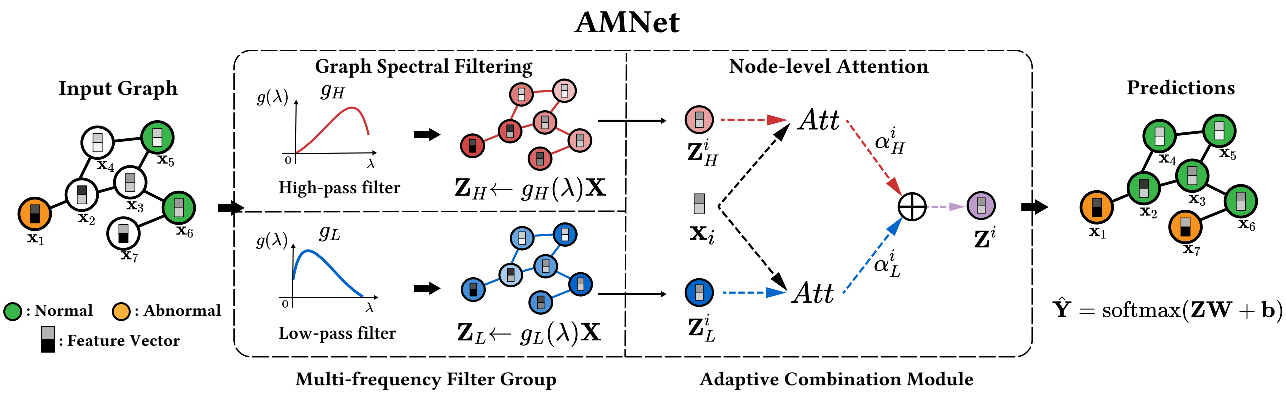

Figure 2: Illustration of AMNet with multi-frequency filter group of two filters. The raw node features are filtered by a learned high-pass filter g_H and a learned low-pass filter g_L, in which a node aggregates its neighborhood information to capture the high-frequency signal Z_H^i and low-frequency signal Z_L^i respectively. Then the attention layer adaptively combines Z_H^i and Z_L^i to obtain the final representation Z^i, which is passed to a feedforward network to get the prediction result in anomaly detection task.

with larger scale will be dominant over the ones with smaller scale, which diminishes the adaptivity to learning filter focusing on different frequencies.

To address aforementioned issues, we introduce the *restricted* Bernstein polynomial parametrization to approximate filters in our multi-frequency group.

Definition 3.1 (Restricted Bernstein polynomial parametrization)**.** Define the graph filter to be parametrized by Berstein polynomials with coefficients θ_m that are restricted in the interval $[0, 1]$:

$$h_\theta(\lambda) = \sum_{m=0}^{M} \theta_m b_m^M(\lambda) = \sum_{m=0}^{M} \theta_m \binom{M}{m} \lambda^m (1-\lambda)^{M-m} \quad (8)$$

Here $b_m(\lambda) = \binom{M}{m}\lambda^m(1-\lambda)^{M-m}$ is the m-th Bernstein basis of order M, and $\theta \in [0,1]^M$ is a learnable vector of polynomial coefficients. Note that $b_m^M(\lambda) \geq 0$ for $\lambda \in [0,1]$, thus avoiding phase shift. Besides, due to $\sum b_m^M(\lambda) = 1$ and coefficients θ are restricted in $[0,1]$, $h(\lambda)$ ranges in $[0,1]$. Then all filters of the filter group share uniform scales. Therefore our method avoid the aforementioned limitations for being combined, while enjoying the same advantages of existing methods such as spatial localization and linear learning complexity. Note that [He *et al.*, 2021] also attempts to show the advantages of Bernstein polynominal parametrization on general graph filter modeling, while this paper focuses on developing unified and combinable graph filters.

Expressive power analysis. We next provide a theoretical analysis to demonstrate that our parametrization enables the filter group to capture multiple frequency graph signals.

Theorem 1. *The restricted Bernstein polynomial parametrization can equivalently express arbitrary graph filter with continuous frequency response function.*

Proof. Let us define U to be the set of restricted Bernstein polynomial that are all in $[0,1]$. We further define V to be the set of polynomials which map the interval $[0,1]$ into $(0,1)$. According to [Qian *et al.*, 2011], we have $V \subset U$. Because the frequency profile of a filter is only determined by the relative absolute value of amplitude, arbitrary frequency response function can be transformed into an equivalent one that map $[0,1]$ to $(0,1)$. According to the Weierstrass Approximation Theorem [Jeffreys and Jeffreys, 1999], let f be a continuous function that map $[0,1]$ to $(0,1)$, for any $\epsilon > 0$, there exists a polynomial function $p \in V$ such that for all x in $[0,1]$, we have $|f(x) - p(x)| < \epsilon$.

Thus, our restricted Bernstein polynomial parametrization could express filters with diverse frequency properties, e.g., low/band/high-pass filters.

3.4 Objective Function

Here we describe the general training objective of AMNet.

Margin-based constraint on attention. Intuitively, to enhance the difference, anomalous nodes need to exploit more high-frequency information. Here we apply a constraint on attention training to capture the intuition which encourages that anomalous nodes and normal nodes focus on different filters, respectively. For example, assuming two filters $\{g_L, g_H\}$ namely with attention value $\{\alpha_L, \alpha_H\}$, the margin-based constraint on attention $\mathcal{L}_a$ can be defined as

$$\mathcal{L}_a = \sum_i max\left(0, r_i\left(\alpha_L^i - \alpha_H^i\right) + \zeta\right) \quad (9)$$

where ζ is slack variable which controls the margin between attention values, and $r_i = 1$ when $\mathbf{Y}_i = 1$, else $r_i = -1$.

Optimization objective. We use the output embedding $\mathbf{Z}$ in Eq. (6) for semi-supervised classification. Suppose the $\hat{\mathbf{Y}} \in \mathbb{R}^{N\times 2}$ denotes the probability of nodes belonging to the anomalous and the normal. Then $\hat{\mathbf{Y}}$ can be calculated with a linear transformation and a softmax function:

$$\hat{\mathbf{Y}} = \text{softmax}(\mathbf{ZW} + \mathbf{b}) \quad (10)$$

Then we have the overall objective function by combining the classification task and constraint on attention:

$$\mathcal{L} = \mathcal{L}_c + \beta\mathcal{L}_a \quad (11)$$

where $\mathcal{L}_c$ represents the loss derived from node classification (e.g, cross entropy) and $\beta \geq 0$ is the parameter that weights the constraint item $\mathcal{L}_a$.

4 Experiments

In this section, we perform evaluations on the effectiveness of (AMNet) under four real-world datasets. More specifically, we aim to answer the following research questions:

- **RQ1:** How does AMNet perform against state-of-the-art baselines on real-world graph anomaly detection tasks?
- **RQ2:** Can AMNet effectively capture both low-frequency and high-frequency information, and fuse both adpatively?
- **RQ3:** Do the components of the AMNet framework work as designed? And how do different modules contribute to the performance of AMNet.

4.1 Experimental Setup

We adopt four real-world datasets that have been used in the previous research to evaluate AMNet. Characteristics of these datasets are summarized in Table 2.

- **Yelp** [Rayana and Akoglu, 2015]: It contains reviews for restaurants in several states of the U.S.. The links are created between two reviews if they are posted by the same user. Our goal is to detect fake reviews here.
- **Elliptic** [Weber *et al.*, 2019]: It is a bitcoin transaction network, where nodes are transactions and edges are the flows of Bitcoin currency. We train and apply our model to predict illicit transcations.
- **FinV** [Yang *et al.*, 2019]: It is a social network provided by FinVolution group, one of the leading fintech platforms in China. Based on the social relationships between users, we aim to predict financial frauds.
- **Telecom** [Yang *et al.*, 2021]: It is a mobile communication network anonymized and provided by China Telecom, the major mobile service providers in China. Our task is to predict telemarketing frauds.

Comparison methods. We compare AMNet with two categories of baselines: 1) general GNNs model, including GCN [Kipf and Welling, 2017], GraphSAGE [Hamilton *et al.*, 2017], GAT [Veličković *et al.*, 2018] and GIN [Xu *et al.*, 2018]. 2) GNN-based anomaly detection model, including DOMINANT [Ding *et al.*, 2019], GeniePath [Liu *et al.*, 2019], GraphConsis [Liu *et al.*, 2020] and CARE-GNN [Dou *et al.*, 2020], which are introduced in Sec 2. Some other relevant GNN-based models like GAS [Li *et al.*, 2019] and Semi-GNN [Wang *et al.*, 2019] are not included in our experiment considering their less effectiveness and efficiency.

Evaluation metrics. We adopt two widely-used and complementary metrics: the Area Under Receiver Operating Characteristic (AUC-ROC) and AUC-PR [Dou *et al.*, 2020; Ding *et al.*, 2021]. The latter pays more attention to the ranking of anomalies than that of normal samples.

Implementation details. All baseline methods are initialized with the same parameters suggested by their official codes and have been carefully fine-tuned. In addition, for baselines that are able to handle heterogeneous graphs, we leverage the possible multi-relation information of the input graph. The filter number K of AMNet is set to 2. For all methods, we report the average results of 10 independent runs.

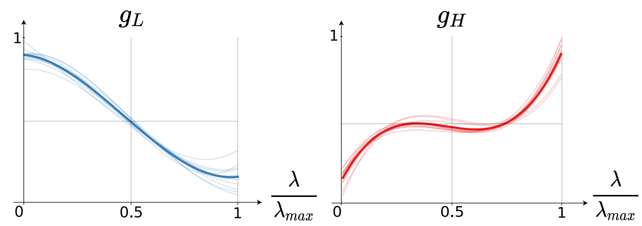

Figure 3: Filters g_L and g_H learned from Elliptic by AMNet.

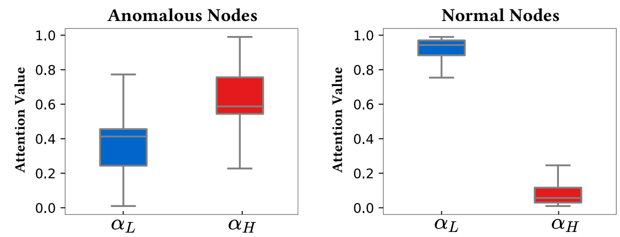

Figure 4: Analysis of attention distribution.

4.2 Effectiveness Results (RQ1)

Table 1 presents the experimental results. Overall, we see that the proposed AMNet outperforms all other baselines in all the datasets. More specifically, it achieves an improvement of 4.81% on AUC-ROC, and 10.2% on AUC-PR. Among all the baselines, DOMINANT, as the state-of-the-art unsupervised gnn-based method, performs the worst due to the lack of supervision. The highlighted results in the table are from AMNet, which is able to exploit both low and high-frequency information, keeping advantage over all the general GNNs and GNN-based graph anomaly detection comparison methods consistently.

4.3 Graph Filters and Adaptive Capability (RQ2)

Visualization of graph filters. To gain a deeper insight into our model, we plot the filter group in Figure 3. It illustrates two filters g_L and g_H learned from real-world dataset Elliptic. We see that g_L exhibits low-pass property while g_H exhibits high-pass property. This phenomena is consistent with the key idea that both low frequency and high frequency information contribute to anomaly detection. It further suggests that AMNet can learn filters that capture multiple frequency signals in an end-to-end manner.

Analysis of attention distribution. We conduct the attention distribution analysis on public real-world dataset Elliptic in Figure 4. We see that normal nodes tend to have dominant attention value on low-pass filter g_L, and therefore preserve stronger low-frequency signals, whereas anomalous nodes emphasize high-frequency signals. This difference effectively sharpens the contrast between normal nodes and anomalous nodes, thus making it easier for the anomaly to be captured by the model. In summary, the experiment demonstrates that our proposed AMNet is able to adaptively adopt graph signals with suitable frequencies for different nodes.

Analysis of attention trend. We next further analyze the changing trends of attention values during the training process in Figure 5, where **x**-axis is the epoch and **y**-axis is the

Methods \ Dataset	Yelp AUC-ROC	Yelp AUC-PR	Elliptic AUC-ROC	Elliptic AUC-PR	FinV AUC-ROC	FinV AUC-PR	TeleCom AUC-ROC	TeleCom AUC-PR
Graph Neural Networks								
GCN	70.97 ± 0.8	29.93 ± 0.6	84.57 ± 0.4	33.17 ± 0.3	64.64 ± 1.1	9.04 ± 0.3	76.69 ± 1.2	59.85 ± 1.2
GAT	74.68 ± 1.3	35.44 ± 1.1	86.03 ± 1.5	56.81 ± 0.9	65.97 ± 1.5	9.44 ± 0.2	79.15 ± 1.8	64.43 ± 0.5
GraphSAGE	73.65 ± 0.8	36.11 ±0.7	85.28 ± 2.1	55.29 ± 1.3	72.13 ± 1.9	16.54 ± 0.9	76.02 ± 1.2	64.07 ± 0.7
GIN	68.50 ± 1.3	31.22 ± 1.3	85.11 ± 1.3	37.34 ± 1.3	67.44 ± 1.3	20.02 ± 1.3	76.51 ± 1.3	59.48 ± 1.3
GNN-based Graph Anomaly Detection Models								
DOMINANT	49.32 ± 0.8	15.58 ± 0.3	16.21 ± 0.3	5.48 ± 0.1	64.59 ± 1.1	8.28 ± 0.3	55.43 ± 0.7	15.68 ± 0.3
GeniePath	75.89 ± 1.8	35.86 ± 0.5	83.14 ± 1.3	44.37 ± 0.8	72.27 ± 1.2	18.43 ± 0.7	83.73 ± 0.7	64.25 ± 0.3
GraphConsis	70.40 ± 1.3	27.02 ± 0.8	86.14 ± 1.1	62.04 ± 1.2	72.82 ± 1.2	27.07 ± 1.0	77.91 ± 1.5	61.82 ± 0.5
CARE-GNN	78.41 ± 1.5	38.90 ± 1.1	85.84 ± 1.2	49.81 ± 1.2	70.31 ± 1.8	23.61 ± 0.3	81.02 ± 0.7	68.06 ± 1.6
AMNet	**85.85 ± 1.1**	**57.77 ± 0.9**	**88.52 ± 1.0**	**74.62 ± 1.4**	**78.38 ± 1.8**	**29.31 ± 0.8**	**87.62 ± 1.3**	**75.18 ± 0.9**

Table 1: Performance of anomaly detection(%).

Dataset	**Yelp**	**Elliptic**	**FinV**	**Telecom**
# nodes	45,954	46,564	11,053	340,751
# edges	3,846,979	73,248	25,944	3,150,996
# features	32	93	8	261
Abnormal(%)	14.53	9.76	4.46	4.62

Table 2: The characteristics of the real-world datasets.

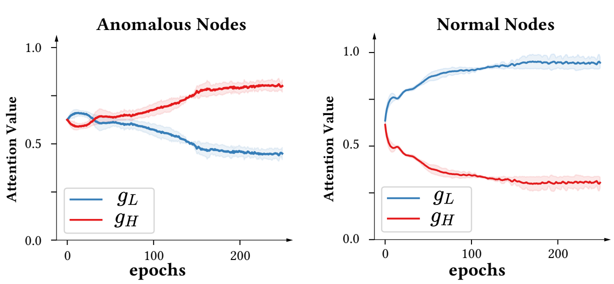

Figure 5: The attention changing trends w.r.t epochs. Y-axis shows attention value with standard deviation over 10 independent runs.

average attention value of nodes. We can see that the attention value for high-pass filter g_H of anomalous nodes gradually increases, while the attention value for low-pass filter g_L keeps decreasing. Meanwhile, the attention value of normal nodes goes in the opposite way. This trend is consistent with the observation in Figure 4, and suggests that AMNet can learn the contribution of different frequency components gradually.

4.4 Ablation Analysis (RQ3)

In this section, we compare AMNet with its two variants on Elliptic and Yelp to validate the effectiveness of the designed modules.

- **AMNet-Cheb**: it replaces the Bernstein polynomial by the Chebyshev polynomial for graph filters in AMNet.
- **AMNet-AC**: it removes the margin-based constraint on attention $\mathcal{L}_a$ in AMNet.

From the results in Table 3, we see that AMNet notably outperforms AMNet-Cheb by an average improvement of 14% on AUC-PR, which is consistent with the analysis in Sec 3.3 on the advantages of our filter over general graph filters for combining different graph signals. And we also find that introducing the attention constraint improves the performance by guiding the anomalous nodes and normal nodes paying more attention on signals of different frequencies, respectively.

Dataset	Elliptic AUC-ROC	Elliptic AUC-PR	Yelp AUC-ROC	Yelp AUC-PR
AMNet-Cheb	86.11	60.44	81.70	44.80
AMNet-AC	87.92	72.27	85.03	56.78
AMNet	88.52	74.62	85.85	57.77

Table 3: The results(%) of ablation study on Elliptic and Yelp.

5 Conclusion

In this paper, we study the problem of can abnormality be detected by graph neural networks. We answer this issue by exploring the nature of GNNs, and analyzing the characteristics of graph signals in anomaly detection scenarios. We conclude that most existing GNNs only consider graph signals with single frequency, whereas abnormality and normal nodes favors different frequency bands. Therefore, to further enhance GNNs' performance in anomaly detection, we proposed AMNet, a novel Adaptive Multi-frequency Graph Neural Network, aiming to adaptively combine multiple frequency signals for each node. Experimental results demonstrate that our model achieves a significant improvement comparing with several state-of-the-art baseline methods.

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