Determining Winners in Elections with Absent Votes

Qishen Han[1] , **Amélie Marian**[2] , **Lirong Xia**[1]
[1]Rensselaer Polytechnic Institute
[2]Rutgers University
hnickc2017@gmail.com, amelie.marian@rutgers.edu, xialirong@gmail.com

Abstract

An important question in elections is determining whether a candidate can be a winner when some votes are absent. We study this determining winner with absent votes (WAV) problem with elections that take top-truncated ballots. We show that the WAV problem is NP-complete for single transferable vote, Maximin, and Copeland, and propose a special case of positional scoring rule such that the problem can be computed in polynomial time. Our results for top-truncated rankings differ from the results in full rankings as their hardness results still hold when the number of candidates or the number of missing votes are bounded, while we show that the problem can be solved in polynomial time in either case.

1 Introduction

In a multi-agent system, voting is one of the most common methods to aggregate preferences and make collective decisions. Voting has been rooted in the democratic procedure while emerging as new techniques to other scenarios including search engines [Dwork *et al.*, 2001], crowdsourcing [Mao *et al.*, 2013], and blockchain governance [Grossi, 2022].

An important question in these scenarios is the need to know possible winners without knowing the preferences of all the voters. There are several common reasons in practice why some votes may not be available right away for tallying: delay of absentee ballots, the forecasting of votes with polling results, or the contestation of the validity of some ballots.

Example 1. Suppose a city runs its mayoral election and adopts the single transferable vote (STV) (or so-called ranked-choice voting (RCV)) as the voting rule. Unfortunately, some of the absentee ballots have experienced substantial delays and are suspected to be lost. The official investigation will take about one month to locate these missing ballots, causing a significant disruption to the usual political proceedings. Is it possible for the city officials to offer a forecast of all potential winners by considering the current votes and estimating the number of undisclosed ballots?

In fact, such examples have happened in practice in U.S. localities that have recently switched to ranked-choice voting. In 2018, the results of the RCV San Francisco mayoral election took a week to be confirmed and tabulated, largely due to the late counting of mail-in ballots. The results of the 2021 New York City primary election were not known until *a full month* after the election due to the large number of absentee ballots. These delays, the lack of transparency, and the incomplete information, or lack thereof, on the outcome of ballots led to distrust in the election process [Ennis, 2023].

Winner determination with absent votes also justifies a candidate's victory in ballots susceptible to manipulation [Baumeister and Hogrebe, 2023]. A proposed heuristic [Jelvani and Marian, 2022] empirically evaluated on NYC election night data shows promises in identifying election winners, or narrowing down the field of possible winners in a single transferrable vote scenario.

From another perspective, determining winners with absent votes is also related to the classic problem of *coalitional manipulation* in computational social choice. In this context, a group of manipulators influence the outcome by strategically adding specific votes to the ballot. Originated from the famous Gibbard-Satterthwaite Theorem [Gibbard, 1973; Satterthwaite, 1975], there have been extensive theoretical studies on this problem in the computational social choice literature from the perspective of coalitional manipulation [Faliszewski and Procaccia, 2010; Faliszewski *et al.*, 2010a]. Subsequent studies characterize the complexity of such problems for different voting rules including STV [Bartholdi and Orlin, 1991], positional scoring rules [Davies *et al.*, 2011; Betzler *et al.*, 2011], Copeland [Faliszewski *et al.*, 2010b], and Maximin [Xia *et al.*, 2009].

However, most previous studies assume that each voter casts a complete linear order, i.e. a *full ranking* toward all the candidates. In contrast, votes where voters cast a few *top preferences* become increasingly common in real-world scenarios. Top-ranked voting is more practical to implement because it simplifies the computation of the winner and prevents voters without full preferences from casting random votes and corrupting the ballot. Moreover, the results of coalitional manipulation under full rankings do not extend to the winner prediction for top-ranking votes. [2014] study vote where top-truncated votes are allowed. [2017] study a weighted version of coalitional manipulation for top-truncated votes. Yet

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their hardness results also do not apply due to the incorporation of weights in the voting process.

Therefore, the following question remains open: **What is the complexity of determining possible winners in top-ranked voting election with absent votes?**

1.1 Our Contribution

We investigate the computational complexity of determining *winner with absent votes (WAV)* under multiple voting rules. We focus on two specific settings of top-truncated rankings. In the *top-ℓ* setting, every voter is asked to provide their top-ℓ preference. And in the *up-to-L* setting, every voter can rank his/her at most L favorite candidates.

We first show that, when the number of candidates or the quantity of absent votes is bounded, the WAV problem can be solved in polynomial time under both top-ℓ and up-to-L settings. This distinguishes our work from the previous studies under full-ranking settings, in which the hardness results hold even for a bounded number of candidates or manipulators.

Subsequently, we show that in STV, Maximin, and Copeland, determining the winner with absent votes is NP-complete for every $\ell \geq 2$ in the top-ℓ setting and every $L \geq 2$ in the up-to-L setting. Conversely, for the positional scoring rule, we show that the winner can be determined in polynomial time under both settings when the scoring vector does not distinguish the second to the ℓ-th (L-th, respectively) rank. This case covers many common voting rules including plurality, veto, and k-approval. A comparison between our results and the results in full rankings in previous works is in Table 1.

We define the problem in the way of determining the winner with absent votes rather than following the convention of coalitional manipulation because the objectives of the two problems are opposite. For winner determination, we hope the problem is easy so that there are efficient ways to provide accurate predictions to the public. In the coalitional manipulation problem, on the other hand, hardness results are more welcomed because they prevent manipulators from corrupting the elections easily [Faliszewski *et al.*, 2010a]. Our motivation aligns with the winner determination problem.

2 Related Works

As mentioned in the introduction, winner determination with absent votes has been extensively explored by the computational social choice community from the perspective of coalitional manipulation. [1973] and [1975] show that all reasonable voting rules suffer from manipulation under some situations. The earliest studies on the complexity of manipulation problems [Bartholdi *et al.*, 1989; Bartholdi and Orlin, 1991] show that determining even if a single manipulator would succeed is NP-hard under some voting rules when the number of candidates is unbounded. A large literature follows the path and develops theoretical results of coalitional manipulation under a spectrum of weighted [Conitzer and Sandholm, 2002; Conitzer *et al.*, 2007; Hemaspaandra and Hemaspaandra, 2007; Zuckerman *et al.*, 2009; Xia *et al.*, 2010] and unweighted [Xia *et al.*, 2009; Betzler *et al.*, 2011; Davies *et al.*, 2011; Narodytska *et al.*, 2011; Faliszewski *et al.*, 2010b] voting rules. However, most of the previous work focuses on featuring full rankings. [2014] study manipulations under up-to-m setting, i.e., voters can rank an arbitrary number of candidates, and most of their results are very different from ours. [2017] study the complexity of weighted coalitional manipulation when top-truncated votes are allowed. However, the incorporation of the weights prevents their intractability results from extending to the unweighted version.

A closely related problem to the winner with absent votes and coalitional manipulation is the *possible winner* problem. The problem takes a set of candidates and a profile of partial orders on the candidates and asks if there is a full-order profile that extends the partial orders and makes a certain candidate a winner. A coalitional manipulation instance can be seen as a possible winner instance where a portion of the profile is full orders and the rest is empty votes. When the number of candidates is bounded, the possible winner problem can be solved in polynomial time in unweighted votes and NP-complete in weighted votes under multiple rules [Conitzer *et al.*, 2007; Pini *et al.*, 2011; Walsh, 2007]. When the number of candidates is unbounded, the problem is P in the Condorcet rule [Konczak and Lang, 2005] yet NP-complete in a large variety of other rules [Bartholdi and Orlin, 1991; Xia and Conitzer, 2008; Betzler and Dorn, 2010; Baumeister *et al.*, 2012; Baumeister *et al.*, 2023]. [2019] shows that the possible winner problem under STV is NP-hard even when the partial profile is restricted to top-2 rankings.

Recent work has also looked at the intersection of voting theory with regulatory frameworks in the context of ranked-choice voting elections. In particular, there has been an interest in defining and computing the margin of victory (MoV), an important robustness measure of elections in Australia, where small margins would trigger elections audits [Blom *et al.*, 2016; Magrino *et al.*, 2011], or potentially result in shifts in the balance of power [Blom *et al.*, 2020a; Blom *et al.*, 2020b], and in election manipulation [Blom *et al.*, 2019] in the ranked-choice voting.

3 Preliminaries

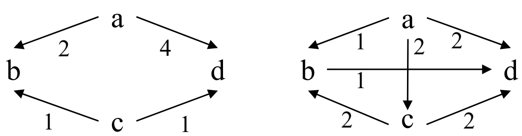
Let M be the set of *candidates* (or *alternatives*). Let $m = |M|$ denote the number of candidates. Given a positive integer ℓ, a top-ℓ ranking R is a ranking on a ℓ-subset of M, where all the unranked candidates are regarded tied with each other and ranked lower than the ranked candidates. Let $\mathcal{L}_\ell(M)$ denote the set of all top ℓ rankings (or linear orders) on M.

There are in total $n + t$ voters in the vote, where n is the number of voters whose votes are known, and t is the number of voters whose votes are absent. In the *top-ℓ setting*, each voter casts a top ℓ ranking $R \in \mathcal{L}_\ell(M)$ to represent their preference, where $a \succ_R b$ means the voter prefers a to b. In the *up-to-L setting*, each agent cast a ranking $R \in (\bigcup_{i=1}^{\ell} \mathcal{L}_i(M))$. The vector of all voters' votes is called a *profile*. Let P denote the profile of known votes and P' denote the profile of absent votes.

We use square brackets to denote votes. For example, given $M = \{a, b, c, d\}$, $[b]$ to denote a vote that prefers candidate b to all other candidates while indifferent among other can-

Rule	Top-ℓ	Up-to-L	Full Ranking
STV	NPC for $\ell \geq 2$ (Thm. 1)	NPC (Thm. 2)	NPC [Bartholdi and Orlin, 1991]
Maximin	NPC for $\ell \geq 2$ (Thm. 3)	NPC for $L \geq 2$ (Thm. 4)	NPC [Xia *et al.*, 2009]
Copeland	NPC for $\ell \geq 2$ (Thm. 5)	NPC for $L \geq 2$ (Thm. 6)	NPC [Faliszewski *et al.*, 2010b]
PSR	P for $\ell = 2$ (Corollary 1) P for $a_2 = \cdots = a_\ell$ (Thm. 7)	P for $\uparrow$-Rd [Menon and Larson, 2017] P for $\downarrow$-Rd and $a_2 = \cdots = a_L$ (Thm. 8)	P for plurality and veto NPC for Borda [Davies *et al.*, 2011]

Table 1: Complexity of predicting winner with absent votes under full ranking, top ℓ ranking, and up to L ranking.



(a) Top-2 (b) Up-to-3

Figure 1: Weighted majority graph of instances in Example 2.

didates, and $[b \succ a]$ is a vote that prefers b the most, a the second, and all other candidates the least.

Given a profile P and two alternatives a and b, $P[a \succ b]$ denotes the votes in P that prefer a to b. The weighted majority graph (WMG) of P is a graph whose vertices are the candidates and weights on $a \to b$ is $\omega_P(a \to b) = P[a \succ b] - P[b \succ a]$.

Example 2. Let $M = \{a, b, c, d\}$, $n = 4$, and $t = 2$.

Let P_1 be a profile of 4 votes under the top-2 setting. P_1 contains two votes for $[c \succ a]$, one vote for $[a \succ d]$, and one vote for $[b \succ a]$.

Let P_2 be a profile of 4 votes under the up-to-3 setting. P_1 contain two votes for $[a \succ c]$, one vote for $[b \succ a \succ d]$, and one vote for $[c]$.

The weighted majority graph of P_1 and P_2 is in Figure 1.

3.1 Voting Rules

A (resolute) voting rule takes a profile as input and outputs a unique candidate as the winner. In the top-ℓ setting, a voting rule $r_\ell : (\mathcal{L}_\ell(M))^* \to M$, and in the up-to-$L$ setting, a voting rule $\overline{r}_L : (\bigcup_{i=1}^{L} \mathcal{L}_i(M))^* \to M$. A voting rule is *anonymous* if the winner is insensitive to the identities of agents.

We focus on the variation of the following common voting rules for the top ℓ or up to L rankings. For a voting rule r, we use r_ℓ to denote its variation in top-ℓ setting and $\overline{r}_L$ to denote its variation in up-to-L setting.

The *single transferable voting* (STV) elects the winner in at most $m-1$ rounds. In each round, each vote contributes 1 score to its most preferred candidate that has not been eliminated, and the candidate with the lowest score is eliminated in that round. A tie-breaking mechanism is applied to select a single loser when necessary. If all candidates ranked in a vote are eliminated, that vote does not contribute to any candidate. The last remaining candidate becomes the winner.

The *Copeland* rule is parametrized by a real number $0 \leq \alpha \leq 1$, denoted by Cd^α. Given a profile P, a candidate a gains 1 score for every other candidate b it beats in the head-to-head competition (the weight on edge $a \to b$ is positive in the WMG) and α score when there is a tie. The candidate with the highest score becomes the winner, and a tie-breaking mechanism is applied to select a single winner if necessary.

In the *Maximin* rule, the *min-score* of a candidate a is the lowest weight of its out-going edges in the weighted majority graph, i.e. $\min_{b \in M \setminus \{a\}} \omega_P(a \to b)$. The candidate with the highest min-score becomes the winner, and a tie-breaking mechanism is applied to select a single winner if necessary.

Remark 1. We follow the definition of the Maximn rule in [Young, 1977]. There is an alternative definition (adopted, for example, in [Menon and Larson, 2017]) in which the min-score of a candidate a is the $\min_{b \neq a} P[a \succ b]$, where $P[a \succ b]$ is the number of votes in P that prefers a to another candidate b. The Maximin rule of two definitions always elect the same winner when P contains only full rankings, but may diverge from each other when P contains top-truncated rankings. See Appendix A for a concrete example of divergence.

We define the *(integer) positional scoring rule* in two settings respectively.

In the top-ℓ setting, a positional scoring rule is characterized by an ℓ-dimension vector $\vec{s}_\ell = (a_1, a_2, \cdots, a_\ell)$ with $a_1 \geq a_2 \geq \cdots \geq a_\ell \geq 0$. Given a top-$\ell$ vote V_i and a candidate c, let $s(V_i, c) = a_j$ where j is the rank of c in V_i or $s(V_i, c) = 0$ if c is not ranked in i. For any profile P, let $s(P, c) = \sum_{V_i \in P} s(V_i, c)$. The candidate c maximizing $s(P, c)$ becomes the winner, and a tie-breaking mechanism is applied to select a single winner if necessary.

In the up-to-L setting, we follow the scheme from [2014] to deal with top-truncated rankings. A positional scoring rule is characterized by an L-dimensional vector, $\vec{s}_L = (a_1, a_2, \cdots, a_L)$ with $a_1 \geq a_2 \geq \cdots \geq a_L \geq 0$, and a *rounding indicator*, denoted by $\uparrow$ or $\downarrow$. In an *up-rounding* ($\uparrow$-Rd for short) scoring rule $\vec{s}_{L\uparrow}$, a candidate c ranked j-th in an ℓ-ranking vote V_i has a score $s(V_i, c) = a_j$. And in a *down-rounding* ($\downarrow$-Rd for short) scoring rule $\vec{s}_{L\downarrow}$, a candidate c ranked j-th in an ℓ-ranking vote V_i has a score $s(V_i, c) = a_{L-\ell+j}$. In both cases, an agent not ranked in a vote gets a score of 0. For any profile P, let $s(P, a) = \sum_{V_i \in P} s(V_i, c)$. The candidate c maximizing $s(P, c)$ becomes the winner, and a tie-breaking mechanism is applied to select a single winner if necessary.

Example 3. We calculate the STV winner for the top-ℓ instance (P_1) in Example 2. In the first round, c gets two votes, a and b get one vote each, and d gets no votes. Therefore, d is eliminated.

In the second round, c gets two votes, and a and b get one vote each. Suppose we use a lexicographic tie-breaking mechanism. Therefore, b is eliminated.

In the third round, $[b \succ a]$ contributes to candidate a. Therefore, both a and c get two votes. Then c is eliminated, and a becomes the winner.

Example 4. We calculate the winner for the up-to-L instance (P_2) in Example 2 under up-rounding and down-rounding positional scoring rules with $\vec{s}_L = (8, 2, 1)$.

In the up-rounding rule, a is ranked the first twice and the second once, so its score is 18. The score of b is 8, the score of c is 10, and the score of d is 1. Therefore, a is the winner.

In the down-rounding rule, a gets 2 points from each $[a \succ c]$ as $L = 3$, and the ranking has a length of 2. a also get two points from $[b \succ a \succ d]$, and has a score of 6, the score of b is 8, the score of c is 3, and the score of d is 1. Therefore, b is the winner.

3.2 Computational Problems

We consider the following computational question: when ℓ (or L, respectively) is a constant, given a set of candidates M, a set of known profiles P of top-ℓ (up-to-L, respectively) votes, the number of absent votes t, and a targeted candidate c, is there a profile P' of t votes that makes c the winner?

We first define the question for the top-ℓ setting. For each constant $\ell \geq 1$ and a voting rule for top-ℓ rankings, we define the following problem.

Definition 1 (WAV-r_ℓ)**. Input**: a set of candidates M, a profile P of known top-ℓ ranking votes, the number of absent votes t, and a candidate c.

Determine: does there exist a profile P' of t top-ℓ ranking votes such that $r_\ell(P \cup P') = c$?

We also consider two variations of the WAV problem with fixed parameters. In WAV with fixed m, the number of the candidates is removed from the input and becomes a predetermined constant. In WAV with fixed t, the quantity of the absent votes becomes a pre-determined constant.

For the up-to-L setting, we follow a similar definition. For each constant $L \geq 1$ and a voting rule $\overline{r}_L$, we define the following problems.

Definition 2 (WAV-$\overline{r}_L$)**. Input**: a set of candidates M, a profile P of known up-to-L ranking votes, the number of absent votes t, and a candidate c.

Determine: does there exist a profile P' of t up-to-L ranking votes ranking votes such that $\overline{r}_L(P \cup P') = c$?

Similarly, we also consider WAV with fixed m and WAV with fixed t in the up-to-L setting.

We focus on $\ell \geq 2$ and $L \geq 2$ cases, as when $\ell = 1$ or $L = 1$, most common voting rules reduce into plurality, and the WAV problem can be computed in polynomial time.

4 Fixed Number of Candidates and Fixed Number of Absent Votes

We first show the easiness result for the variation of a fixed number of candidates and a fixed number of absent votes under both settings.

Proposition 1. For any $\ell \geq 2$ and any anonymous voting rule r_ℓ, both WAV-r_ℓ with any fixed $m \geq 2$ and WAV-r_ℓ with any fixed t can be solved in polynomial time if the winner of r_ℓ can be computed in polynomial time.

Proof Sketch. **Fixed** m**.** We enumerate all possible *anonymous* profiles P' of t votes. There are $\frac{m!}{(m-\ell)!} = O(m^\ell) = O(1)$ different top-ℓ rankings. The numbers of these rankings sum up to t. Therefore, there will be at most $O(t^{m^\ell}) = poly(t)$ many anonymous profiles.

Fixed t**.** We enumerate all possible profiles P' of t votes. For each vote, there are $O(m^\ell)$ different top-ℓ rankings. Therefore, the number of all possible P' is at most $O(m^{t\ell})$. □

Proposition 2. For any $L \geq 2$ and any anonymous voting rule $\overline{r}_L$, both WAV-$\overline{r}_L$ with any fixed $m \geq 2$ and WAV-$\overline{r}_L$ with any fixed t can be solved in polynomial time if the winner of $\overline{r}_L$ can be computed in polynomial time.

Proposition 1 and 2 imply that previous results for full-rankings votes (where $\ell = m$ is variable) do not apply to our top-ℓ and up-to-L settings, as their hardness results hold even when t is fixed, including Copeland when $t = 2$ [Faliszewski *et al.*, 2010b], Maximin for any $t \geq 2$ [Xia *et al.*, 2009], and STV when $t = 1$ [Bartholdi and Orlin, 1991].

In the rest of the paper, we focus on the problem with variables t and m under common voting rules.

5 Single Transferable Vote

Theorem 1. For every constant $\ell \geq 2$, WAV-STV$_\ell$ is NP-complete.

Proof sketch. The membership of NP is held by running the vote and checking the winner. The hardness is proved by a reduction from *restricted exact three-cover (RXC3)* that follows the spirit of the reduction in the hardness proof for the manipulation problem under STV [Bartholdi and Orlin, 1991]. Here we only present the case of $\ell \geq 4$. The full proof, including the case of $\ell = 2$ and $\ell = 3$ is in Appendix B.

Definition 3 (RXC3 [Gonzalez, 1985])**. Input**: (1) a set of q-elements, denoted by $X = \{x_1, x_2, \ldots, x_q\}$, where q is divisible by 3; (2) q sets $\mathcal{S} = \{S_1, S_2, \ldots, S_q\}$ such that for every $j \leq q$, $S_j \subseteq X$ and $|S_j| = 3$. For every $i \leq q$, x_i is in exactly three sets in $\mathcal{S}$. Without loss of generality, we assume that q is an even number. If q is odd, then we use an instance with duplicate X and $\mathcal{S}$.

Determine: if there exists a subset $\mathcal{S}^* \subseteq \mathcal{S}$ such that for every $x_i \in X$, there exists exactly one $S_j \in \mathcal{S}^*$ such that $x_i \in S_j$. We call $\mathcal{S}^*$ an *exact 3-cover* of X. Note that if such $\mathcal{S}^*$, there exists $|\mathcal{S}^*| = \frac{q}{3}$.

For an arbitrary RXC3 instance $(X, \mathcal{S})$, where $X = \{x_1, x_2, \ldots, x_q\}$ and $\mathcal{S} = \{S_1, S_2, \ldots, S_q\}$. We construct the following WAV-STV$_\ell$ instance.

Candidates: there are $3q + 3$ alternatives $\{w, c\} \cup \{d_0, d_1, \ldots, d_q\} \cup \{b_1, \overline{b}_1, \ldots, b_q, \overline{b}_q\}$. We assume that $d_0 \succ d_1 \succ d_2 \succ \cdots \succ d_q \succ b_1 \succ \overline{b}_1 \succ b_2 \succ \overline{b}_2 \succ \cdots \succ b_q \succ \overline{b}_q$ in tie-breaking.

Absent Votes: $t = q/3$.

Known votes: The profile P consists of the following votes, of which only the top preferences are specified. We'll show that either w or c is the winner, therefore, the votes can be filled to top-ℓ ranking arbitrarily without affecting the proof. Both i and j in the list are in $\{1, 2, \ldots, q\}$.

• $\boldsymbol{P_1}$: There are $12q$ votes of $[c \succ w]$.
• $\boldsymbol{P_2}$: There are $12q-1$ votes of $[w \succ c]$.
• $\boldsymbol{P_3}$: There are $10q+2q/3$ votes of $[d_0 \succ w \succ c]$.
• $\boldsymbol{P_4}$: For every i, there are $12q-2$ votes of $[d_i \succ w \succ c]$.
• $\boldsymbol{P_5^1}$: For every i, there are $6q+4i-6$ votes of $[b_i \succ \bar{b}_i \succ w \succ c]$; and $\boldsymbol{P_5^2}$: for every i and every j such that $x_j \in S_i$, there are two votes of $[b_i \succ d_j \succ w \succ c]$.
• $\boldsymbol{P_6^1}$: For every i, there are $6q+4i-2$ votes of $[\bar{b}_i \succ b_i \succ w \succ c]$; and $\boldsymbol{P_6^2}$: for every i, there are two votes of $[\bar{b}_i \succ d_0 \succ w \succ c]$.

First, no matter what rankings P' contains, **the winner will be either** c **or** w. This is because once one of c or w is eliminated in any round, the remaining other will get all $24q-1$ votes from P_1 and P_2. On the other hand, all other alternatives cannot have such a high score and be the winner.

Suppose RXC3 is a YES instance. $\mathcal{S}^*$ is an exact 3-cover of X, and $I = \{i \mid S_i \in \mathcal{S}^*\}$ be the index set of $\mathcal{S}^*$. Then we construct P' as follows: for each $i \in I$, there is one vote of $[\bar{b}_i \succ b_i \succ c \succ w]$. In the first q round of voting, for each $i \le q$, if $i \in I$, b_i is eliminated; otherwise $\bar{b}_i$ is eliminated. At the beginning of the $q+1$ round, the plurality scores of the remaining alternatives are as in the following table. Therefore,

Rd.	w	c	b_i or $\bar{b}_i$	d_0	d_i
$q+1$	$12q-1$	$12q$	$12q+8i-1$ or $12q+8i-5$	$12q$	$12q$

Table 2: Plurality score in the $q+1$ round.

w is eliminated in round $q+1$, and c will become the winner.

Suppose WAV-STV$_\ell$ is a YES instance. We prove that RXC3 is a YES instance in the following steps.

Step 1. In the first q rounds, exactly one of b_i and $\bar{b}_i$ is eliminated for every $i \le q$. Firstly, the initial score of b_i and $\bar{b}_i$ is at most $6q+4i+q/3 \le 10q+q/3$, while the score of other alternatives is at least $10q+2/3q$. On the other hand, once one of b_i and $\bar{b}_i$ is eliminated, the other gets the transferred votes and has a score of more than $12q$. Therefore, in the first q round, in each round, either b_i or $\bar{b}_i$ is eliminated for a distinct i.

Step 2. Let $I = \{i : b_i \text{ is eliminated in the first } q \text{ rounds.}\}$. Then I must be the index set of an RXC3 solution. Firstly, for each $i \in I$, $\bar{b}_i$ needs at least one vote for P' to win b_i, and each vote in P' can contribute to at most one $\bar{b}_i$. Therefore, there are at most $q/3$ of $\bar{b}_i$ that beat b_i.

Then if I is not (the index set of) an RXC3 solution, there must exist some $x_j \in X$ that is not covered, and the corresponding d_j does not get any transferred votes. Then in the $q+1$ round, such d_j will be eliminated with $12q-2$ votes, and its vote will be transferred to w. Then w has a score of at least $24q-3$ which exceeds c all the time. Therefore, c cannot be the winner.

Therefore, once WAV-STV$_\ell$ is a YES instance, the index set of eliminated b_i in the first q rounds forms the index set of a solution to the RXC3, and RXC3 is a YES instance. □

Theorem 2. For every constant $L \ge 2$, WAV-$\overline{STV}_L$ is NP-complete.

The proof for the up-to-L case follows the top-ℓ case by replacing all ℓ to L. The construction of the WAV instance requires P' to make use of all L positions in every vote to make c the winner.

6 Maximin

For Maximin and Copeland, we leverage the following lemma to construct the instance in the reduction.

Lemma 1. For any constant $\ell \ge 2$, an arbitrary set of candidates M with $m \ge \ell$, and two arbitrary candidates $a, b \in M$, there exists a voting profile P with $poly(m)$ of top-ℓ ranking votes such that the weighted majority graph of P contains only one non-zero-weighted edge of $a \to b$ with weight 2.

Lemma 1 enables us to construct an arbitrary WMG with even edge weights in polynomial-many votes.

Proof. Our construction of P follows the spirit of McGarvey [McGarvey, 1953]. It takes two steps:
Step 1: We first construct a slightly different profile P'. For any ℓ-subset M_ℓ of M, and any permutation σ_{M_ℓ} on M_ℓ, P' contains a vote for $[\sigma_{M_\ell}(1) \succ \sigma_{M_\ell}(2) \succ \cdots \succ \sigma_{M_\ell}(\ell)]$. The number of votes in P' is $A_\ell^m = O(m^\ell)$. Due to symmetricity, all the candidates are tied in P', and the weights of all the edges are 0 in the WMG of P'.
Step 2: Pick one vote in P' such that b is ranked the top and a is ranked the second. P is constructed by swapping a and b in this vote while keeping all other votes unchanged in P'. Since the only change is the relative position between a and b in one vote, the WMG of P contains only one edge which is $a \to b$ with weight 2. And P also contains $O(m^\ell)$ edges. □

Theorem 3. For all constant $\ell \ge 2$, WAV-Maximin$_\ell$ is NP-complete.

Remark 2. Theorem 3 does not contradict Theorem 11 in [Menon and Larson, 2017]. The two papers adopt different definitions of the Maximin rule which may lead to different winners under top-truncated votes, as discussed in Remark 1.

Proof sketch. The membership of NP is held by running the vote and checking the winner. For the hardness, we give a reduction from RXC3. For an RXC3 instance $X = \{x_1, x_2, \ldots, x_q\}$ and $\mathcal{S} = \{S_1, S_2, \ldots, S_q\}$, we construct the following WAV-Maximin$_\ell$ instance.
Candidates. There are $2q+\ell$ candidates: $X \cup \mathcal{S} \cup \{c\} \cup W$, where $W = \{w_1, w_2, \cdots, w_{\ell-1}\}$. We assume $c \succ x_1 \succ x_2 \succ \cdots \succ x_q$ in tie-breaking.
Absent votes. $t = \frac{q}{3(\ell-1)}$. (Without loss of generality, we assume that q can be divided by $6(\ell-1)$. With not, we duplicate both X and $\mathcal{S}$ for $6(\ell-1)$ times.)
Known votes. The WMG of P is shown in Figure 2. (There are no edges inside W, X, or S in $WMG(P)$.) P can be constructed via Lemma 1 and adding one vote for $[c \succ w_1 \succ \cdots \succ w_{\ell-1}]$. Then in profile P, the min score of c is $-(q + \frac{q}{3(\ell-1)} + 1)$, of each w_i is $-(2q+1)$, of each x_i is $-q$ (from $S_j \ni x_i$), and for each S_j is $-2q$. The details of the construction can be found in Appendix C.

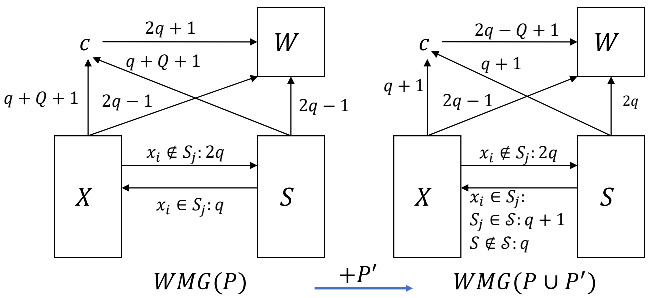


Figure 2: WMG for Maximin. $Q = \frac{q}{3(\ell-1)}$.

Intuition of construction. The min-score of each x_i comes from three $S_j \ni x_i$, and is exactly $\frac{q}{3(\ell-1)} + 1$ higher than c's min-score . To make c the winner, c appears in all votes in P' to increase its min-score by $\frac{q}{3(\ell-1)}$. Moreover, the S_j that appears in P' should consist of an exact 3-cover so that the min score of every x_i decreases by 1. If not, c will be beaten by some x_i not covered.

Suppose RXC3 is a YES instance. And let $\mathcal{S}^*$ be the exact 3-cover of X. Then we construct P' as follows: c is ranked the top and followed by $\ell - 1$ of S_j in each vote. The set of all the S_j ranked the second to the ℓ-th in P' (which is exactly $\frac{q}{3(l-1)} \times (l-1) = \frac{q}{3}$ of S_j) is exactly $\mathcal{S}^*$. In $P \cup P'$, the min-score of c is $(-q-1)$. For any x_i, since $\mathcal{S}^*$ is an exact 3-cover, there exists a $S_j \in \mathcal{S}^*$ such that $x_i \in S_j$. Since there is one vote that ranked S_j higher than x_i in P', the min-score of x_i in $P \cup P'$ is $-q-1$. The min-score of any S_j or any w_i will not exceed $-2q+1$. Therefore, c becomes the winner.

Suppose WAV-Maximin$_\ell$ is a YES instance. And let P' be a profile of t votes such that Maximin$_\ell(P \cup P') = c$. We proceed with the proof in two steps.

Step 1. Without loss of generality, we can assume that c is ranked the first in all votes in P'. If this is not the case, we can lift c to the first and keep the order of other candidates in every vote. Then the min-score of c will not decrease, and the min-score of any other candidate will not increase. Therefore, the new profile is also a solution.

Step 2. For every vote in P', the second to ℓ-th rank is some $S_j \in \mathcal{S}$, and the set of all these S_j (denoted by $\mathcal{S}^*$) is an exact 3-cover of X. First, the min-score of c in $P \cup P'$ is $-q-1$ since c is ranked top in all votes in P'. Suppose $\mathcal{S}^*$ is not an exact 3-cover of X, then there exists an x_i no covered by $\mathcal{S}^*$, and the min-score of x_i will be $-q$ from $S_j \ni x_i$, which is higher than c's min-score. Therefore, c cannot be the winner. Consequently, RXC3 is a YES instance with solution S^*.

The full proof is in Appendix C. □

Theorem 4. For any constant $L \geq 2$, WAV-$\overline{\text{Maximin}}_L$ is NP-complete.

The proof follows the top-ℓ case by replacing ℓ with L.

7 Copeland

Theorem 5. For any constant $\ell \geq 2$ and any $\alpha \in [0,1]$, WAV-Cd$_\ell^\alpha$ is NP-complete.

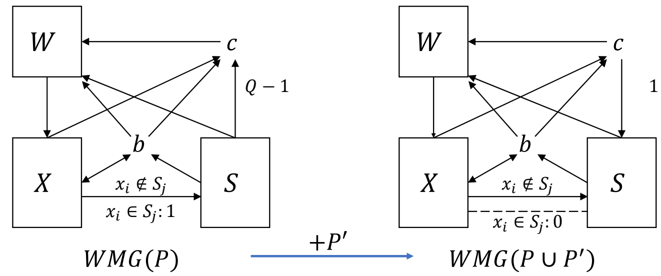


Figure 3: The WMG for Copeland. $Q = \frac{q}{3(\ell-1)}$.

Proof sketch. The membership of NP is held by running the vote and checking the winner. For the hardness, we give a reduction from RXC3. We apply a similar but slightly more complicated construction as in Maximin's proof. We present the construction for $\alpha < 1$ (more precisely, $\alpha < \frac{q-3}{q}$, which converges to 1 as q increases). The full proof including how to modify the construction for $\alpha = 1$ is in Appendix D. We assume that q can be divided by $6(\ell-1)$.

Candidates. There are $2q + \frac{q}{2} + 3$ candidates: $X \cup \mathcal{S} \cup \{c, b\} \cup W$, where $W = \{w_1, w_2, \cdots, w_{\frac{q}{2}+1}\}$. W.l.o.g, we assume $x_1 \succ x_2 \succ \cdots \succ x_q \succ c \succ w_1 \succ \cdots \succ w_{\frac{q}{2}+1}$ in tie-breaking.

Absent votes. $t = \frac{q}{3(\ell-1)}$.

Known votes. The weighted majority graph (with weights of some key edges) of P is in Figure 3. For edges inside S and X respectively (not shown in the figure), each candidate beats about half of the other candidates inside the group in the head-to-head competition and is beaten by the other half. The edges inside W will not affect the winner. The Copeland score for each candidate in P is as follows: c has $(\frac{q}{2}+1)$; x_i has $(q + \frac{q}{2} + 1)$; S_j has at most $(q+4)$; w_i has at most $(q + \frac{q}{2})$; and b has $(q+2)$.

Intuition of construction. The only edges that can be flipped by P' are $S_j \to c$ and $x_i \to S_j$ for $x_i \in S_j$. We set the weights so that c needs to win every S_j to become the winner, which requires every vote in P' to include c. On the other hand, every x_i needs to be tied with or be beaten by some $S_j \ni x_i$ to make c the winner. Therefore, the rest $\frac{q}{3}$ positions in P' will be taken by S_j that forms an exact 3-cover.

Suppose RXC3 is a YES instance with solution $\mathcal{S}^*$. We construct P': c is ranked the top and followed by $\ell - 1$ of S_j in each vote, and the set of all the S_j ranked second to the ℓ-th in P' is exactly $\mathcal{S}^*$. Then c becomes the candidate with the unique highest score $(q + \frac{q}{2} + 1)$ and becomes the winner.

Suppose WAV-Cd$_\ell^\alpha$ is a YES instance with solution P'. We could first show that all votes in P' must contain c. Then all S_j ranked in P' must form an exact 3-cover of X. Otherwise, there will be some x_i not covered, and the Copeland score of such x_i will also be equal to c. Then according to the tie-breaking rule, c cannot be the winner.

The full proof is in Appendix D. □

Similar to STV and Maximin, this proof also applies to the up-to-L setting by replacing ℓ with L.

Theorem 6. For any constant $L \geq 2$ and any $\alpha \in [0,1]$, WAV-$\overline{\text{Cd}}_L^\alpha$ is NP-complete.

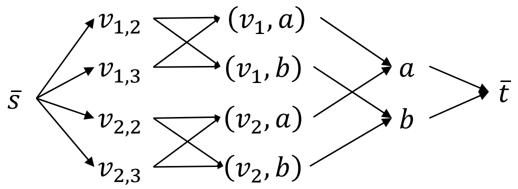


Figure 4: An illustration of the network when $M = \{a, b, c\}$, $t = 2$, and $\ell = 3$.

8 Positional Scoring Rules (PSR)

For scoring rules $\vec{s}_\ell = (a_1, a_2, \cdots, a_\ell)$, We show that the WAV problem is in P for the special case $a_2 = \cdots = a_\ell$.

Theorem 7. For any $\ell \geq 2$ and $\vec{s}_\ell$ such that $a_2 = \cdots = a_\ell$, WAV-$\vec{s}_\ell$ can be determined in polynomial time.

Proof sketch. We convert the problem into a maximum flow problem with a similar idea in [Baumeister *et al.*, 2012]. Given an instance of WAV-$\vec{s}_\ell$, we construct the following network flow. An illustration is in Figure 4.
Nodes: $\{\bar{s}, \bar{t}\} \cup T \cup TM \cup M$.

• $\bar{s}$ is the source node, and $\bar{t}$ is the sink node.

• T contains $(\ell - 1)t$ nodes. For each absent vote v, and $d = 2, \cdots, \ell$, there is a node v_d for the$d$-th position of v.

• TM contains $t(m-1)$ nodes. For each absent vote v and each candidate $a \neq c$, there is a node (v, a) in TM.

• M contains $m - 1$ nodes, each representing a candidate other than c.

Edges: $E_1 \cup E_2 \cup E_3 \cup E_4$.

• For each node $v_d \in T$, there is an edge $s \to v_d$ with capacity 1.

• For each vote v, each position d, and each candidate a, there is an edge $v_d \to (v, a)$ with capacity 1.

• For each vote v and each candidate $a \neq c$, there is an edge $(v, a) \to a$ with capacity 1.

• Let $a_2 = \cdots = a_\ell = A$. Let $\vec{s}(P, a)$ be the score of a from profile P. For each node $a \in M$, there is an edge $a \to \bar{t}$ with capacity $\lfloor \frac{\vec{s}(P,c) - \vec{s}(P,a) + t \cdot a_1}{A} \rfloor$. (We assume the capacities are non-negative. Otherwise, it is a NO instance.)

Interpretation of network. For a flow f:

• $f(v_d \to (v, a)) = 1$ stands for that a denotes that candidate a is ranked at d-th in vote v.

• $f((v, a) \to a) = 1$ stands for that a appears in the top-ℓ ranking in v. The capacity ensures that every candidate appears at most once in v.

• $A \cdot f(a \to \bar{t})$ stands for the total score a gets from P'.

The capacities in E_1, E_2, and E_3 guarantee that the profile P' is valid. The capacities in E_4 ensure that the total score of a does not exceed the total score of c from $P \cup P'$.

When the max-flow f is $(\ell-1)t$ (we assume f is an integer flow without loss of generality [Ford and Fulkerson, 1956]), it means that there is a way to fill all $(\ell - 1)t$ positions (2 to ℓ in t votes) by some candidates and form a valid profile P' such that no other candidates' scores exceed c. Therefore, c becomes the winner in $P \cup P'$. When WAV-$\vec{s}_\ell$ is a YES instance with solution P', we could assume that c is ranked the top at all votes $v \in P'$. Then we can construct a flow f of $(\ell - 1)t$ by setting the corresponding edge flows to 1.

The full proof is in Appendix E. □

Theorem 7 directly indicates that WAV-$\vec{s}_\ell$ is in P for any $\vec{s}_\ell$ when $\ell = 2$.

Corollary 1. WAV-$\vec{s}_2$ can be determined in polynomial time for any $\vec{s}_2$.

In the up-to-L setting, whether c can be a winner under an up-rounding scoring rule can be verified by checking the case when all votes in P' are $[c]$, i.e. rank c alone.

Proposition 3 ([Menon and Larson, 2017], Theorem 1)**.** For any constant $L \geq 1$ and any scoring vector $\vec{s}_L$, WAV-$\vec{s}_{L\uparrow}$ can be computed in polynomial time.

For the down-rounding scoring rule, we show a similar result as the top-ℓ setting.

Theorem 8. For any constant $L \geq 2$ and $\vec{s}_L$ such that $a_2 = \cdots = a_L$, WAV-$\vec{s}_{L\downarrow}$ can be determined in polynomial time.

Proof Sketch. Firstly, if a solution P' exists, we could assume without loss of generality that P' contains only top-1 ranking and top-L ranking, and c is ranked the top in all the votes. If this is not the case, we could substitute all non-top-L votes into $[c]$, and rank c the top of all the top-L votes while keeping the order of other candidates unchanged. In this way, the scoring of c is strictly increasing, while the score of all other candidates is non-increasing.

Then we give the algorithm outline. First, we enumerate the number of top-1 votes and top-L votes in P'. The sum of two kinds of votes is t. Therefore, there are in total $t+1$ cases. For each case, we set all top-1 votes to be $[c]$, and construct a maximum flow instance as in the proof of Theorem 7. If there is some case where the maximum flow is above the threshold, then we output YES. Otherwise, when all cases the maximum flow is below its threshold, we output NO. □

9 Conclusion and Future Work

We investigate the computational complexity of determining winners with absent votes when the votes are top-truncated. We have shown that the problem is in P when the number of candidates or the quantity of absent votes is bounded. In the unbounded cases, we show that the problem is NP-complete for STV, Maximin, and Copeland. We also give a special case of scoring rules where the problem can be computed in polynomial time. Winner determination with absent votes is closely related to the classic coalitional manipulation problem in social choice, yet previous results on full rankings do not directly extend to top-truncated settings.

A question that remains open in our paper is the complexity of WAV for general positional scoring rules. In the full-ranking setting, the complexity of coalitional manipulation is regarded as a challenging task. There are hardness results under an artificially constructed scoring vector [Xia *et al.*, 2010] and Borda [Betzler *et al.*, 2011]. Another related topic is to reduce the possible winners by eliciting extra information from voters via, for example, a query model.[1] We would care about protocols that may predict the final winner most accurately conditioned on query constraints.

[1]We thank an anonymous reviewer for proposing this idea

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