Airdrop Games

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Abstract

Launching a new blockchain system or application is frequently facilitated by a so called *airdrop*, where the system designer chooses a pre-existing set of potentially interested parties and allocates newly minted tokens to them with the expectation that they will participate in the system — such engagement, especially if it is of significant level, facilitates the system and raises its value and also the value of its newly minted token, hence benefiting the airdrop recipients. A number of challenging questions befuddle designers in this setting, such as how to choose the set of interested parties and how to allocate tokens to them. To address these considerations we put forward a game-theoretic model for such *airdrop games*. Our model can be used to guide the designer's choices based on the way the system's value depends on participation (modeled by a "technology function" in our framework) and the costs that participants incur. We identify both bad and good equilibria and identify the settings and the choices that can be made where the designer can influence the players towards good equilibria in an expedient manner.

1 Introduction

Launching a new blockchain system is challenging as it requires the upfront contributions of different parties, without any guarantee that the system will be successful. The characteristics of such launches are as follows:

- There is a set of possibly interested parties. Participating incurs some *cost*, hence the system designer performs an *airdrop* of tokens to entice the participants: a certain amount of the available tokens are distributed in advance to potential contributors, regardless of their (future) individual contribution to the system [Allen *et al.*, 2023].[1] Identifying the potential contributors typically piggybacks on an existing blockchain system e.g., as in "restaking" in Ethereum where new tokens are allocated based on existing staked ether holdings [EigenLabs, 2014], but more direct approaches have also been attempted, e.g., in worldcoin [Worldcoin, 2025], prospective users scan their retina in order to receive tokens.
- The eventual success of the system depends on the actual contributions and level of participation, which, in turn, reflects on the monetary value of the tokens received via the airdrop. The higher the overall participation of the players, the higher is the value of the new token, and thus also the value of the airdrop allocation received initially. The dependency between system value and participation can be modeled by an underlying *technology function* that we make explicit below.

Potential contributors thus face a dilemma: If they contribute, they incur a cost but (potentially) increase the value of their token allocation. Naturally, contributors should act strategically and contribute in a way that maximizes utility. Several equilibria exist: in good equilibria, "enough participation" is achieved and the launch of the system "succeeds" while in the bad equilibria a complete breakdown of the system is possible.

From the designer's perspective, some fundamental questions need to be addressed in order to understand how a project can be successfully launched:

> *What is the level of contribution of the parties that we can reasonably expect, given a specific allocation? How can this be influenced by different "tokenomics" policies that award larger or smaller amounts of tokens as part of the airdrop allocation? What kind of technology functions are more favorable in terms of facilitating a successful launch?*

In this work, we formally address these questions via a novel game-theoretic model and its analysis. To illustrate the nature of the problems, consider the following technology function:

Example 1 (Threshold Technologies.)**.** *Consider a system technology that requires contributions from at least 50% of the contributors (the total number being $N = 10$). If this* threshold *is met, the system operates correctly, and the token's value is* high*, \$10. Conversely, if the threshold is not reached, the system fails, and the token's value drops to* low*, at 0. With an airdrop granting each participant 1 token and contribution cost $\alpha = 1$, two equilibria emerge: (i) no one contributes, since an individual contribution alone does not*

[1] The nature of participation or contribution should be interpreted broadly and includes holding tokens, participating in governance, or actively running bespoke software that performs system functions.

increase the token's value but incurs a cost of $1, and (ii) exactly 50% contribute, that is 5 players. If any one of them contemplated not contributing, that would cause the token's value to drop from $10 to 0 (a net profit change from 10 - 1= 9 to 0). Also none of the 5 players not contributing has any incentive to contribute, since they are already enjoying the high value without any cost. Clearly, the latter equilibrium is preferable, and its existence for higher costs is guaranteed only if the designer sets the airdrop properly (e.g., for participation costs of $\alpha = 20$ *USD, an individual airdrop of at least 2 tokens would be required).*

The threshold-based technology described above is natural, but we can consider *other types* of technologies determining the system's value. For instance, [Alabi, 2017] suggests that several systems follow *Metcalfe's Law*: the value of a network is proportional to the square of the number ℓ of contributors (number of entities *holding* the native token in a wallet). In this case, the token's value follows $t(\ell) = q \cdot \ell^2$, with q a rescaling constant. The equilibria emerging with this technology are *different* from the example above, leading to different trade-offs when deciding on the number of tokens to airdrop.

1.1 Our Contribution

Our contributions can be summarized as follows.

Game-theoretic model (Section 2). We propose a game-theoretic model for airdrops. The model incorporates the key feature that token allocations are issued in some *new* token whose value: (i) is not determined at launch but, (ii) is affected by the actual participation or contribution of the potentially interested parties allocated these tokens according to some *technology function*. The analysis of equilibria in our model (see below) is informative to the designer and answers basic questions like "*Who receives tokens/how many?*" as posed in [Fröwis and Böhme, 2019]. Intuitively, the designer sets some "eligibility" criterion based on past information, which determines the number and the individual costs ("who receives tokens and why"), and the corresponding airdrop allocation ("how many tokens").

Analysis of equilibria (Section 3). We characterize the set of pure Nash equilibria for the general setting, as a function of *amount of rewarded tokens*, the *number of potential contributors*, their *individual costs*, and the *technology* which "converts" individual contributions into system value. We show that the model's general version corresponds to a *potential game* (Theorem 1). Thus, pure Nash equilibria always exist and are reached via simple best response dynamics. We further characterize the set of pure Nash equilibria in Section 3.1. It is worth noting that (i) we consider *heterogeneous* costs, that is, players have different costs in general, (ii) pure Nash equilibria do *not* require players to know about others' costs but only about others' strategies (contributions), and (iii) these equilibria are quite natural as they arise from simple best response (as opposed to *mixed* Nash equilibria for which no "simple" dynamics exist, and whose empirical support is comparably limited at the level of individual player behavior in the lab or field). These considerations are also fundamental for the designer, as a target.

Refined unique (logit) equilibria (Section 3.2). For some technologies, *bad* equilibria where no player contributes coexist with *good* equilibria where a sufficiently high level of players contribution is reached, thus making the system valuable. We consider a well-known class of "noisy" best response dynamics, termed *logit dynamics* [Blume, 1993; Blume, 2003] (see Section 1.2 for further discussion) for our model. These seem natural in our context and do not require excessive sophistication from the players. We first show that, in the so-called *vanishing noise regime*, these dynamics select only *stochastically stable* pure Nash equilibria, characterized by Theorem 3, thus discarding *bad* equilibria in several cases (see below). We also consider the so-called *finite noise* regime and the corresponding *unique* stationary equilibrium [Auletta *et al.*, 2011]. Under some mild restrictions on the model (Section 4), we provide tight bounds on the *time* for the dynamics to reach its equilibrium or a particular level of contribution (Section 4.1). Time is also a fundamental aspect for the designer, as the system needs to reach a "sufficiently good" state within the given launch period (after which the system is supposed to start its normal autonomous operations).

Applications to relevant technology functions (Section 5). Our model accommodates generic technology functions to express the token value depending on the actual contributions of the tokens' receivers. We apply our results to the important class of *threshold* technologies, a natural, simple description of systems which need an "initial minimal base" to succeed [Chaidos *et al.*, 2023; Arieli *et al.*, 2018; Yan and Chen, 2021; Jiménez-Jiménez *et al.*, 2021; Chang, 2020; Shao *et al.*, 2023; Wang *et al.*, 2023]. Threshold technologies are a typical example where *bad* equilibria with zero participation always exist, along with *good* equilibria, so there is a need for theory to explain how successful systems can reach good equilibria. Threshold technologies represent a "hard case" in our setting, in the sense that bad equilibria persist for any airdrop reward amount. Logit dynamics provide a formal argument that good equilibria are more likely to be chosen (Theorem 6 and Corollary 2) also in this hard case. The analysis further indicates when it is optimal for the designer to perform an airdrop at all (and the optimal rewards) in both the vanishing- and non-vanishing regimes (Sections 5.1 and 5.2).

The analysis of threshold technologies highlights the need for *low costs* to converge to a desired state within reasonable time (Section 5.3). This highlights the benefits of recent technological developments like Ethereum's restaking [EigenLabs, 2014] or Cardano's *partnerchain* framework [Ward, 2024], which "reuse" contributors from a mainchain who are likely to incur lower costs (see [Georganas *et al.*, 2025]).

We stress that our results can be applied to other technology functions, including Meltcafe's Law and other examples in the literature (details in [Georganas *et al.*, 2025]).

1.2 Related Work

Airdrops. Airdrops are costly as an action, as recording them on an existing chain incurs transaction fees, necessitating the designer to implement *simple* allocation strategies [Fröwis and Böhme, 2019; Lommers *et al.*, 2023]. Empirical studies suggest simple airdrops will remain common for "projects without established on-chain activity" [Allen, 2024], while others [Messias *et al.*, 2023] recommend "scaling rewards with costs", aligning with our theoretical findings. Additional work on practical features and goals of airdrops includes [Makridis *et al.*, 2023; Yaish and Livshits, 2024; Fan *et al.*, 2023; Lommers *et al.*, 2023].

Related Games and Models. Our model can be seen as a variant of *blockchain participation games* [Chaidos *et al.*, 2023] and the *combinatorial agency* model [Babaioff *et al.*, 2012] in contract theory. In the former, players receive a *monetary* reward, contrary to our setting where the rewards are tied to the system value (the variant of universal payments with no retraction is the closest to ours, whereas in other variants rewards are even more loosely tied to the system's success); the system value is a threshold technology of (eligible) players actively contributing. In the combinatorial agency model, players receive again a *monetary* reward conditioned on the success of the project, expressed by some "success probability" function on the contributing players. Consequently, the equilibria in these two models are different from ours and these results do not apply to our setting.

A closely related class of problems is *crowdfunding* games [Arieli *et al.*, 2018; Yan and Chen, 2021; Jiménez-Jiménez *et al.*, 2021; Chang, 2020; Shao *et al.*, 2023; Wang *et al.*, 2023]. Similar studies concern public funding projects [Soundy *et al.*, 2021; Bilò *et al.*, 2023] and public goods projects on networks [Bramoullé and Kranton, 2007; Galeotti *et al.*, 2010; Dall'Asta *et al.*, 2011; Yu *et al.*, 2020]. These models are mathematically equivalent to our games in the case where the designer *cannot change the rewards*. Specific problems correspond to technology functions being S-shaped [Buragohain *et al.*, 2003] or a threshold function [Galeotti *et al.*, 2010]. Stochastic stability is studied in [Boncinelli and Pin, 2012]. All these problems (and ours) belong to rich class of public goods games. These are generally computationally hard, even when the underlying (technology) function is specified by a network [Papadimitriou and Peng, 2021; Gilboa and Nisan, 2022; Klimm and Stahlberg, 2023; Do Dinh and Hollender, 2024; Galeotti and Goyal, 2010].

Logit Dynamics. Logit dynamics have been largely studied in the context of games and equilibrium selection problem, that is, as a refinement of pure Nash equilibrium (see e.g. [Blume, 1993; Blume, 2003; Montanari and Saberi, 2009; Asadpour and Saberi, 2009; Alós-Ferrer and Netzer, 2010; Alós-Ferrer and Netzer, 2015; Auletta *et al.*, 2011; Auletta *et al.*, 2012; Auletta *et al.*, 2013b; Auletta *et al.*, 2013a; Okada and Tercieux, 2012; Coucheney *et al.*, 2014; Ferraioli *et al.*, 2016; Mamageishvili and Penna, 2016; Ferraioli and Ventre, 2017; Alós-Ferrer and Netzer, 2017; Penna, 2018; Kleer, 2021]). This is usually done in two ways. The first is to consider the so-called *vanishing noise* regimes and a resulting set of *stable equilibria* (see e.g. [Blume, 1993; Alós-Ferrer and Netzer, 2010; Alós-Ferrer and Netzer, 2015]). The second is to consider *non-vanishing noise* regimes and the corresponding unique stationary distribution of the process as the equilibrium concept [Auletta *et al.*, 2011]. The logit response model also finds applications in economics [Costain and Nakov, 2019], in pricing algorithms [Müller *et al.*, 2021; van de Geer and den Boer, 2022], and coalitional bargaining [Sawa, 2019].

2 Modelling Airdrop Games

The model captures the following key aspects of airdrops: (i) The designer chooses the amount of tokens to be airdropped to potential contributors. (ii) Potential contributors decide whether to perform a (costly) task for the system. The resulting system value depends on the total contribution and the underlying "technology" of the project. (iii) Contributors maximize utility, resulting in an *equilbrium* and system value.

The designer faces a tradeoff between the airdrop amount and the resulting system value (too small airdrops do not incentivize enough contributors and thus the project fails, while giving away all tokens is not optimal either because it minimizes profit).

Parameters and Underlying Game

Contributors. There is a set of n potential contributors (players): Each contributor chooses her actual contribution (strategy) $a_i \in A_i \subseteq \mathbb{R}^+$, incurring in a cost of $c_i \cdot a_i$ where c_i denotes the per unit cost of i.

System (technology) value. The overall value of the system depends on each individual effort or contribution, i.e., on the strategy profile $a = (a_1, \ldots, a_n)$ and it is equal to $V(a)$ for some monotone non-decreasing function (higher contributions yield the same or higher value).

Token value (and total supply). Given the token total supply (the overall number of tokens of in system) T_{tot}, the value or price of the token is

$$t(a) := \frac{V(a)}{T_{tot}} . \tag{1}$$

Airdrop (Token) Allocation. The designer allocates some constant fraction $\rho \in [0, 1]$ of the overall token supply as an airdrop, i.e., to be distributed equally among the players. Thus, each player receives γ tokens, where

$$\gamma := \frac{\rho \cdot T_{tot}}{n} \qquad\qquad \rho \in [0, 1] . \tag{2}$$

(For the sake of simplicity, we allow γ to be a fractional number, and ignore rounding effects.) It is worth noting that the monetary reward (number of tokens times the token price) is *independent* on the token total supply T_{tot}, and it equals to a fraction ρ/n of the system value,

$$\gamma \cdot t(a) \overset{(1)+(2)}{=} \frac{\rho \cdot T_{tot}}{n} \cdot \frac{V(a)}{T_{tot}} = \frac{\rho}{n} \cdot V(a) . \tag{3}$$

Utilities. Players' utilities equals the monetary reward received (number of tokens times the token value) minus the incurred cost

$$u_i(a) := \gamma \cdot t(a) - c_i \cdot a_i \overset{(3)}{=} \frac{\rho}{n} \cdot V(a) - c_i \cdot a_i . \tag{4}$$

Equilibria. A strategy profile $a = (a_1, \ldots, a_n)$ is a pure Nash equilibrium if no player i can increase her utility by changing her strategy a_i, that is,

$$u_i(a) \geq u_i(a') \quad \overset{(4)}{\iff} \quad \frac{\rho}{n} \cdot (V(a) - V(a')) \geq c_i \cdot (a_i - a'_i) \tag{5}$$

for all i and all $a' = (a_1, \ldots, a_{i-1}, a'_i, a_{i+1}, \ldots, a_n)$.

Logit dynamics. Logit dynamics [Blume, 1993; Blume, 2003] are a kind of "noisy" best response dynamics where players have some *inverse noise* or *learning rate* $\beta \geq 0$ and each of them responds according to a so-called *logit response*

$$p_i^\beta(a_i|a_{-i}) = \frac{\exp(\beta u_i(a_i, a_{-i}))}{Z_i^\beta} \tag{6}$$

where Z_i^β is a normalizing factor so that the above formula is a probability distribution, and $(x, a_{-i}) := (a_1, \ldots, a_{-i}, x, a_{i+1}, \ldots, a_n)$. For $\beta = 0$, players choose a strategy at random with uniform distribution, while for $\beta \to \infty$, they tend to the best response rule.[2] At each step of the dynamics, a randomly chosen player revises her current strategy according to the logit response above (6). Logit dynamics converge to a *unique equilibrium* π^β given by the *stationary distribution* of the underlying Markov chain: $\pi^\beta(a)$ is the probability of players selecting profile a after sufficiently many steps of revisions ("learning") have been performed. Note that this depends on the parameter β. For the class of exact potential games, in the vanishing noise regime ($\beta \to \infty$), equilibrium π^β concentrates on the *subset* of pure Nash equilibria whose potential is optimal [Blume, 1993; Blume, 2003]. The *mixing time* of the underlying Markov chain is the time required by the dynamics to reach the equilibrium π^β starting from *any* state [Levin *et al.*, 2006; Auletta *et al.*, 2011].

Objectives and Metrics

There are different (possibly conflicting) metrics to evaluate system performance, given contributions, costs, the designer's profit etc.

System Value. This is the value specified by the technology function $V(a)$ as a function of all contributions.

Social Cost. This is the sum of all players costs,

$$SC(a, c) := \sum_i c_i \cdot a_i \; . \tag{7}$$

Users' Welfare. This is the sum of all players' utilities (4),

$$UW(a, \rho, c) := \sum_i u_i(a) \overset{(4)+(7)}{=} \rho \cdot V(a) - SC(a, c) \; . \tag{8}$$

Profit. This is the value of the remaining tokens remaining with the designer, after subtracting the airdropped tokens and the cost d_V for developing the technology $V(\cdot)$,

$$\mathsf{profit}(a, \rho) = (1 - \rho) \cdot V(a) - d_V \; . \tag{9}$$

The designer strategically chooses the airdrop allocation ρ aiming to maximize its profit (utility) defined as in (9).[3] We consider which system values can be achieved given the underlying technology, players' costs, and their utility maximizing strategies. Note that the designer's profit can be *negative* (also when the former does not provide any reward, $\rho = 0$). This corresponds to inherently "bad" projects that are destined to fail and should not be implemented. More generally, we consider a technology "implementable" or "profitable" if there is some equilibrium (also in randomized sense – Section 3.2) which yields a positive profit to the designer.

Special Cases

We shall sometime consider the following relevant restrictions on the technology functions, the possible contribution levels, the possible costs, and combinations thereof.

Anonymous Technologies. It is natural to assume that the value of the system simply depends on the overall level of contribution $\ell = \sum_i a_i$, that is, $V(a) = V(a')$ whenever $\sum_i a_i = \sum_i a'_i$. We refer to this case as the *anonymous* technology function. With slight abuse of notation, we write $V(\ell)$ instead of $V(a)$.

Binary Contributions. In some settings where potential contributors have only two options (strategies), either to contribute ($a_i = 1$) or to not contribute ($a_i = 0$), we refer to this restriction as *binary contributions*.

Uniform Costs. It is natural to consider equal cost for all (e.g. if they are fully determined by the type of hardware/resources required). This means $c_i = \alpha$, with $\alpha > 0$.

3 Airdrop Games: Main Characteristics

In this section, we consider games with utilities in (4), airdrop allocations (2) in full generality. We show that these games are always potential games (Theorem 1), implying that (i) best/better response dynamics always converge to pure Nash equilibria and (ii) logit dynamics equilibria can be characterized in terms of potential, implying that bad equilibria are not selected under vanishing noise (Section 3.2).

Theorem 1. *For airdrop allocation* (2)*, the game in* (4) *with arbitrary efforts and any technology function is an exact potential game with potential function*

$$\phi(a) := \gamma \cdot t(a) - SC(a) = \frac{\rho}{n} \cdot V(a) - SC(a) \; . \tag{10}$$

3.1 Characterization of pure Nash equilibria

The next theorem characterizes the set of pure Nash equilibria. Intuitively, airdrop allocations should be neither too high (otherwise players can benefit by increasing their contribution) nor too low (otherwise they can benefit by reducing their contribution). Note an important *asymmetry* between the two cases: increasing the contribution is never beneficial when the system's value does not change, while decreasing the contribution is always advantageous under these conditions.

Theorem 2. *For any technology function and arbitrary efforts, and for airdrop allocations* (2)*, the set of pure Nash equilibria is given by the strategy profiles a such that the following two conditions hold for all i:*

1. *For all $a^+ = (a_i^+, a_{-i})$ with $a_i^+ > a_i$:* $\frac{\rho}{n} \leq c_i \cdot \frac{a_i^+ - a_i}{V(a^+) - V(a)}$ *whenever* $V(a) < V(a^+)$.
2. *For all $a^- = (a_i^-, a_{-i})$ with $a_i^- < a_i$:* $\frac{\rho}{n} \geq c_i \cdot \frac{a_i - a_i^-}{V(a) - V(a^-)}$ *and* $V(a) > V(a^-)$.

[2] In case multiple best response exist, the corresponding player chooses any of them with uniform distribution.

[3] Note that we still assume the designer to move first by announcing the airdrop ρ, and then the (other) players will reach some equilibrium accordingly.

The next corollary provides a more convenient characterization for certain restrictions of interest.

Corollary 1. *For any technology function and binary efforts, and for airdrop allocations* (2)*, the set of pure Nash equilibria is given by the strategy profiles a such that the following two conditions hold:*

1. *For all i such that $a_i = 0$: $\frac{\rho}{n} \leq \frac{c_i}{V(1,a_{-i})-V(a)}$ or $V(1, a_{-i}) = V(a)$.*
2. *For all j such that $a_j = 1$: $\frac{\rho}{n} \geq \frac{c_j}{V(a)-V(0,a_{-j})}$ and $V(0, a_{-j}) < V(a)$.*

Moreover, for any anonymous technology function and binary efforts, the set of strategy profiles a which are an equilibrium corresponds to those satisfying these two conditions:

1. *For $\ell < n$: $\frac{\rho}{n} \leq \frac{c^{(0)}_{\min}}{V(\ell+1)-V(\ell)}$ or $V(\ell + 1) = V(\ell)$.*
2. *For $\ell > 0$: $\frac{\rho}{n} \geq \frac{c^{(1)}_{\max}}{V(\ell)-V(\ell-1)}$ and $V(\ell - 1) < V(\ell)$.*

where $c^{(0)}_{\min} = \min\{c_i : a_i = 0\}$ is the smallest cost among players not contributing, and $c^{(1)}_{\max} = \max\{c_i : a_i = 0\}$ is the largest cost among players contributing.

Example 2 (linear technology with heterogeneous costs)**.** *For linear technologies, $V(\ell) = \lambda_V \cdot \ell$ with $\lambda_V > 0$, the set of pure Nash equilibria is characterized by the ℓ^* such that (w.l.o.g. assume $c_1 \leq c_2 \leq \cdots \leq c_n$) these inequalities hold: $\frac{c_{\ell^*}}{\lambda_V} \leq \frac{\rho}{n} \leq \frac{c_{\ell^*+1}}{\lambda_V}$. The optimal strategy (profit maximizing) for the designer is to choose the minimum ρ satisfying the above condition, thus making the first inequality tight: $\rho^* = argmax_{\ell\in[n]}(1 - \rho^*_\ell) \cdot \lambda_V \cdot \ell\, , \rho^*_\ell = n \cdot \frac{c_{\ell^*}}{\lambda_V}$, which is equivalent to set $\rho^* = argmax_{\ell\in[n]}(\lambda_V - n \cdot c_\ell) \cdot \ell$.*

3.2 Logit response equilibria

We next consider logit dynamics and the corresponding equilibria. Specifically, for the vanishing noise regime ($\beta \to \infty$), the dynamics selects a subset of so called *stochastically stable* pure Nash equilibria (see e.g. [Asadpour and Saberi, 2009]).

Theorem 3. *For any technology function and arbitrary efforts, for airdrop allocations* (2)*, and for vanishing noise ($\beta \to \infty$), the dynamics converges with probability which tends to 1 to states of maximal potential.* [4] *In particular, the corresponding stationary distribution π_ρ, depending on the airdrop allocation ρ, satisfies*

$$\lim_{\beta\to\infty}(\pi_\rho(a)) = \begin{cases} \frac{1}{|\mathsf{POTMAX}_\rho|} & \text{for } a \in \mathsf{POTMAX}_\rho \\ 0 & \text{for } a \notin \mathsf{POTMAX}_\rho \end{cases} \tag{11}$$

where $\mathsf{POTMAX}_\rho := argmax_a\{\phi(a)\}$ is a subset of equilibria which depends on the airdrop allocation ρ as follows:

$$\mathsf{POTMAX}_\rho = argmax_a \left\{ \frac{\rho}{n} \cdot V(a) - SC(a) \right\} . \tag{12}$$

The next example shows that, the bad equilibria in which no player contributes are selected with *vanishing probability*, provided the airdrop allocation ρ is sufficiently high.

[4]Note that in this work we do not change sign in the definition of exact potential game and related dynamics.

Example 3 (rule out bad equilibria)**.** *We consider a two-player game with a simple anonymous technology function, binary contributions, and uniform costs ($c_i = \alpha$). The technology function is an* AND *technology where a high (non-zero) value is achieved only if both players contribute, $V(0) = V(1) = 0$ and $V(2) > 0$, which implies that $(0, 0)$ and $(1, 1)$ are the only two Nash equilibria. Theorem 3 says that logit-response dynamics with vanishing noise ($\beta \to \infty$) the good equilibrium $(1, 1)$ is reached with probability tending to 1 if and only if the airdrop allocation ρ satisfies $\frac{\rho}{2} \cdot (V(2) - V(0)) > 2\alpha$. Also note that, since $\rho \leq 1$, this holds only for $V(2) - V(0) > 4\alpha$.*

To repeat the intuition here, when contributors are prone to some experimentation, instead of just picking best responses, the system is likely to end up in the high value equilibrium instead of the low one.

Profit and optimal airdrops revisited (logit dynamics)
We observe the same tradeoff regarding the optimal choice of airdrop ρ. A too small ρ may result in a bad equilibrium in which none cooperates, thus a system with small value $t(0)$. A very high ρ, on the other hand, will leave the system designer with only a tiny fraction of the system ownership (value). In order to deal with the logit (randomized) equilibrium π_ρ we simply consider the expected value of the system, and extend the definition of profit (9) in the natural way:

$$\begin{aligned} \mathsf{profit}(\rho) &= (1-\rho)V(\pi_\rho) - d_V, \\ \text{where } V(\pi_\rho) &:= \mathop{\mathbb{E}}_{a\sim\pi_\rho}[V(a)] . \end{aligned} \tag{13}$$

4 Binary Efforts and Uniform Costs

We consider a *binary effort* scenario where players contribute or not ($a_i \in \{0, 1\}$) and costs are *uniform* ($c_i = \alpha$ for all i). Under these restrictions, our model is characterized by three parameters: (i) α is the cost per player when contributing; (ii) β is the rationality level of the players; (iii) ρ is the airdrop allocation – the corresponding number of tokens γ is given by (2). While the rationality level β is exogenous to the system, the designer can change ρ, and costs α are part of the airdrop design (see [Georganas *et al.*, 2025] for practical examples where the designer can reduce α).

4.1 Logit equilibria and convergence time

In this section, we make use of *birth and death processes* to analyze the case of binary effort and airdrop rewards, when also the technology function is *symmetric*. In this case, the dynamics boils down to a birth and death process, where we keep track of the number ℓ of actively participating players in a given profile a, i.e., $\ell = \sum_i a_i$ and $a_i \in \{0, 1\}$. Hence, the birth and death process has $n + 1$ states, $\ell \in [0, n]$. The stationary distribution is thus

$$\hat{\pi}(\ell) := \binom{n}{\ell} \cdot \pi(\ell) \tag{14}$$

where $\pi(\ell)$ is the stationary distribution of a generic state a with $\sum_i a_i = \ell$, and the binomial coefficient counts the number of such states in the original Markov chain which are "grouped together" in the birth and death process. The birth and death process has transition probabilities $p(\ell)$ and $q(\ell)$ of moving "up by one" or "down by one", respectively, given by

$$p(\ell) = \frac{n-\ell}{n} \cdot p_i^\beta(1|a_{-i}) \ , \quad q(\ell) = \frac{\ell}{n} \cdot p_i^\beta(0|a_{-i}) \quad (15)$$

where $p_i^\beta(\cdot)$ is the logit response (6).

Since our process in (15) is an irreducible birth and death chain, the following "sharp" bound on the *mixing time* holds.

Theorem 4 (Theorem 1.1 in [Chen and Saloff-Coste, 2013]). *Let ℓ_0 be a state satisfying $\hat{\pi}([0,\ell_0]) \geq 1/2$ and $\hat{\pi}([\ell_0, n]) \geq 1/2$, where $\hat{\pi}(I) := \sum_{\ell \in I} \hat{\pi}(\ell)$, and set*

$$\mathcal{T}_{\mathsf{cutoff}} = \max \left\{ \sum_{\ell=0}^{\ell_0 - 1} \frac{\hat{\pi}([0,\ell])}{\hat{\pi}(\ell) p(\ell)}, \sum_{\ell=\ell_0+1}^{n} \frac{\hat{\pi}([\ell,n])}{\hat{\pi}(\ell) q(\ell)} \right\} . \quad (16)$$

Then the mixing time of the logit dynamics satisfies $\mathcal{T}_{\mathsf{mix}} = \Theta(\mathcal{T}_{\mathsf{cutoff}})$ and, in particular, the following bounds hold:

$$(1/24) \cdot \mathcal{T}_{\mathsf{cutoff}} \leq \mathcal{T}_{\mathsf{mix}} \leq 288 \cdot \mathcal{T}_{\mathsf{cutoff}} \ . \quad (17)$$

Note that such an ℓ_0 always exists (sum up all $\hat{\pi}(\ell)$ from 0 until the smallest ℓ_0 where the sum of these probabilities is at least $1/2$). We next introduce useful definitions to analyze the *hitting time* of a target ℓ, based on the technology's "local steepness".

Definition 1. *We define the drift at location ℓ as the ratio $d(\ell) := \hat{\pi}(\ell+1)/\hat{\pi}(\ell)$. We also say that technology function V is s-steep at some interval $I = [\ell_1, \ell_2]$ if $V(\ell+1) - V(\ell) \leq s$ for all $\ell \in I \setminus \{\ell_2\}$.*

Intuitively, the drift describes the tendency of the process to "move down" (drift < 1) or "move up" (drift > 1). The following theorem states that the hitting time for a target value ℓ grows exponentially with the length of any interval, preceding the target value, where the tendency to move down persists. The theorem further connects the drift to the "flatness" of the technology function (see the threshold function in Section 5). Its proof is based on bounds in [Palacios and Tetali, 1996].

Theorem 5. *The hitting time $\mathcal{T}_{\mathsf{hitting}}(\ell)$ of the logit dynamics to reach a state with contribution level ℓ starting tom the state with contribution level $\ell = 0$ can be bounded as follows:*

1. *If the drift in some interval $I = [\ell_1, \ell_2]$ is at most d_I, then $\mathcal{T}_{\mathsf{hitting}}(\ell) \geq (1/d_I)^{|I|}$ for all $\ell > \ell_2$.*
2. *If V is s-steep in some interval $I = [\ell_1, \ell_2]$, then for all $\ell > \ell_2$ it holds that $\mathcal{T}_{\mathsf{hitting}}(\ell) \geq \left(\exp\left(-\beta\left(\frac{\rho}{n} \cdot s - \alpha\right)\right) \cdot \frac{\ell_1+1}{n-\ell_1}\right)^{\ell_2-\ell_1}$.*

5 Application to Threshold Technologies

We analyze a *threshold technology*, modelling scenarios in which the system is either highly valuable if the overall contribution of the players reaches a certain threshold τ, and less valuable otherwise:

$$V(\ell) = \begin{cases} V_{\mathsf{low}} & \ell < \tau \\ V_{\mathsf{high}} & \ell \geq \tau \end{cases} \qquad V_{\mathsf{low}} < V_{\mathsf{high}} \ , \quad (18)$$

where ℓ is the number of actively participating players, i.e., $\ell = \sum_i a_i$ and $a_i \in \{0,1\}$. The corresponding token values according to (1) are thus $t_{\mathsf{low}} = V_{\mathsf{low}}/T_{tot}$ and $t_{\mathsf{high}} = V_{\mathsf{high}}/T_{tot}$. We are interested in the probability that the underline dynamics selects the high value (optimal) outcome,

$$p_{\mathsf{high}}(\rho) := \Pr_{a \sim \pi_\rho}[V(a) = V_{\mathsf{high}}] \ . \quad (19)$$

5.1 Stochastic stability ($\beta \to \infty$ regime)

Theorem 6. *For any threshold technology* (18) *with airdrop rewards* (2) *and vanishing noise ($\beta \to \infty$), the probability of selecting the high value outcome* (19) *undergoes a sharp transition given by the rewards ρ:*

$$\lim_{\beta\to\infty} p_{\mathsf{high}}(\rho) = \begin{cases} 1 & \rho > \rho_c \\ 0 & \rho < \rho_c \end{cases}, \quad \rho_c := \frac{\alpha \cdot n \cdot \tau}{V_{\mathsf{high}} - V_{\mathsf{low}}} \ . \quad (20)$$

For the edge case where $\rho = \rho_c$, the probability satisfies $\lim_{\beta\to\infty} p_{\mathsf{high}}(\rho) = 1/\left(1 + \binom{n}{\tau}\right)$.

An immediate corollary of the previous result follows. Intuitively, the corollary states that there exists three regions: (i) for very high cost, the probability of selecting the good outcome vanishes no matter how we set the rewards, and thus the optimal strategy of the designer is to set no airdrop, (ii) for intermediate costs, though it is possible to set $\rho > 0$ such that the probability of selecting the good outcome tends to one, the designer still prefer to set $\rho = 0$, and (iii) for low costs there is $\rho > 0$ maximizing the designer's profit and making the probability of selecting the good outcome going to one.

Corollary 2. *For any threshold technology* (18) *with airdrop rewards* (2) *and vanishing noise ($\beta \to \infty$), the probability of selecting the high value outcome* (19) *is as follows:*

1. *For $\alpha \cdot n \cdot \tau > V_{\mathsf{high}} - V_{\mathsf{low}}$ the probability of selecting the high value outcome vanishes for any airdrop reward ρ. Hence, and the best strategy for the designer is to give no airdrop rewards, which guarantees a profit of $V_{\mathsf{low}} - d_V$.*
2. *For $\alpha \cdot n \cdot \tau < V_{\mathsf{high}} - V_{\mathsf{low}}$ the probability of selecting the high value outcome tends to 1 for any airdrop reward $\rho > \rho_c$. The optimal strategy (profit maximizing) for the designer is as follows:*
   - *(a) For $\alpha \cdot n \cdot \tau \geq (V_{\mathsf{high}} - V_{\mathsf{low}}) \cdot (1 - V_{\mathsf{low}}/V_{\mathsf{high}})$ it is still optimal for the designer to give no rewards (causing the probability of selecting the good outcome to vanish).*
   - *(b) For $\alpha \cdot n \cdot \tau < (V_{\mathsf{high}} - V_{\mathsf{low}}) \cdot (1 - V_{\mathsf{low}}/V_{\mathsf{high}})$ the best strategy for the designer is to set airdrop rewards slightly above $\rho_c < 1$, which guarantees a profit of $(1 - \rho_c - \epsilon)V_{\mathsf{high}} - d_V$ for any small $\epsilon > 0$.*

Note that the "intermediate" regime in part 2a of the corollary above occurs only for $V_{\mathsf{low}} > 0$. Here high rewards *could* make the system succeed, but they are not optimal for the designer. For $V_{\mathsf{low}} = 0$ we have a single transition (either provide no airdrop or set the airdrop slightly above ρ_c).

5.2 Non-vanishing noise (β finite regime)

In this section, we analyze logit dynamics for threshold technologies in the case of non-vanishing inverse noise $\beta > 0$. Research suggest that in practice people respond according to some specific value of β which is approximately the same across different games and situations they face (details in

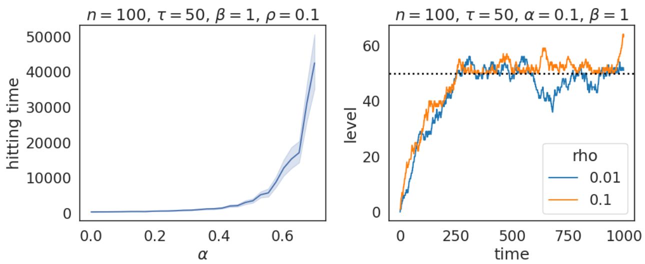

Figure 1: On the left, larger costs α increase the hitting time (100 repetitions 95% confidence). On the right, larger rewards values ρ help to maintain the dynamics above the threshold once it is reached.

[Georganas *et al.*, 2025]). The next result provides useful bounds on the probability that the high value outcome is selected at equilibrium by the dynamics.

Theorem 7. *For any threshold technology* (18) *with airdrop rewards* (2) *and any inverse noise parameter* $\beta > 0$*, the probability of selecting the high value outcome* (19) *is monotone increasing in the rewards* ρ *and, in particular, it has the following form:*

$$p_{\mathsf{high}}(\rho) = \frac{1}{1 + C \cdot \exp(-\rho B)}, \quad B = \frac{\beta}{n} \cdot (V_{\mathsf{high}} - V_{\mathsf{low}}) \,,$$

where $C = C(\alpha\beta, n, \tau) = \frac{1-p_{\mathsf{high}}(0)}{p_{\mathsf{high}}(0)}$ *does not depend on rewards* ρ *nor on the values* V_{low} *and* V_{high}*.*

Based on the result above, we are able to characterize the optimal airdrop rewards for the designer.

Theorem 8. *For any threshold technology* (18) *with* $V_{\mathsf{low}} = 0$*, and with airdrop rewards* (2) *and any inverse noise parameter* $\beta > 0$*, the designer's profit* (13) *is*

$$\mathsf{profit}(\rho) = V_{\mathsf{high}} \cdot \frac{1-\rho}{1 + C \cdot \exp(-\rho \cdot B)} - d_V \,, \qquad (21)$$

where quantities C *and* $B = \frac{\beta}{n} \cdot V_{\mathsf{high}}$ *are defined as in Theorem 7. Moreover the following holds:*

1. *For* $n \geq \beta \cdot V_{\mathsf{high}}$ *the optimal strategy (profit maximizing) for the designer is to give no airdrop rewards, which guarantees a profit of* $V_{\mathsf{high}} \cdot p_{\mathsf{high}}(0) - d_V$*.*
2. *For* $n < \beta \cdot V_{\mathsf{high}}$ *the optimal strategy (profit maximizing) for the designer is to set an airdrop reward* $\rho \leq \bar{\rho} := 1 - 1/B = 1 - \frac{n}{\beta V_{\mathsf{high}}}$*. The probability of selecting the high value outcome for the designer's optimal rewards* ρ^* *is bounded as follows:* $p_{\mathsf{high}}(\rho^*) \leq p_{\mathsf{high}}(\bar{\rho}) = \frac{1}{1+C\cdot\exp(1-B)} = \frac{1}{1+C\cdot\exp(1-\beta\cdot V_{\mathsf{high}}/n)}$*.*
3. *For* $n < \beta \cdot V_{\mathsf{high}} \cdot (1 - p_{\mathsf{high}}(0))$ *the optimal strategy (profit maximizing) for the designer is to give strictly positive airdrop rewards.*

5.3 Convergence time

In this section, we study the *time* for logit dynamics to converge to its equilibrium (stationary distribution) and to the good outcome of the threshold function ($\ell \geq \tau$). Specifically, we provide tight bounds on the *mixing time* (Theorem 9) and on the *hitting time* of a target value (Theorem 10).

Some intuition first. We observe experimentally (Figure 1) that lower costs α *accelerates* convergence to the desired "high value" region, while increasing rewards ρ helps to *maintain* the desired equilibrium (but it does *not* accelerate convergence). Intuitively, the dynamics converge quickly to an "average" contribution level ℓ^* which depends *only* on $\alpha\beta$:

$$\ell^* = n \cdot p_{\alpha\beta} \,, \qquad p_{\alpha\beta} := \frac{1}{1 + \exp(\alpha\beta)} \,. \qquad (22)$$

Then, convergence to the desired "high value" region $\ell \geq \tau$ is fast for $\tau \leq \ell^*$ but becomes *slow* for $\tau > \ell^*$. This suggests that the convergence time grows with the *gap* $\tau - \ell^*$ and the hard case is when $\ell^* \ll \tau$.

Formal analysis As for the mixing time, we leverage on the bounds for birth and death chains (Theorem 4). To this end, we note that in this particular case of threshold functions, the birth and death probabilities (15) assume a special form. This leads to the next theorem, which provides a lower bound on the mixing time for the "useful" scenario, that is, when the success probability is larger than the failure probability.

Theorem 9 (mixing time)**.** *For any threshold technology* (18) *and airdrop rewards* (2)*, if* $p_{\mathsf{high}}(\rho) > 1/2$*, then the mixing time can be bounded as follows:* $\mathcal{T}_{\mathsf{mix}} = \Theta(\mathcal{T}_{\mathsf{cutoff}})$ *and*

$$\mathcal{T}_{\mathsf{cutoff}} \geq \sum_{\ell=0}^{\tau} \frac{\hat{\pi}([0,\ell])}{\hat{\pi}(\ell)p(\ell)} \geq \exp(\alpha\beta) \cdot \frac{\exp(\tau-1)}{\binom{n}{\tau-1}} \,. \qquad (23)$$

We next provide bounds on the *hitting time* of the good value, that is, to reach a contribution level $\ell = \tau$. The first part of the next theorem says that the dynamics converge quickly to a contribution level ℓ^* given by (22).

Theorem 10 (hitting time)**.** *For any threshold technology* (18) *and airdrop rewards* (2)*, let* $\mathcal{T}_{\mathsf{hitting}}(\ell)$ *be the expected time for the logit dynamics to reach state* ℓ *starting from state* 0*. Then, for* ℓ^* *defined as in* (22)*, the following holds:*

1. *(Upper Bound).* $\mathcal{T}_{\mathsf{hitting}}(\ell^*) \leq O\left(n^2 \cdot \frac{\ell^*}{n-\ell^*}\right)$.
2. *(Lower Bound).* $\mathcal{T}_{\mathsf{hitting}}(\tau) \geq \left(\exp(\alpha\beta) \cdot \frac{\ell+1}{n-\ell}\right)^{\tau-\ell}$*, for all* $0 \leq \ell \leq \tau$*. This implies,* $\mathcal{T}_{\mathsf{hitting}}(\tau) \geq (1 + 1/\ell^*)^{\tau-\ell^*-1}$.

The second part of the theorem states that the time to reach the "high value" region increases with larger $\alpha\beta$, growing exponentially with the gap between ℓ^* and $\tau > \ell^*$.

6 Conclusions and Future Work

This paper presents a game-theoretic framework to address the dynamics involved in launching a new blockchain, specifically focusing on how contributions can be incentivized through token rewards and the possibility of support from an established mainchain. The analysis provided, offers both theoretical and practical insights into the design of blockchain launches, airdrop mechanisms, and the use of mainchain resources to achieve successful outcomes.

References

[Alabi, 2017] Ken Alabi. Digital blockchain networks appear to be following Metcalfe's Law. *Electronic Commerce Research and Applications*, 24:23–29, 2017.

[Allen *et al.*, 2023] Darcy WE Allen, Chris Berg, and Aaron M Lane. Why airdrop cryptocurrency tokens? *Journal of Business Research*, 163:113945, 2023.

[Allen, 2024] Darcy WE Allen. Crypto airdrops: An evolutionary approach. *Journal of Evolutionary Economics*, pages 1–24, 2024.

[Alós-Ferrer and Netzer, 2010] Carlos Alós-Ferrer and Nick Netzer. The logit-response dynamics. *Games and Economic Behavior*, 68(2):413–427, 2010.

[Alós-Ferrer and Netzer, 2015] Carlos Alós-Ferrer and Nick Netzer. Robust stochastic stability. *Economic Theory*, 58:31–57, 2015.

[Alós-Ferrer and Netzer, 2017] Carlos Alós-Ferrer and Nick Netzer. On the convergence of logit-response to (strict) nash equilibria. *Economic Theory Bulletin*, 5:1–8, 2017.

[Arieli *et al.*, 2018] Itai Arieli, Moran Koren, and Rann Smorodinsky. The one-shot crowdfunding game. In *Proc. of the ACM Conference on Economics and Computation (EC)*, page 213–214, 2018.

[Asadpour and Saberi, 2009] Arash Asadpour and Amin Saberi. On the inefficiency ratio of stable equilibria in congestion games. In *Proc. of the 5th International Workshop on Internet and Network Economics (WINE)*, pages 545–552. Springer, 2009.

[Auletta *et al.*, 2011] Vincenzo Auletta, Diodato Ferraioli, Francesco Pasquale, Paolo Penna, and Giuseppe Persiano. Convergence to equilibrium of logit dynamics for strategic games. In *Proc. of the 23rd annual ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*, pages 197–206, 2011.

[Auletta *et al.*, 2012] Vincenzo Auletta, Diodato Ferraioli, Francesco Pasquale, and Giuseppe Persiano. Metastability of logit dynamics for coordination games. In *Proc. of the 23rd annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pages 1006–1024. SIAM, 2012.

[Auletta *et al.*, 2013a] Vincenzo Auletta, Diodato Ferraioli, Francesco Pasquale, Paolo Penna, and Giuseppe Persiano. Logit dynamics with concurrent updates for local interaction games. In *Proc. of the 21st Annual European Symposium on Algorithms (ESA)*, pages 73–84. Springer, 2013.

[Auletta *et al.*, 2013b] Vincenzo Auletta, Diodato Ferraioli, Francesco Pasquale, and Giuseppe Persiano. Mixing time and stationary expected social welfare of logit dynamics. *Theory of Computing Systems*, 53:3–40, 2013.

[Babaioff *et al.*, 2012] Moshe Babaioff, Michal Feldman, Noam Nisan, and Eyal Winter. Combinatorial agency. *Journal of Economic Theory*, 147(3):999–1034, 2012.

[Bilò *et al.*, 2023] Vittorio Bilò, Laurent Gourvès, and Jérôme Monnot. Project games. *Theor. Comput. Sci.*, 940(Part):97–111, 2023.

[Blume, 1993] Lawrence E Blume. The statistical mechanics of strategic interaction. *Games and economic behavior*, 5(3):387–424, 1993.

[Blume, 2003] Lawrence E. Blume. How noise matters. *Games and Economic Behavior*, 44(2):251–271, 2003.

[Boncinelli and Pin, 2012] Leonardo Boncinelli and Paolo Pin. Stochastic stability in best shot network games. *Games and Economic Behavior*, 75(2):538–554, 2012.

[Bramoullé and Kranton, 2007] Yann Bramoullé and Rachel Kranton. Public goods in networks. *Journal of Economic theory*, 135(1):478–494, 2007.

[Buragohain *et al.*, 2003] Chiranjeeb Buragohain, Divyakant Agrawal, and Subhash Suri. A game theoretic framework for incentives in P2P systems. In *Proc. of the 3rd International Conference on Peer-to-Peer Computing (P2P)*, pages 48–56. IEEE, 2003.

[Chaidos *et al.*, 2023] Pyrros Chaidos, Aggelos Kiayias, and Evangelos Markakis. Blockchain participation games. In *Proc. of the 19th International Conference on Web and Internet Economics (WINE)*, pages 169–187. Springer, 2023.

[Chang, 2020] Jen-Wen Chang. The economics of crowdfunding. *American Economic Journal: Microeconomics*, 12(2):257–280, 2020.

[Chen and Saloff-Coste, 2013] Guan-Yu Chen and Laurent Saloff-Coste. On the mixing time and spectral gap for birth and death chains. *ALEA-Latin American Journal of Probability and Mathematical Statistics*, 10(1):293–321, 2013.

[Costain and Nakov, 2019] James Costain and Anton Nakov. Logit price dynamics. *Journal of Money, Credit and Banking*, 51(1):43–78, 2019.

[Coucheney *et al.*, 2014] Pierre Coucheney, Stéphane Durand, Bruno Gaujal, and Corinne Touati. General revision protocols in best response algorithms for potential games. In *7th Int. Conference on NETwork Games, COntrol and OPtimization (NetGCoop)*, pages 239–246. IEEE, 2014.

[Dall'Asta *et al.*, 2011] Luca Dall'Asta, Paolo Pin, and Abolfazl Ramezanpour. Optimal equilibria of the best shot game. *Journal of Public Economic Theory*, 13(6):885–901, 2011.

[Do Dinh and Hollender, 2024] Jérémi Do Dinh and Alexandros Hollender. Tight inapproximability of nash equilibria in public goods games. *Information Processing Letters*, 186:106486, 2024.

[EigenLabs, 2014] EigenLabs. The universal intersubjective work token. *White paper*, 2014. url: https://www.blog.eigenlayer.xyz/eigen/.

[Fan *et al.*, 2023] Sizheng Fan, Tian Min, Xiao Wu, and Wei Cai. Altruistic and profit-oriented: Making sense of roles in web3 community from airdrop perspective. In *Proc. of the Conference on Human Factors in Computing Systems (CHI)*. Association for Computing Machinery, 2023.

[Ferraioli and Ventre, 2017] Diodato Ferraioli and Carmine Ventre. Social pressure in opinion games. In *Proc. of the 26th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 3661–3667, 2017.

[Ferraioli *et al.*, 2016] Diodato Ferraioli, Paul W Goldberg, and Carmine Ventre. Decentralized dynamics for finite opinion games. *Theoretical Computer Science*, 648:96–115, 2016.

[Fröwis and Böhme, 2019] Michael Fröwis and Rainer Böhme. The operational cost of ethereum airdrops. In *Data Privacy Management, Cryptocurrencies and Blockchain Technology: ESORICS 2019 Int. Workshops, DPM 2019 and CBT 2019*, pages 255–270, 2019.

[Galeotti and Goyal, 2010] Andrea Galeotti and Sanjeev Goyal. The law of the few. *American Economic Review*, 100(4):1468–92, September 2010.

[Galeotti *et al.*, 2010] Andrea Galeotti, Sanjeev Goyal, Matthew O Jackson, Fernando Vega-Redondo, and Leeat Yariv. Network games. *The review of economic studies*, 77(1):218–244, 2010.

[Georganas *et al.*, 2025] Sotiris Georganas, Aggelos Kiayias, and Paolo Penna. Airdrop games. *arXiv preprint arXiv:2505.03428*, 2025.

[Gilboa and Nisan, 2022] Matan Gilboa and Noam Nisan. Complexity of public goods games on graphs. In *Proc. of the 15th International Symposium on Algorithmic Game Theory (SAGT)*, pages 151–168. Springer, 2022.

[Jiménez-Jiménez *et al.*, 2021] Francisca Jiménez-Jiménez, Maria Virtudes Alba-Fernández, and Cristina Martínez-Gómez. Attracting the right crowd under asymmetric information: A game theory application to rewards-based crowdfunding. *Mathematics*, 9(21), 2021.

[Kleer, 2021] Pieter Kleer. Sampling from the gibbs distribution in congestion games. In *Proc. of the 22nd ACM Conference on Economics and Computation (EC)*, pages 679–680, 2021.

[Klimm and Stahlberg, 2023] Max Klimm and Maximilian J. Stahlberg. Complexity of equilibria in binary public goods games on undirected graphs. In *Proc. of the 24th ACM Conference on Economics and Computation (EC)*, page 938–955, 2023.

[Levin *et al.*, 2006] David A. Levin, Yuval Peres, and Elizabeth L. Wilmer. *Markov chains and mixing times*. American Mathematical Society, 2006.

[Lommers *et al.*, 2023] Kristof Lommers, Christos Makridis, and Lieven Verboven. Designing airdrops. *Available at SSRN 4427295*, 2023.

[Makridis *et al.*, 2023] Christos A. Makridis, Michael Fröwis, Kiran Sridhar, and Rainer Böhme. The rise of decentralized cryptocurrency exchanges: Evaluating the role of airdrops and governance tokens. *Journal of Corporate Finance*, 79(C), 2023.

[Mamageishvili and Penna, 2016] Akaki Mamageishvili and Paolo Penna. Tighter bounds on the inefficiency ratio of stable equilibria in load balancing games. *Oper. Res. Lett.*, 44(5):645–648, 2016.

[Messias *et al.*, 2023] Johnnatan Messias, Aviv Yaish, and Benjamin Livshits. Airdrops: Giving money away is harder than it seems. *arXiv:2312.02752*, 2023.

[Montanari and Saberi, 2009] Andrea Montanari and Amin Saberi. Convergence to equilibrium in local interaction games. In *Proc. of the 50th annual Symp. on Foundations of Computer Science (FOCS)*, page 303–312, 2009.

[Müller *et al.*, 2021] David Müller, Yurii Nesterov, and Vladimir Shikhman. Dynamic pricing under nested logit demand. *arXiv preprint arXiv:2101.04486*, 2021.

[Okada and Tercieux, 2012] Daijiro Okada and Olivier Tercieux. Log-linear dynamics and local potential. *Journal of Economic Theory*, 147(3):1140–1164, 2012.

[Palacios and Tetali, 1996] JoséLuis Palacios and Prasad Tetali. A note on expected hitting times for birth and death chains. *Statistics & Probability Letters*, 30(2):119–125, 1996.

[Papadimitriou and Peng, 2021] Christos Papadimitriou and Binghui Peng. Public goods games in directed networks. In *Proc. of the 22nd ACM Conference on Economics and Computation (EC)*, page 745–762, 2021.

[Penna, 2018] Paolo Penna. The price of anarchy and stability in general noisy best-response dynamics. *Int. J. Game Theory*, 47(3):839–855, 2018.

[Sawa, 2019] Ryoji Sawa. Stochastic stability under logit choice in coalitional bargaining problems. *Games and economic behavior*, 113:633–650, 2019.

[Shao *et al.*, 2023] Qi Shao, Man Hon Cheung, and Jianwei Huang. Crowdfunding with cognitive limitations. *IEEE/ACM Trans. Netw.*, 31(6):2714–2729, may 2023.

[Soundy *et al.*, 2021] Jared Soundy, Chenhao Wang, Clay Stevens, and Hau Chan. Game-theoretic analysis of effort allocation of contributors to public projects. 2021.

[van de Geer and den Boer, 2022] Ruben van de Geer and Arnoud V den Boer. Price optimization under the finite-mixture logit model. *Management Science*, 68(10):7480–7496, 2022.

[Wang *et al.*, 2023] Lu Wang, Xue Wang, and Hang Wei. Bundling and pricing strategies in crowdfunding. *Available at SSRN 4432999*, 2023.

[Ward, 2024] Mike Ward. Partner chains are coming to cardano. *IOHK blog post*, November 2024.

[Worldcoin, 2025] Worldcoin, 2025. Description available at https://world.org/worldcoin-token.

[Yaish and Livshits, 2024] Aviv Yaish and Benjamin Livshits. Tierdrop: Harnessing airdrop farmers for user growth, 06 2024.

[Yan and Chen, 2021] Xiang Yan and Yiling Chen. Optimal crowdfunding design. In *Proc. of the 20th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS)*, pages 1704–1706, 2021.

[Yu *et al.*, 2020] Sixie Yu, Kai Zhou, Jeffrey Brantingham, and Yevgeniy Vorobeychik. Computing equilibria in binary networked public goods games. In *Proc. of the AAAI Conference on Artificial Intelligence*, volume 34, pages 2310–2317, 2020.