Parameterised Gradual Semantics Dealing with Varied Degrees of Compensation

Dragan Doder[1] , **Leila Amgoud**[2] and **Srdjan Vesic**[3]
[1]Utrecht University
[2]CNRS – IRIT
[3]CRIL – CNRS – Univ. Artois
d.doder@uu.nl, leila.amgoud@irit.fr, vesic@cril.fr

Abstract

Compensation is a strategy that a semantics may follow when it faces dilemmas between quality and quantity of attackers. It allows several weak attacks to compensate one strong attack. It is based on compensation degree, which is a tuple that indicates (i) to what extent an attack is weak and (ii) the number of weak attacks needed to compensate a strong one. Existing principles on compensation do not specify the parameters, thus it is unclear whether semantics satisfying them compensate at only one degree or several degrees, and which ones. This paper proposes a parameterised family of gradual semantics, which unifies multiple semantics that share some principles but differ in their strategy regarding solving dilemmas. Indeed, we show that the two semantics taking the extreme values of the parameter favour respectively quantity and quality, while all the remaining ones compensate at some degree. We define three classes of compensation degrees and show that the novel family is able to compensate at all of them while none of the existing gradual semantics does.

1 Introduction

An argumentation system is made of an argumentation graph and a semantics. The former is a graph whose nodes are arguments, and the edges represent attacks between arguments. Each argument may have a basic weigh, which represents its intrinsic strength, for example the degree of trustworthiness of the argument's source [da Costa Pereira *et al.*, 2011]. A semantics is a formal method that evaluates the strength of every argument in the graph. This evaluation is crucial since it determines the outcomes of the system. Consequently, a plethora of semantics have been proposed in the literature. The very first ones are the *extension* semantics introduced in [Dung, 1995]. They calculate sets of jointly acceptable arguments, called extensions, which they use to classify arguments as sceptically accepted, credulously accepted and rejected. In [Cayrol and Lagasquie-Schiex, 2005] another type of semantics, called *gradual*, has been introduced with the purpose of refining the above cited semantics. Examples of such semantics are Trust-based [da Costa Pereira *et al.*, 2011], $h-$Categorizer [Besnard and Hunter, 2001; Pu *et al.*, 2014], social semantics [Leite and Martins, 2011].

Due to the large number of semantics in the literature, formal properties, or *principles*, have been proposed to analyse the underpinnings of each semantics and to compare pairs of semantics in [Amgoud *et al.*, 2017]. Three of those properties, namely *Cardinality Precedence* (CP), *Quality Precedence* (QP) and *Compensation*, describe three incompatible strategies that a semantics may follow whenever it faces a dilemma when comparing an argument that is attacked by a *few strong attackers* with another argument that is attacked by *numerous weaker attackers*.

Consider the argumentation graph $\mathbf{G}_1$ in Table 1 on a multiple criteria decision problem where A, B stand for buying the houses h_1 and h_2 respectively, and argument Z for the house h_2 being very expensive. Furthermore, let Y_1, X_1, X_2 stand for: the house h_1 is far from the school s, there is a metro station just in front of h_1, there is another school in the neighbourhood of h_1. Assume that the remaining arguments Y_2, Y_3 refer to other criteria in the same spirit as Y_1 and they are attacked by $X_3, \ldots, X_6$. The arguments Y_1, Y_2 and Y_3 are attacked while Z is not. Hence, a reasonable semantics would declare Z as stronger than any Y_i, $i = 1, 3$. It follows that A has more attackers than B but its attackers are weaker than the sole attacker of B. The question is which of the two arguments A and B is stronger. A semantics which follows (CP) (resp. (QP)) would declare A weaker than B (resp. A stronger than B). A semantics satisfying Compensation might rather declare the two arguments equally strong.

Compensation is clearly based on a *degree*, which is a tuple that indicates: i) to what extent an attacker is weak, ii) the number of weak attackers needed to compensate a strong one. However, its corresponding principles in [Amgoud *et al.*, 2017; Amgoud *et al.*, 2022] do not refer to any compensation degree. They state that in case of dilemma, a semantics *may* compensate without specifying when this may happen, which makes them the less informative properties in the literature. It has been shown in [Amgoud *et al.*, 2022] that there are two gradual semantics in the literature that satisfy the property: Trust-based [da Costa Pereira *et al.*, 2011] and weighted h-categorizer [Amgoud *et al.*, 2017]. However, no insight is given on their degree(s) of compensation.

In [Amgoud *et al.*, 2016], the authors focused on a specific class of degrees of compensation, and proposed a parame-

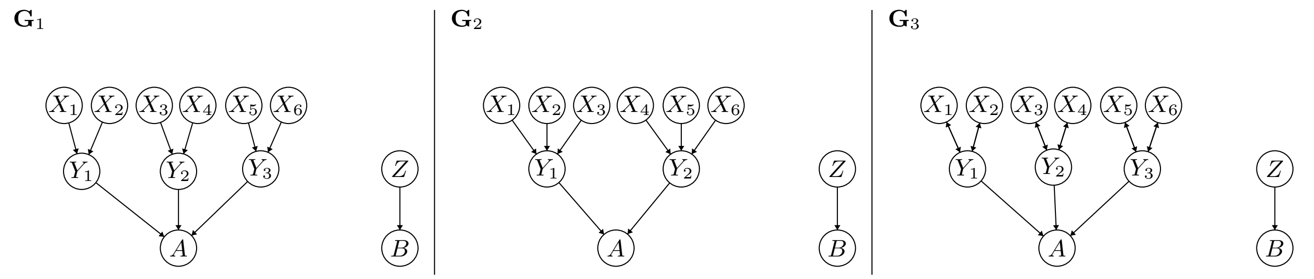

Table 1: Examples of Argumentation Graphs

terised family of gradual semantics that extend h-Categorizer [Pu *et al.*, 2014]. The aim is that every value of the parameter defines a semantics that compensates at a degree from the class. However, it turns out that the semantics satisfy the principle only when the number of attackers of A (in $\mathbf{G}_1$) is fixed to be greater than the number of attackers of each Y_i. Hence, compensation is possible in this graph while it is not possible in $\mathbf{G}_2$. This seems ad-hoc since there is no intuitive reason for such a restriction. Furthermore, it is not clear what semantics do in such graphs.

This paper investigates gradual semantics that satisfy compensation at various degrees. Its contributions are five-fold:

1. It proposes a large parameterised family of gradual semantics. The family is based on a parameter α, which takes values from the interval $(0, +\infty)$, each of which leads to a different semantics. The smaller its value, the bigger the influence of the number of attacks. Conversely, the greater its value, the bigger the influence of the quality of attackers. Thus, α determines the strategy that a semantics follows to solve dilemmas.

2. It shows that the family is the first which *unifies* multiple semantics that share some principles but differ in their behaviour regarding solving dilemmas. Indeed, it proves that when α approaches 0, the corresponding semantics satisfies (CP) while when it approaches $+\infty$, the semantics satisfies (QP). All the other values of α guarantee (Compensation) with different degrees.

3. It proves that the new family encompasses the semantics from [Amgoud *et al.*, 2016] which compensate.

4. It discusses three classes of compensation situations, i.e., three parameterised compensation degrees, and shows that the new semantics satisfies all of the variants of Compensation at some degree.

5. It shows that neither of the existing semantics satisfies all the variants of compensation, meaning that the new family has the **greatest compensatory power**.

The paper is organised as follows: Section 2 recalls the background, Section 3 defines the novel family of semantics, Section 4 introduces three classes of compensation degrees, Section 5 analyses existing gradual semantics, and the last section is devoted to some related work and concluding remarks. All proofs are provided in supplementary material.

2 Background

In the paper, we are interested in *weighted argumentation graphs*. Their nodes are *arguments*, each of which has a *basic weight*, and edges represent *attacks* (i.e., conflicts) between arguments. For the sake of simplicity, weights of arguments are elements of the unit interval $[0, 1]$. The greater the value, the stronger the argument.

Definition 1 (Weighted Graph)**.** *A* weighted argumentation graph *is a tuple* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$*, where* $\mathcal{A}$ *is a non-empty finite set of arguments,* $w : \mathcal{A} \to [0, 1]$*, and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*. Let* $\mathtt{AG}$ *denote the set of all weighted graphs.*

For $a, b \in \mathcal{A}$, $w(a)$ is the basic weight of a, $(a, b) \in \mathcal{R}$ means *a attacks b*.

Notations: Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$ and $a \in \mathcal{A}$. We denote by $w \equiv 1$ the case where all arguments have a weight equal to 1 and we call such a graph *flat*. $\mathtt{Att}_{\mathbf{G}}(a)$ denotes the set $\{b \in \mathcal{A} \mid (b, a) \in \mathcal{R}\}$ of *direct attackers* of a in $\mathbf{G}$. Let $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle \in \mathtt{AG}$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. $\mathbf{G} \oplus \mathbf{G}' = \langle \mathcal{A} \cup \mathcal{A}', w'', \mathcal{R} \cup \mathcal{R}' \rangle \in \mathtt{AG}$ such that $\forall x \in \mathcal{A}$ (resp. $x \in \mathcal{A}'$), $w''(x) = w(x)$ (resp. $w''(x) = w'(x)$).

A gradual semantics is a function that assigns a value from a given ordered scale to each argument. Different scales can be used, but as in most papers on gradual semantics, we use the unit interval of reals $[0, 1]$ with the interpretation: the greater the value, the stronger the argument.

Definition 2 (Gradual Semantics)**.** *A* gradual semantics *is a function* $\mathbf{S}$ *assigning to any* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$ *a weighting* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}$ *on* $\mathcal{A}$*, i.e.,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}} : \mathcal{A} \to [0, 1]$*. For any* $a \in \mathcal{A}$*,* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a)$ *is the* strength *of* a *in* $\mathbf{G}$ *under semantics* $\mathbf{S}$.

Let us now discuss the possible strategies that a semantics may follow when it faces a conflict between the *strength* and the *quantity* of attackers. Three strategies have been identified in [Amgoud *et al.*, 2017; Amgoud *et al.*, 2022], they depend on whether quantity or quality is more important. The first, called *Cardinality Precedence*, states that a great number of attackers has more effect on an argument than just a few. In the graph $\mathbf{G}_1$, the argument A would be weaker than B.

Principle 1 (Cardinality Precedence)**.** *A semantics* $\mathbf{S}$ *satisfies* cardinality precedence *(CP) iff, for any* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$*,* $\quad \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$*,*
- $|\{x \in \mathtt{Att}_{\mathbf{G}}(a) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) > 0\}| < |\{y \in \mathtt{Att}_{\mathbf{G}}(b) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > 0\}|$*,*

	A	B	Y_1	Y_2	Z	X_1	X_2	X_3	X_4	X_5	X_6
$\alpha = 2$	0.659	0.5	0.366	0.366	1	1	1	1	1	1	1
$\alpha = 0.5$	**0.50**	**0.50**	0.250	0.250	1	1	1	1	1	1	1

Table 2: The values of arguments in $\mathbf{G}_2$ under semantics $\mathbf{S}_\alpha$, where $\alpha \in \{0.5, 2\}$.

	A	B	Y_1	Y_2	Y_3	Z	X_1	X_2	X_3	X_4	X_5	X_6
$\alpha = 2$	0.527	0.5	0.518	0.518	0.518	1	0.659	0.659	0.659	0.659	0.659	0.659
$\alpha = 0.5$	0.348	0.5	0.389	0.389	0.389	1	0.616	0.616	0.616	0.616	0.616	0.616

Table 3: The values of arguments in $\mathbf{G}_3$ under semantics $\mathbf{S}_\alpha$, where $\alpha \in \{0.5, 2\}$.

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Quality Precedence gives more importance to the strength of attackers. In the graph $\mathbf{G}_1$, the argument A would be stronger than B.

Principle 2 (Quality Precedence)**.** *A semantics* $\mathbf{S}$ *satisfies* quality precedence *(QP) iff, for any* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$*, for all* $a, b \in \mathcal{A}$*, if*

- $w(a) = w(b)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$,
- $\exists y \in \mathtt{Att}_{\mathbf{G}}(b)$ *such that* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > 0$ *and* $\forall x \in \mathtt{Att}_{\mathbf{G}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

The third strategy is based on compensation. Its basic idea is that several weak attacks have the same impact as few strong attacks on an argument. For instance, in the graph $\mathbf{G}_1$, the three *weak* attackers of A may compensate the *strong* attacker of B, and thus A is as strong as B. Compensation is very specific compared to other principles. It is clearly based on a degree which is a tuple indicating i) to what extent an attack is weak, and ii) the number of weak attackers needed to compensate a strong one. An exact definition of compensation would fix the degree, however this is not an obvious task since there are a lot of possibilities. Leaving the degree unfixed, as in the definition below from [Amgoud *et al.*, 2022], leads to a principle with a high degree of granularity, but it does not state when a semantics compensates.

Principle 3 (Compensation)**.** *A semantics* $\mathbf{S}$ *satisfies* compensation *iff it violates both CP and QP.*

Remark: It is worth noticing that (CP), (QP) and Compensation compare two groups of arguments (attackers). In [Amgoud and Ben-Naim, 2013], the so-called *group comparison* (GC) has been proposed for ranking pairs of arguments on the basis of their attackers. It states that an argument A is ranked at least as high as B if the attackers of B are at least as numerous and well-ranked as those of A. This relation is not applicable in the context of compensation. Consider for instance the graph $\mathbf{G}_1$. (GC) declares the sets $\{Y_1, Y_2, Y_3\}$ and $\{Z\}$ as incomparable. Similarly, both in $\mathbf{G}_2$ and $\mathbf{G}_3$ the groups of attackers would be incomparable with respect to (GC). In fact, (GC) declares two sets as equally strong only when they have the same cardinality and their arguments are equally strong. Hence, it is orthogonal to compensation.

3 Novel Family of Gradual Semantics

This section introduces a novel family of gradual semantics which compensate. The family evaluates the strengths of arguments in weighted argumentation graphs. It is based on a parameter α which takes values from $(0, +\infty)$, where each value defines a semantics. We define the strength of an argument in an iterative way, starting from its basic weight in the considered weighted graph, and updating its value in each step, using the values of its attackers from the previous step.

Definition 3 (Parameterised Semantics $\mathbf{S}_\alpha$)**.** *Let* $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$*, and let us assume an ordering on* $\mathcal{A}$*, i.e., let* $\mathcal{A} = \{a_1, \ldots, a_n\}$*. Let us define the sequence of* $n-$*tuples* $s_0(\alpha, \mathbf{G}), s_1(\alpha, \mathbf{G}), s_2(\alpha, \mathbf{G}), \ldots$ *in the following way:*

- $s_0(\alpha, \mathbf{G}) = (w(a_1), w(a_2), \ldots, w(a_n))$
- $s_{i+1}(\alpha, \mathbf{G}) = F^\alpha(s_i(\alpha, \mathbf{G}))$*, where the* update *operator*

$$F^\alpha : [0,1]^n \to [0,1]^n, \ F^\alpha = (F^\alpha_1, \ldots, F^\alpha_n)$$

is defined by:

$$F^\alpha_k(x_1, \ldots, x_n) = \frac{w(a_k)}{1 + \left(\sum_{j : a_j \in \mathtt{Att}(a_k)} x_j^\alpha\right)^{\min\{1, 1/\alpha\}}} \tag{1}$$

- $s^*(\alpha, \mathbf{G}) = \lim_{n \to +\infty} s_n(\alpha, \mathbf{G})$.

Then $\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}$ *is defined by*

$$(\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(a_1), \ldots, \mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(a_n)) = s^*(\alpha, \mathbf{G}).$$

It is trivial to check that the values of $\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}$ do not depend on the ordering of the set $\mathcal{A}$. The exponent $\min\{1, 1/\alpha\}$ in the function that defines F^α_k is our **technical solution** of the research problem: define a family of semantics that can compensate at every degree (see Section 4), while the extreme values of α favor quantity and quality (Theorem 5) and the semantics satisfy desirable principles (Theorem 6).

The following result shows that $\mathbf{S}_\alpha$ is always well defined.

Theorem 1. *For every* $\alpha \in (0, +\infty)$ *and every* $\mathbf{G} \in \mathtt{AG}$*, the sequence* $\{s_n(\alpha, \mathbf{G})\}_{n=0}^{+\infty}$ *converges.*

Example 1. *The tables 2 and 3 summarise the strengths (*$\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(.)$*) of arguments in the graphs* $\mathbf{G}_2$ *and* $\mathbf{G}_3$ *respectively, for* $\alpha = 2$ *and for* $\alpha = 0.5$*. Note that the semantics* $\mathbf{S}_{0.5}$ *considers the two arguments* A *and* B *as equally strong.*

In what follows, we show that $\mathbf{S}_\alpha$ can be equivalently defined in a more intuitive way, by calculating the strength of an argument based on its basic weight and strengths of its attackers in the way defined by the set of equations (2) (one equation per argument). Our choice to define $\mathbf{S}_\alpha$ as above is because Definition 3 directly provides an algorithm for calculation the strength with arbitrary precision.

Theorem 2. *For every $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$ and every $a \in \mathcal{A}$, the following holds:*

$$\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(a) = \frac{w(a)}{1 + \left(\sum_{b \in \mathtt{Att}(a)} (\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(b))^\alpha\right)^{\min\{1, 1/\alpha\}}} \quad (2)$$

We strengthen Theorem 2 by showing that $\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}$ is the only function satisfying the equation (2). This **characterisation result** means that (2) is an alternative definition of the semantics, which is useful for checking some of its properties in a convenient way.

Theorem 3. *Let $\alpha \in (0, +\infty)$, $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$ and $D : \mathcal{A} \to [0, 1]$. If $D(a) = \frac{w(a)}{1+\left(\sum_{b \in \mathtt{Att}(a)} (D(b))^\alpha\right)^{\min\{1,1/\alpha\}}}$, for all $a \in \mathcal{A}$, then $D \equiv \mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}$.*

We consider two semantics as *compatible* if they return the same ranking of arguments on each graph.

Definition 4 (Compatibility)**.** *Let $\mathbf{S}$ and $\mathbf{S}'$ be two gradual semantics. We say that $\mathbf{S}$ and $\mathbf{S}'$ are* compatible *if for every graph $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, for all $a, b \in \mathcal{A}$, the following holds:*
$\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$ *iff* $\mathtt{Deg}^{\mathbf{S}'}_{\mathbf{G}}(a) \geq \mathtt{Deg}^{\mathbf{S}'}_{\mathbf{G}}(b)$.

We show that each two distinct semantics from the new family are incompatible, which is expected since they are designed to satisfy the compensation at different degrees. This result shows that each (even a small) change in parameter α impacts the way the semantics ranks the arguments.

Theorem 4. *Let $\alpha, \beta \in (0, +\infty)$. If $\alpha \neq \beta$, then the semantics $\mathbf{S}_\alpha$ and $\mathbf{S}_\beta$ are not compatible.*

In what follows, we **characterize the two semantics that take the extreme values of** α. Let us first consider the weighted max-based semantics from [Amgoud *et al.*, 2017]. This semantics is defined (if we adopt the terminology from this paper) in the same way as $\mathbf{S}_\alpha$, with the difference that the update function F^α from Definition 3 is replaced with the function $F^{\max} : [0, 1]^n \to [0, 1]^n$, $F^{\max} = (F^{\max}_1, \ldots, F^{\max}_n)$, defined by

$$F^{\max}_k(x_1, \ldots, x_n) = \frac{w(a_k)}{1 + \max_{j: a_j \in \mathtt{Att}(a_k)} x_j}. \quad (3)$$

It was shown in [Amgoud *et al.*, 2017] that the weighted max-based semantics satisfies (QP).

Next we introduce a simplification of a weighted card-based semantics from [Amgoud *et al.*, 2017]. We define that semantics as in Definition 3, by replacing F^α with the function F^c defined by

$$F^c_k(x_1, \ldots, x_n) = \frac{w(a_k)}{1 + \ |\{x_j \mid a_j \in \mathtt{AttF}_{\mathbf{G}}(a_k), x_j \neq 0\}|}$$

where $\mathtt{AttF}_{\mathbf{G}}(a)$ is the set of attackers of a that have strictly positive basic weight. Observe that the fixed point of this function is reached after one iteration[1], and from the form of F^c it is clear that the resulting semantics satisfies (CP).

Now we formally show that in the limit cases, when α approaches 0 (resp. $+\infty$) we obtain semantics that satisfy (CP) (respectively (QP)).

Theorem 5. *The following two results hold.*

- $\lim_{\alpha \to +\infty} F^\alpha \equiv F^{\max}$
- $\lim_{\alpha \to 0^+} F^\alpha \equiv F^c$.

We prove next that all of our semantics behave well with respect to principles for gradual semantics proposed in [Amgoud *et al.*, 2017]. We recall those principles in Appendix A. Note that there are other principles in the literature (eg., [Amgoud *et al.*, 2022; Baroni *et al.*, 2019; Bonzon *et al.*, 2021]), but due to lack of space, we focus on the ones that highlight the underpinnings of our semantics.

Theorem 6. *For every $\alpha \in (0, +\infty)$, $\mathbf{S}_\alpha$ satisfies all the principles from Appendix A, and compensation.*

To sum up, this section introduced a novel parameterised family which unifies multiple gradual semantics that share some principles (in Appendix A) but differ in their behaviour regarding solving dilemmas.

4 Classes of Compensation Degrees

We have seen previously that every semantics (not taking one of the two extreme values of α) satisfies compensation. However, the latter does not inform precisely in which way a semantics compensates as it leaves the compensation degree unfixed. The aim of this section is to shed light on **various situations where semantics of the family compensate**. There are certainly a lot of possibilities but we focus here on three classes of degrees. The first one has been introduced in [Amgoud *et al.*, 2016] for *flat* graphs. It defines a strong attack as an attack from an unattacked argument, and a weak attack as one that comes from an argument that is attacked by i unattacked arguments. Before recalling the property, let us introduce a useful notation.

Notation: For every $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\mathcal{C}^i(\mathbf{G}) = \{a \in \mathcal{A}$ such that $|\mathtt{Att}(a)| = i$ and $\forall b \in \mathtt{Att}(a), \mathtt{Att}(b) = \emptyset\}$.

Let us now recall the way Compensation was defined by Amgoud *et al.* (2016) for flat graphs of the form $\langle \mathcal{A}, w \equiv 1, \mathcal{R} \rangle$. It says that an argument receiving one strong attack is as good as an argument receiving n weak attacks.

Definition 5 ((n,i)-Compensation)**.** *Let $n, i \in \{1, 2, 3, \ldots\}$. A semantics $\mathbf{S}$ satisfies (n,i)*-compensation *iff for any $\mathbf{G} = \langle \mathcal{A}, w \equiv 1, \mathcal{R} \rangle \in \mathtt{AG}$, for all $a, b \in \mathcal{A}$, the following holds: if*

- $|\mathtt{Att}(a)| = n$, $\mathtt{Att}(a) \subseteq \mathcal{C}^i(\mathbf{G})$*, and*
- $|\mathtt{Att}(b)| = 1$, $\mathtt{Att}(b) \subseteq \mathcal{C}^0(\mathbf{G})$*,*

[1]Namely, after assigning the basic weights, the iterative application of F^c_k on an argument yields 0 in all iterations if and only if that argument's initial weight is 0. For other arguments (those having strictly positive initial weight), we obtain the final result in the first iteration, since it is depends only on the number of attackers having positive acceptability degree, and that number does not change.

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

This definition is somehow limited since it ignores arguments that are attacked by arguments which are themselves attacked. In the graph $\mathbf{G}_3$, every attacker Y_i of A is attacked by two arguments which are themselves attacked. Hence, the above definition does not apply on A and B even if there is a conflict between the number and the strength of their attackers. The notion of weakness is too specific that it ignores reasonable cases of weak arguments. Furthermore, several weak attackers can only compensate a single strong attacker. We propose another class of degrees in which several weaker attackers may compensate a smaller group of stronger attackers, and the two parameters of compensation are fixed.

Definition 6 ($(m, c, n, d)-$Compensation)**.** *Let $m, n \in \mathbb{N}$ such that $m > n$ and $0 < c < d \leq 1$. A semantics $\mathbf{S}$ satisfies $(m, c, n, d)-$compensation iff for any $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, for all $a, b \in \mathcal{A}$ such that $w(a) = w(b)$, if*

- $|\mathtt{Att}(a)| = m$
- *for every $x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = c$*
- $|\mathtt{Att}(b)| = n$
- *for every $x \in \mathtt{Att}(b)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = d$*

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

This class is also limited since it assumes that all attackers of an argument have the same strength. In what follows, we relax this constraint. The idea is that two different sets of attackers compensate as long as one set contains more attackers (following Principle 1, we exclude worthless attackers, i.e., those attackers whose degree is 0), while the other contains the arguments that are stronger than any of them.

Definition 7 ($(m, \bar{c}, n, \bar{d})-$Compensation)**.** *Let $m, n \in \mathbb{N}$ such that $m > n$ and $\bar{c} = (c_1, \ldots, c_m) \in [0, 1]^m$ and $\bar{d} = (d_1, \ldots, d_n) \in [0, 1]^n$ such that $\max c_i < \max d_j$ and $\min c_i > 0$. A semantics $\mathbf{S}$ satisfies $(m, \bar{c}, n, \bar{d})-$compensation, $\bar{c} = (c_1, \ldots, c_m)$ and $\bar{d} = (d_1, \ldots, d_n)$, iff for any $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, for all $a, b \in \mathcal{A}$ such that $w(a) = w(b)$, the following holds: if*

- $\{z \in \mathtt{Att}(a) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(z) > 0\} = \{x_1, \ldots, x_m\}$
- $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x_1), \ldots, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x_m)) = (c_1, \ldots, c_m)$
- $\{z \in \mathtt{Att}(b) \mid \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(z) > 0\} = \{y_1, \ldots, y_n\}$
- $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y_1), \ldots, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y_n)) = (d_1, \ldots, d_n)$

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

It is immediate to see that any of the three classes implies Compensation.

Proposition 1. *If a semantics $\mathbf{S}$ satisfies $(n, i)-$ (resp. $(m, c, n, d)-$, $(m, \bar{c}, n, \bar{d})-$) Compensation, then $\mathbf{S}$ satisfies Compensation.*

We show next that semantics of our family satisfy $(n, i)-$Compensation. We only exclude the case $n = 1$, since one attacked argument does not compensate one non-attacked argument.

Theorem 7. *For every $n, i \in \mathbb{N}$ such that $n > 1$ there exists a unique $\alpha \in (0, +\infty)$ such that $\mathbf{S}_\alpha$ satisfies $(n, i)-$Compensation.*

Instances of the novel family of semantics can satisfy all possible degrees of compensation as specified in Definition 6. This result shows that a *smaller number* of attackers of some fixed degree can always compensate a greater number of attackers of some fixed *weaker* (but still positive) degree.

Theorem 8. *For every quadruple (m, c, n, d) which satisfies the conditions of Definition 6, there exists a unique $\alpha \in (0, +\infty)$ such that $\mathbf{S}_\alpha$ satisfies $(m, c, n, d)-$compensation.*

Finally, we prove that of our family of semantics can cover all possible degrees of compensation as defined in Definition 7. This result is more general and corresponds to the idea that quantity can always be compensated by quality. Intuitively, it says that any given set of attackers can always be compensated by another set containing an attacker which is stronger than each of them individually, regardless of exact strengths of other attackers.

Theorem 9. *For every quadruple $(m, \bar{c}, n, \bar{d})$ which satisfies the conditions of Definition 7, there exists $\alpha \in (0, +\infty)$ such that $\mathbf{S}_\alpha$ satisfies $(m, \bar{c}, n, \bar{d})-$compensation.*

To sum up, this section provided three classes of compensation degrees which are guaranteed by semantics of the novel family. This sheds light on various compensation situations.

5 Analyzing Existing Gradual Semantics

This section analyses existing semantics that satisfy Compensation. There are three such semantics: the $\alpha-\mathtt{BBS}$ *semantics* from [Amgoud *et al.*, 2016], *Trust-based* from [da Costa Pereira *et al.*, 2011] and weighted $h-$Categorizer from [Amgoud *et al.*, 2017]. Note that any gradual semantics, like *Iterative Schema* from [Gabbay and Rodrigues, 2015], which violates Compensation does not satisfy (n, i)-compensation, (m, c, n, d)-compensation nor $(m, \bar{c}, n, \bar{d})-$compensation due to Proposition 1.

$\alpha-\mathtt{BBS}$ semantics. α-BBS is a family of gradual semantics introduced in [Amgoud *et al.*, 2016]. It evaluates arguments of **flat** graphs, i.e., of the form $\mathbf{G} = \langle \mathcal{A}, w \equiv 1, \mathcal{R} \rangle$. It is based on a parameter α taking values from the interval $(0, +\infty)$. Each value gives birth to one semantics, which assigns a numerical value from $[1, +\infty)$ to every argument $a \in \mathcal{A}$ as follows: If $\mathtt{Att}_{\mathbf{G}}(a) = \emptyset$, then $s_\alpha(a) = 1$, else:

$$s_\alpha(a) = 1 + \left(\sum_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \frac{1}{(s_\alpha(b))^\alpha} \right)^{1/\alpha}$$

Note that this semantics uses the scale $[1, +\infty)$ instead of $[0, 1]$ (see Def. 2). Furthermore, the value $s_\alpha(a)$ represents the burden of the argument a (i.e., how heavily it is attacked), and therefore follows the intuition that the smaller the value, the stronger the argument.

Example 2. *Consider the argumentation graph $\mathbf{G}_1$ from Table 1 and let all the arguments have the weight 1. For $\alpha = 1$, we obtain $s_\alpha(A) = s_\alpha(B) = 2$.*

α-BBS semantics have been proposed with the aim of ensuring $(n, i)-$Compensation (Definition 5) at any degree. It has been shown that the parameter α of those semantics is related to the degree (n, i) as follows.

Property 1 ([Amgoud *et al.*, 2016])**.** *For every $n, i \in \mathbb{N}$ such that $n > i$ there exists a unique $\alpha \in (0, +\infty)$ such that $\alpha-$*BBS *satisfies $(n, i)-$Compensation.*

Furthermore, it is easy to check that a stronger result holds, namely that the α from the previous property is greater or equal to 1.

Property 2. *For every $n, i \in \mathbb{N}$ such that $n > i$ there exists a unique $\alpha \in [1, +\infty)$ such that $\alpha-$*BBS *satisfies $(n, i)-$Compensation.*

Together, the previous two results also imply that for $n > i$, there exists no $\alpha \in (0, 1)$ such that $\alpha-$BBS satisfies $(n, i)-$Compensation.

However, the properties remain silent about cases where $n \leq i$ (as in the graph $\mathbf{G}_2$). In what follows, we show that there is no $\alpha-$BBS semantics that satisfies $(n, i)-$Compensation when $n \leq i$.

Theorem 10. *For every $n, i \in \mathbb{N}$ such that $i \geq n$ there does not exist $\alpha \in (0, +\infty)$ such that $\alpha-$*BBS *satisfies $(n, i)-$Compensation.*

We can see that the values of α from $(0, 1)$ are never used by $\alpha-$BBS to satisfy $(n, i)-$Compensation, and the values from $[1, +\infty)$ can only compensate only for (n, i) such that $n > i$.

Unlike $\alpha-$BBS, our novel family has more compensation capabilities as its semantics compensate at any degree (n, i) even when $n \leq i$, as stated by Theorem 7.

Example 3. *Consider the graph $\mathbf{G}_2$ from Table 1. According to Theorem 10, α-BBS cannot compensate on $\mathbf{G}_2$. However, Theorem 7 shows that there exists a unique α that yields compensation. Indeed, for $\alpha = 0.5$, arguments A and B in graph $\mathbf{G}_2$ have exactly the same degree, namely* $\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}_2}(A) = \mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}_2}(B) = 0.5$.

The following result states that for every $\alpha \geq 1$, the semantics $\mathbf{S}_\alpha$ of our family corresponds to the inverse of the corresponding $\alpha-$BBS semantics. However, this result does not hold for any $\alpha < 1$.

Theorem 11. *Let $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in$* AG *such that $w \equiv 1$, and let $a \in \mathcal{A}$. For every $\alpha \in [1, +\infty)$,*

$$\mathtt{Deg}^{\mathbf{S}_\alpha}_{\mathbf{G}}(a) = \frac{1}{s_\alpha(a)}.$$

This result shows that the novel family improves the family proposed in [Amgoud *et al.*, 2016] by keeping the semantics that satisfy $(n, i)-$Compensation and replacing the remaining ones with semantics that satisfy the principle (as shown in Theorem 7). Consequently, instances of α-BBS where $\alpha \in [1, +\infty)$ inherit the properties of our family regarding the two other classes of compensation degrees.

Trust-based semantics. Trust-based semantics (TB) has been proposed in [da Costa Pereira *et al.*, 2011] to evaluate the strength of arguments in weighted argumentation graphs, where the basic weight assigned to an argument represents the degree of reliability of its source. For any graph $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, any $a \in \mathcal{A}$, the strength of a is the limit reached by the scoring function f defined as follows:

$$\mathtt{Deg}^{TB}_{\mathbf{G}}(a) = \lim_{i \to +\infty} f_i(a), \text{ where}$$

$$f_i(a) = \frac{1}{2} f_{i-1}(a) + \frac{1}{2} \min[w(a), 1 - \max_{b \mathcal{R} a} f_{i-1}(b)] \quad (4)$$

While it has been shown in [Amgoud *et al.*, 2022] that (TB) satisfies Compensation, the following result shows that it is not able to compensate at any of the three classes of compensation degrees.

Theorem 12. *The following properties hold:*

- *There exist no $n, i \in \{1, 2, 3, \ldots\}$ such that Trust-based semantics satisfies $(n, i)-$Compensation.*
- *There exist no m, c, n, d such that Trust-based semantics satisfies $(m, c, n, d)-$Compensation.*
- *There exist no m, c, n, d such that Trust-based semantics satisfies $(m, \bar{c}, n, \bar{d})-$Compensation.*

Weighted h-Categoriser Introduced in [Amgoud *et al.*, 2017], weighted h-Categoriser evaluates arguments in weighted graphs as follows: For any graph $\mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, any $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(a) = w(a)$ if $\mathtt{Att}_{\mathbf{G}}(a) = \emptyset$, else:

$$\mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(a) = \frac{w(a)}{1 + \sum\limits_{b \in \mathtt{Att}_{\mathbf{G}}(a)} \mathtt{Deg}^{\mathtt{Hbs}}_{\mathbf{G}}(b)}.$$

It has been shown in [Amgoud *et al.*, 2022] that Hbs satisfies Compensation. It has also been shown in [Amgoud *et al.*, 2016] that in case of flat graphs, it satisfies $(n, i)-$Compensation when $n = i + 1$.

Property 3 ([Amgoud *et al.*, 2016; Amgoud *et al.*, 2022])**.**

- Hbs *satisfies Compensation.*
- Hbs *satisfies $(n, i)-$Compensation if and only if $n = i + 1$.*

The following result shows that under some conditions, Hbs can compensate at the two other classes of degrees.

Theorem 13. *The following properties hold:*

- Hbs *satisfies $(m, c, n, d)-$compensation if and only if $m \cdot c = n \cdot d$*
- Hbs *satisfies $(m, \bar{c}, n, \bar{d})-$compensation if and only if $\sum_{i=1}^{m} c_i = \sum_{j=1}^{n} d_j$*

To sum up, this section showed that the novel family offers semantics with greater compensation capabilities. They allow compensation at more degrees than the existing gradual semantics. Furthermore, the family encompasses some semantics of $\alpha-$BBS as well as Hbs.

6 Related Work and Conclusion

This paper investigated gradual semantics, which have been initiated by Cayrol and Lagasquie-Schiex (2005). Several semantics and approaches have been proposed since then (eg., [Besnard and Hunter, 2001; da Costa Pereira *et al.*, 2011;

Leite and Martins, 2011; Rago *et al.*, 2016; Potyka, 2018; Amgoud *et al.*, 2022]) and several efforts have also been made to study principles underlying gradual semantics (eg., [Amgoud and Ben-Naim, 2013; Bonzon *et al.*, 2016; Baroni *et al.*, 2019; Yun *et al.*, 2020; Amgoud *et al.*, 2022]. This paper focused on one of them, namely compensation. The latter is one of the possible strategies to follow when facing a dilemma between the quantity and the quality of attackers.

In this paper we pointed out some limits of existing principles that formalise the notion of compensation, and proposed ways for getting rid of them. The idea is to have insight on the ways semantics compensate. For that purpose, we defined three classes of compensation degrees. We also proposed a large family of gradual semantics which encompasses semantics that are able to compensate at any of those degrees. We have shown that there is no other gradual semantics in the literature that is able to do the same job.

Let us draw the reader's attention to the fact that the notion of compensation is different from of accrual [Prakken, 2005; Lucero *et al.*, 2009]. Compensation is related to the fact that the effect of few strong attackers can be similar to that of more weaker attackers. Accrual focuses on claims, and shows how different reasons supporting the same claim may be aggregated into a single argument.

Viana and Alcântara (2021) defined two new principles: Quality Compensation and Cardinality Compensation, which are weakened versions of Quality Precedence and Cardinality Precedence, respectively. They allow, for example, to use the quality of attacks to decide which argument is stronger in case the overall sum of attacks on two arguments is equal. We do not provide a more fine-grained discussion regarding these principles because our principles are neither inspired by nor related to them.

As a future work, we plan to investigate other reasonable degrees of compensation, and apply our formal approach in multiple criteria decision making.

Appendix A: Principles for Semantics

In what follows, we recall some properties from [Amgoud *et al.*, 2017]. Let $\mathbf{S}$ be a semantics.

Anonymity: $\mathbf{S}$ satisfies *anonymity* iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle \in \mathtt{AG}$, for any isomorphism f from $\mathbf{G}$ to $\mathbf{G}'$, the following holds: $\forall\, a \in \mathcal{A}, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(f(a))$.

Independence: $\mathbf{S}$ satisfies *independence* iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$, $\mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle \in \mathtt{AG}$ s.t $\mathcal{A} \cap \mathcal{A}' = \emptyset$, the following holds: $\forall\, a \in \mathcal{A},\ \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G} \oplus \mathbf{G}'}(a)$.

Directionality: $\mathbf{S}$ satisfies *directionality* iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, $\forall \mathbf{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle \in \mathtt{AG}$, s.t.

- $\mathcal{A}' = \mathcal{A}$,
- $w' = w$,
- $\mathcal{R}' = \mathcal{R} \cup \{(a, b)\}$,

it holds that: $\forall x \in \mathcal{A}$, if there is no path from b to x, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}'}(x)$.

Equivalence: $\mathbf{S}$ satisfies equivalence iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if

- $w(a) = w(b)$, and
- there exists a bijective function f from $\mathtt{Att}(a)$ to $\mathtt{Att}(b)$ s.t. $\forall x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(f(x))$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Maximality: $\mathbf{S}$ satisfies maximality iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$, if $\mathtt{Att}(a) = \emptyset$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = w(a)$.

Neutrality: $\mathbf{S}$ satisfies neutrality iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if

- $w(a) = w(b)$, and
- $\mathtt{Att}(b) = \mathtt{Att}(a) \cup \{x\}$ s.t. $x \in \mathcal{A} \setminus \mathtt{Att}(a)$ and $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) = 0$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Weakening: $\mathbf{S}$ satisfies weakening iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$, if

- $w(a) > 0$, and
- $\exists b \in \mathtt{Att}(a)$ s.t. $w(b) > 0$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) < w(a)$.

Proportionality: $\mathbf{S}$ satisfies proportionality iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if

- $\mathtt{Att}(a) = \mathtt{Att}(b)$,
- $w(a) > w(b)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Resilience: $\mathbf{S}$ satisfies resilience iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) = 0$ iff $w(a) = 0$.

Reinforcement: $\mathbf{S}$ satisfies reinforcement iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if

- $w(a) = w(b)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$,
- $\mathtt{Att}(a) \setminus \mathtt{Att}(b) = \{x\}$, $\mathtt{Att}(b) \setminus \mathtt{Att}(a) = \{y\}$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(y) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x)$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Counting: $\mathbf{S}$ satisfies counting iff $\forall \mathbf{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle \in \mathtt{AG}$, $\forall a, b \in \mathcal{A}$, if

- $w(a) = w(b)$,
- $\mathtt{Att}(b) = \mathtt{Att}(a) \cup \{x\}$ with $x \notin \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(x) > 0$, and $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > 0$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{G}}(b)$.

Acknowledgements

Srdjan Vesic was supported by the project AGGREEY ANR-22-CE23-0005 from the French National Research Agency (ANR). Leila Amgoud was supported by the ANR (ANR-19-PI3A-0004) through the AI Interdisciplinary Institute, ANITI, as a part of"Investing for the Future — PIA3" program.

References

[Amgoud and Ben-Naim, 2013] Leila Amgoud and Jonathan Ben-Naim. Ranking-based semantics for argumentation frameworks. In *Conference on Scalable Uncertainty Management (SUM'13)*, pages 134–147, 2013.

[Amgoud *et al.*, 2016] L. Amgoud, J. Ben-Naim, D. Doder, and S. Vesic. Ranking arguments with compensation-based semantics. In *International Conference on Principles of Knowledge Representation and Reasoning KR'16*, 2016.

[Amgoud *et al.*, 2017] Leila Amgoud, Jonathan Ben-Naim, Dragan Doder, and Srdjan Vesic. Acceptability semantics for weighted argumentation frameworks. In *International Joint Conference on Artificial Intelligence (IJCAI'17)*, pages 56–62, 2017.

[Amgoud *et al.*, 2022] Leila Amgoud, Dragan Doder, and Srdjan Vesic. Evaluation of argument strength in attack graphs: Foundations and semantics. *Artificial Intelligence*, 302:103607, 2022.

[Baroni *et al.*, 2019] Pietro Baroni, Antonio Rago, and Francesca Toni. From fine-grained properties to broad principles for gradual argumentation: A principled spectrum. *International Journal of Approximate Reasoning*, 105:252–286, 2019.

[Besnard and Hunter, 2001] Philippe Besnard and Anthony Hunter. A logic-based theory of deductive arguments. *Artificial Intelligence*, 128(1-2):203–235, 2001.

[Bonzon *et al.*, 2016] Elise Bonzon, Jérôme Delobelle, Sébastien Konieczny, and Nicolas Maudet. A comparative study of ranking-based semantics for abstract argumentation. In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence*, pages 914–920, 2016.

[Bonzon *et al.*, 2021] Elise Bonzon, Jérôme Delobelle, Sébastien Konieczny, and Nicolas Maudet. A parametrized ranking-based semantics compatible with persuasion principles. *Argument and Computation*, 12(1):49–85, 2021.

[Cayrol and Lagasquie-Schiex, 2005] Claudette Cayrol and Marie-Christine Lagasquie-Schiex. Graduality in argumentation. *Journal of Artificial Intelligence Research (JAIR)*, 23:245–297, 2005.

[da Costa Pereira *et al.*, 2011] Célia da Costa Pereira, Andrea Tettamanzi, and Serena Villata. Changing one's mind: Erase or rewind? In *International Joint Conference on Artificial Intelligence IJCAI'11*, pages 164–171, 2011.

[Dung, 1995] Phan Minh Dung. On the Acceptability of Arguments and its Fundamental Role in Non-Monotonic Reasoning, Logic Programming and n-Person Games. *Artificial Intelligence*, 77:321–357, 1995.

[Gabbay and Rodrigues, 2015] Dov M. Gabbay and Odinaldo Rodrigues. Equilibrium states in numerical argumentation networks. *Logica Universalis*, 9(4):411–473, 2015.

[Leite and Martins, 2011] João Leite and João Martins. Social abstract argumentation. In *International Joint Conference on Artificial Intelligence IJCAI'11*, pages 2287–2292, 2011.

[Lucero *et al.*, 2009] Mauro Javier Gómez Lucero, Carlos Iván Chesñevar, and Guillermo Ricardo Simari. On the accrual of arguments in defeasible logic programming. In *IJCAI 2009, Proceedings of the 21st International Joint Conference on Artificial Intelligence*, pages 804–809, 2009.

[Potyka, 2018] Nico Potyka. Continuous dynamical systems for weighted bipolar argumentation. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Sixteenth International Conference, KR*, pages 148–157, 2018.

[Prakken, 2005] Henry Prakken. A study of accrual of arguments, with applications to evidential reasoning. In Giovanni Sartor, editor, *The Tenth International Conference on Artificial Intelligence and Law, Proceedings of the Conference, June 6-11, 2005, Bologna, Italy*, pages 85–94. ACM, 2005.

[Pu *et al.*, 2014] Fuan Pu, Jian Luo, Yulai Zhang, and Guiming Luo. Argument ranking with categoriser function. In Robert Buchmann, Claudiu Vasile Kifor, and Jian Yu, editors, *Knowledge Science, Engineering and Management - 7th International Conference, KSEM 2014, Sibiu, Romania, October 16-18, 2014. Proceedings*, volume 8793 of *Lecture Notes in Computer Science*, pages 290–301. Springer, 2014.

[Rago *et al.*, 2016] Antonio Rago, Francesca Toni, Marco Aurisicchio, and Pietro Baroni. Discontinuity-free decision support with quantitative argumentation debates. In *International Conference on Principles of Knowledge Representation and Reasoning (KR'16)*, pages 63–73, 2016.

[Viana and Alcântara, 2021] Henrique Viana and João F. L. Alcântara. On the refinement of compensation-based semantics for weighted argumentation frameworks. In André Britto and Karina Valdivia Delgado, editors, *Intelligent Systems - 10th Brazilian Conference, BRACIS 2021, Virtual Event, November 29 - December 3, 2021, Proceedings, Part I*, volume 13073 of *Lecture Notes in Computer Science*, pages 344–358. Springer, 2021.

[Yun *et al.*, 2020] Bruno Yun, Srdjan Vesic, and Madalina Croitoru. Ranking-based semantics for sets of attacking arguments. In *The Thirty-Fourth AAAI Conference on Artificial Intelligence, AAAI*, pages 3033–3040, 2020.