Diffusion Incentives in Cooperative Games

Yao Zhang
ShanghaiTech University
zhangyao1@shanghaitech.edu.cn

Abstract

We study a cooperative game setting where we want to gather more players through their social connections. Social connections can be modeled as a graph, and initially, only a subset of the players are in the game. We want to introduce diffusion incentives in such a cooperative game, i.e., incentivize the players to use their connections to invite more players to join the game. Our goal cannot be achieved by existing classical solutions, such as the Shapley value. Hence, to combat this problem, we have already proposed a solution called weighted permission Shapley value. Under this solution, for each player, inviting all of her neighbors is a dominant strategy in all monotone games. As one special application of the diffusion cooperative game, we also consider the diffusion incentives in query networks and the weighted permission Shapley value successfully characterizes the solution to the query network. Furthermore, we also characterize a Sybil-proof solution to the query network called the double geometric mechanism.

1 Introduction

The cooperative game is a classical topic in game theory, which focuses on reward schemes for a fixed coalition of players [Young, 1985; Peleg and Sudhölter, 2007]. Here we focus on monotone games, i.e., more players will create more value, and all players will collaborate in the grand coalition [Driessen, 2013]. Since more involved players means more created values, we consider a new problem of gathering additional participants who are not already in the game.

To gather more participants, utilizing social connections and incentivizing players to invite their social neighbours is a new trend in the field of mechanism design [Zhao, 2021]. Previous literature mainly focused on non-cooperative games. For example, Li et al. [2017; 2022] proposed the very first diffusion mechanisms to attract buyers in auctions via social networks. We share a similar motivation in which players are connected and one cannot become aware of and participate in the collaboration without being invited by their participating neighbors. In particular, we aim to create a reward distribution mechanism that incentivizes players in the current coalition to ask their neighbors to collaborate.

Classical cooperative game solutions will not work in this new environment because there may be competition among inviters and invitees to share the reward. For example, Shapley value, one of the most well-known solutions, which computes the average marginal contributions of a player to join all possible sub-coalitions [Shapley, 1953], fails to give diffusion incentives in a simple counterexample. Consider two players, and assume that, initially, only one of them is in the game, and she can invite the other player to the game. However, the other player can provide the same contribution as the initial player. In this example, the Shapley value of the initial player after inviting the other player is only half of what it would be if the initial player kept herself alone in the game.

To combat this problem, a very first idea called layered Shapley value [Zhang et al., 2020] was introduced. We then have modelled the diffusion cooperative games and proposed a solution called the weighted permission Shapley value [Zhang and Zhao, 2022] inspired by the permission structure [Gilles et al., 1992] and the weighted Shapley value [Kalai and Samet, 1987]). The new model can also describe the classical model of query networks [Kleinberg and Raghavan, 2005], and the new solution can well explain the winning solution in the DARPA 2009 Red Balloon Challenge [Pickard et al., 2011], which is a classic application of the query network. We also invented and characterized the Double Geometric Mechanism, a Sybil-proof mechanism for the deployment of query networks [Zhang et al., 2021].

The completed research described above, as well as the target of the remaining work, will be briefly introduced in the following sections.

2 Contributions

2.1 Completed Research

We model the social connections of players as a network presented by a graph. Each edge indicates that one player can invite the other. There is a special player set called the initial set, who are in the game/coalition initially without invitation, and the invitations have to start from the initial players. The social connections, namely the set of neighbours, are private

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information of the players, and we treat them as the players’
type. Then the invitation process can be mathematically con-
sidered as players’ reporting their types once rather than an
iterated process. According to the reported types, we can in-
duce a directed graph that indicates how players invite each
other. Since we actually consider a diffusion model, only the
set of players who can be reached by at least one player from
the initial set in the directed graph can contribute to the game
and share the reward. In practice, this means that the others
will not be informed about the game at all.

The main goal is to establish a reward distribution mecha-
nism that efficiently distributes the value created by all the
involved players. The key property we emphasize here is
diffusion incentive compatibility, which requires that inviting
all neighbours is a dominant strategy for all players. A first
idea for this purpose is to distribute the marginal contribution
in each layer, which is layered Shapley value, but this can-
not provide positive incentives to players for their invitation.
We then utilize a permission structure to represent the priori-
ties between an inviter and an invitee, and assign different
weights to them to control the importance of their priorities.
Finally, we have a solution called weighted permission Shap-
ley value. We proved that if a player’s weight only depends
on her shortest distances to initial players and is monotone
non-decreasing, then the solution satisfies diffusion incentive
compatibility.

As an application of the diffusion cooperative game, the
query incentive network, where a requester tries to find an an-
swer to a specific problem by diffusing the request in the net-
cwork, can also be solved by our solution. We have shown that
the famous solution given by the winning team from MIT in
the DARPA 2009 Red Balloon challenge can be represented
by a special case of our solution. More importantly, we have
also found that the weighted permission Shapley value is the
only solution to the query incentive network that is anony-
mous, strongly individually rational, and efficient.
Sybil-proofness is another important property in our set-
ing, which ensures that players cannot cheat for more re-
wards by creating fake identities. We have already proposed a
solution called double geometric mechanism that is diffusion
incentive compatible and Sybil-proof for the special case of
the query network. This mechanism cannot be represented
by the weighted permission Shapley value since it is not effi-
cient.

2.2 Ongoing Work and Future Directions
Based on the current results, we are going to characterize
all efficient and diffusion incentive compatible solutions for
diffusion cooperative games. This will give us a more gen-
eral solution and a full understanding to diffusion cooperative
games. Stability, such as core, is another future direction in
our new setting, which prevents sub-coalitions from leaving
the game. With the constraint of social connections, we may
consider the case where only connected players can leave to-
gether. Finally, we have only solved Sybil-proofness in query
networks. Hence, we also want to achieve Sybil-proofness in
all diffusion cooperative games.

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