Parameterized and Streaming Algorithms for Euclidean Fair k-Center Clustering

Zeyu Lin[1] , **Chaoqi Jia**[2] , **Longkun Guo**[1*] and **Chao Chen**[2]
[1]School of Mathematics and Statistics, Fuzhou University, Fuzhou 350116, China
[2]School of Accounting, Information Systems and Supply Chain, RMIT University, VIC 3000, Australia
zeyu_lin@foxmail.com, chaoqi.jia@rmit.edu.au, longkun.guo@gmail.com, chao.chen@rmit.edu.au

Abstract

Motivated by the growing importance of fairness in machine learning, fair k-center clustering has attracted considerable research attention as a fundamental problem. In this problem, a dataset is partitioned into m disjoint groups, and the objective is to select k data points as centers, subject to upper bounds on the number of centers chosen from each group, aiming to minimize the maximum distance between any data point and its assigned center. Focusing on Euclidean spaces, which are ubiquitous in machine learning applications, we first develop a parameterized approximation algorithm for Euclidean fair k-center with an approximation ratio of 2.732. By incorporating this algorithm as a post-processing stage into a one-pass streaming framework for large-scale data, we obtain an approximation ratio of 4.464. These ratios can be further respectively improved to 2.414 and 3.828 with a runtime exponential on k. To ensure polynomial-time complexity, we further design a one-pass streaming algorithm with an approximation ratio of 4.732, which can be further improved to 4.42, outperforming the state-of-the-art ratio. Finally, extensive experiments show that our methods significantly outperform state-of-the-art approaches in terms of clustering accuracy.

1 Introduction

Center-based clustering is a typical unsupervised learning technique in machine learning (ML) with a wide range of applications, including data summarization [Gadekar *et al.*, 2025], text analysis [Jia *et al.*, 2026b], and decision-making [Huang *et al.*, 2019]. In particular, the k-center problem aims to minimize the maximum distance from any point to its nearest center and has been extensively studied. In general metric spaces, a tight 2-approximation is achievable [Gonzalez, 1985; Hochbaum and Shmoys, 1985], whereas in Euclidean spaces, it is NP-hard to obtain a polynomial-time approximation with a factor better than 1.82 [Feder and Greene, 1988].

Many applications have proposed k-center problem with constraints including instance-level constraints [Guo *et al.*, 2024; Guo *et al.*, 2025] for improve the clustering accuracy and *fairness* constraints to mitigate representation bias [Chierichetti *et al.*, 2017; Bera *et al.*, 2022]. In particular, we focus on data summarization fairness which can limit the number of older messages included in a user's feed digest [Mahabadi and Trajanovski, 2023] or enforce genre diversity in movie recommendations. For data summarization fairness, the dataset S is partitioned into m disjoint groups $S = S_1 \cup \cdots \cup S_m$ and it requires that at most k_l centers be selected from each group S_l. This constraint in the fair k-center ensures balanced representation by sensitive attributes.

Under this fairness setting, there has been significant research interest in improving approximation ratios for the problem across different metrics, such as general metrics [Guo *et al.*, 2026b], Euclidean metrics [Guo *et al.*, 2026a] and doubling metrics [Ceccarello *et al.*, 2024], as well as under different computational models, including the offline setting [Kleindessner *et al.*, 2019; Jones *et al.*, 2020] and the streaming model [Chiplunkar *et al.*, 2020; Lin *et al.*, 2024; Guo *et al.*, 2026b].

Inspired by Nagarajan *et al.*, we recently achieved a $(1 + \sqrt{3})$-approximation for the Euclidean k-supplier problem, improving over the long-standing factor-3 barrier. In Euclidean space, we combine with both Jones *et al.* and Guo *et al.* to design algorithms for the fair k-center problem in the streaming model and achieve a $(3 + \sqrt{3} + \epsilon)$-approximation guarantee. After that, we further aim to improve the approximation ratio. To this end, we propose the first fixed-parameter tractable (FPT) algorithm for fair k-center in Euclidean space, breaking the existing approximation barrier while handling both online model and offline setting.

1.1 Related Work

FPT k-Center under Constraints and Specialized Metrics. Fixed-parameter tractable (FPT) approximation algorithms have been extensively studied for the k-center problem in metrics. Agarwal and Procopiuc gave an approximation scheme with running time $O(n \log k) + (k/\epsilon)^{O(k^{1-1/d})}$ where n is the input size and d is the dimension. More recently, Goyal and Jaiswal established a unified FPT framework for a broad class of constrained k-center problem, obtaining 2-approximation algorithms with runtime $k^{O(k)} n^{O(1)}$. For constrained vari-

*Corresponding Author

ants, Bandyapadhyay *et al.* obtained a 3-approximation for non-uniform k-center with a runtime $2^{O(k \log k)}n^2$. Wu *et al.* further studied fair k-center with outliers and obtained an approximation scheme with runtime $f(k, z, \epsilon) \cdot n^{O(1)}$, where f is a computable function depending on k, z and ϵ, z denotes the maximum number of outliers. For Euclidean spaces, Bandyapadhyay *et al.* also developed an approximation scheme for Euclidean non-uniform k-center with runtime $2^{O((k \log k)/\epsilon)}dn$. For FPT algorithms beyond metric spaces, Zhou *et al.* studied k-center on general graphs and proposed a 3-approximation algorithm running in $O(2^{\alpha} \cdot n^5)$ time as well as a 13-approximation algorithm running in $\beta^{O(\beta)} \cdot poly(n)$ where α is the number of vertices participating in triangle-inequality violations and β is the minimum number of vertices whose removal transforms the graph into a metric graph.

Data Summarization Fair k-Center. Fair clustering under group representation constraints, specifically, upper bounds on the number of centers per group, was formalized in the context of data summarization by Kleindessner *et al.*, who gave a 5-approximation for two groups and a $(3 \cdot 2^{m-1} - 1)$-approximation for m groups. This was significantly improved by Jones *et al.*, who achieved a 3-approximation for arbitrary m via a reduction to matching. Chen *et al.* later studied a closely related diversity-aware fair k-supplier problem and also obtained a 5-approximation using maximum matching. These offline results establish 3 as the state-of-the-art approximation ratio in general metrics, and a natural question is whether this barrier can be broken in structured spaces $\mathbb{R}^d$.

However, all known algorithms achieving ratios below 5 are either offline or require multiple passes; none operate in the one-pass streaming regime while exploiting Euclidean geometry or fixed-parameter tractability.

Streaming Algorithms for Fair Clustering. Streaming fair clustering poses additional challenges due to the irrevocable nature of decisions and limited memory. Early work on streaming k-center with outliers [Charikar *et al.*, 2003; Matthew McCutchen and Khuller, 2008] laid foundational techniques, but fairness constraints require more sophisticated summaries. Lin *et al.* recently proposed a one-pass streaming algorithm for fair k-center achieving a $(7 + \epsilon)$-approximation, while Chiplunkar *et al.* gave a two-pass streaming algorithm with ratio 3. By leveraging a λ-independent center set and reducing the hardest case to a constrained vertex cover problem, Guo *et al.* presented a one-pass streaming algorithm that achieved a $(5 + \epsilon)$-approximation for fair k-center in general metric spaces, improving over the previous $7 + \epsilon$ bound. For Euclidean space, Guo *et al.* further improved the approximation ratio to $(1+2\sqrt{3}+\epsilon) \approx 4.464$. Notably, none of the prior streaming approaches leverage FPT subroutines or geometric coreset constructions to achieve better ratios, leaving a gap between offline Euclidean advances and streaming practice.

To the best of our knowledge, our work is the first to combine FPT techniques, Euclidean geometry, and streaming models to achieve better approximation ratio for fair k-center in a one-pass streaming setting.

1.2 Our Contribution

Motivated by the importance of Euclidean space in machine learning, we propose a suite of parameterized, polynomial-time approximation and streaming algorithms. The contribution can be summarized as follows:

- Devise a parameterized approximation algorithm with ratio $1 + \sqrt{3} \approx 2.732$ and integrate it into the streaming framework to obtain an approximation ratio $1+2\sqrt{3}+\epsilon \approx 4.464$ for Euclidean fair k-center. These guarantees can be further improved to $1 + \sqrt{2} \approx 2.414$ and $1 + 2\sqrt{2} + \epsilon \approx 3.828$, respectively.
- Propose a polynomial-time one-pass streaming algorithm for Euclidean fair k-center that uses $O(k \log \alpha)$ memory and achieves an approximation ratio of $3 + \sqrt{3} + \epsilon \approx 4.732$ by integrating the milestone algorithm of [Jones *et al.*, 2020], where α is the aspect ratio, defined as the ratio between the maximum and minimum pairwise distances.
- Conduct extensive experiments on real-world datasets to evaluate the practical performance of our algorithms, demonstrating that they significantly outperform all state-of-the-art methods in terms of solution quality.

Moreover, with a higher memory complexity of $O(h \cdot k \log \alpha)$, the approximation ratio of our streaming algorithm can be further improved to $3 + \sqrt{2 + \frac{2}{h}} \approx 4.42$ by setting $h = 122$, which slightly improves upon the state-of-the-art ratio of 4.464 due to Guo *et al.*.

2 Preliminary

Let S be a finite set of n data points distributed in an Euclidean space, where the distance function $d : S \times S \to \mathbb{R}_{\geq 0}$. For a given parameter $k \in \mathbb{N}$, the traditional k-center clustering problem seeks to select a set of k centers $C \subseteq S$ such that $\max_{s \in S} d(s, C)$ is minimized, where $d(s, C) = \min_{c \in C} d(s, c)$ denotes the distance from a point s to its nearest center in C.

2.1 Problem Statement

The fair k-center clustering problem extends this by imposing additional fairness constraints. Specifically, the dataset S is divided into m disjoint groups: $S = S_1 \cup S_2 \cup \cdots \cup S_m$. Each group S_l ($l \in [m]$, where $[m] = \{1, 2, \ldots, m\}$) has an associated upper bound k_l on the number of centers that can be selected from it. These constraints ensure that the total number of chosen centers across all groups equals k: $\sum_{l=1}^{m} k_l = k$. This formulation aims to balance the representation of each group in the final clustering solution, thereby mitigating potential biases and ensuring fairness. Then, the fair k-center clustering problem is to find a center set C satisfying the formulation as follows:

$$\begin{aligned} \min_{C \subseteq S} \quad & \max_{s \in S} d(s, C) \\ s.t. \quad & \mid C \cap S_l \mid \leq k_l, \forall l \qquad (1) \\ & \mid C \cap S \mid = \sum_{l=1}^{m} \mid C \cap S_l \mid \leq k. \qquad (2) \end{aligned}$$

We assume that the optimal radius of the fair k-center problem is known as r^* (i.e., $r^* = \max_{s \in S} d(s, C^*)$ for the optimum center set C^*). Note that we actually do not know the exact value of r^*. Nevertheless, we devise r^* following the previous works [Jia *et al.*, 2026a] for the offline algorithm and [Guha, 2009] for the online algorithm.

2.2 λ-Independent Center Set in Euclidean Space

A key structural tool in this paper is the λ-independent center set, a concept formalized in the streaming setting by [Guo *et al.*, 2026b]. Its definition is as follows:

Definition 1. *(λ-independent center set) $\Gamma \subseteq S$ is a λ-independent center set of S, if and only if it satisfies the following two conditions:*

1) *For any two points $p, q \in \Gamma$, the distance between them is larger than λ, i.e. $d(p, q) > \lambda$.*
2) *For any point $p \in S$, there exists a point $q \in \Gamma$, such that $d(p, q) \le \lambda$.*

Recall that C^* is the optimal center set and r^* is the optimum radius. Then we have:

Lemma 1. *Assume $\Gamma \subseteq S$ is a λ-independent center set in Euclidean space. If $\lambda = \sqrt{3}r^*$, then $|\Gamma| \le 2k$.*

Proof. We first show that the number of points in Γ within distance r^* from each $c^* \in C^*$ is at most two. Suppose that there exists a center c^* with three points $c_1, c_2, c_3 \in \Gamma$. Then we have $d(c^*, c_i) \le r^*$ for $i \in \{1, 2, 3\}$. That is, the circle centered at c^* with radius r^* has $\{c_1, c_2, c_3\}$ in its interior. Then, by the geometric property of any inner triangle within a circle, we have

$$\max \min_{\forall i,j \in \{1,2,3\}} d(c_i, c_j) = \sqrt{3}r^*,$$

where the maximum is attained when all the three points c_1, c_2, c_3 are exactly on the circle and the distance between any two points equals $\sqrt{3}r^*$. This contradicts the fact that the distance between any pair of points in Γ is strictly greater than $\sqrt{3}r^*$. So there are at most two points of Γ that are within distance r^* from any $c^* \in C^*$. Therefore, the size of Γ is at most $2|C^*| = 2k$. □

Moreover, when $\lambda = 2r^*$, we can prove $|\Gamma| \le k$ similarly.

3 Parameterized and Streaming Algorithms for Euclidean Fair k-Center

In this section, we first show that the λ-independent center set can lead to a parameterized approximation algorithm with approximation ratio $1 + \sqrt{3} \approx 2.732$ in Euclidean space, which is better than 3 for the metric space when requiring a runtime exponential on k. Then, we propose incorporating the parameterized algorithm into the post-processing stage of the streaming framework for fair k-center clustering, achieving an overall approximation ratio of $1 + 2\sqrt{3} + \epsilon$, which is approximately 4.464 for small ϵ. Moreover, we show that the ratio of the offline parameterized approximation algorithm can be further improved to 2.414, and consequently, the ratio of the streaming algorithm is then improved to 3.828. The proof of the improvement will be provided in the full version.

3.1 Offline Parameterized Approximation

First, we show that λ-independent center sets with $\lambda = \sqrt{3}r^*$ can be employed to derive a parameterized approximation algorithm as $\bigcup_{l=1}^m \Gamma_l$ contains a desirable approximation solution, where Γ_l is a λ-independent center set for S_l with $\lambda = \sqrt{3}r^*$:

Theorem 1. *For $\lambda = \sqrt{3}r^*$, there exists $\Gamma \subseteq \bigcup_{l=1}^m \Gamma_l$ for which the following three conditions hold: (1) for each point $s \in S$, there must exist a point $c \in \Gamma$ with $d(s, c) \le (1 + \sqrt{3})r^*$; (2) $|\Gamma| \le k$; (3) $|\Gamma \cap S_l| \le k_l$. In other words, there exists Γ in $\bigcup_{l=1}^m \Gamma_l$ that is a $(1 + \sqrt{3})$-approximation solution for the fair k-center problem.*

Proof. Let C^* be an optimal solution. We first give a method to construct a Γ from $\bigcup_{l=1}^m \Gamma_l$ according to C^*, and then show that the constructed Γ satisfies all three conditions.

For the first, our construction simply proceeds as in the following: for each center $c^* \in C^*$, if $c^* \in C^*$ is an element of $\bigcup_{l=1}^m \Gamma_l$, then add c^* to Γ; Otherwise, find group S_l that contains c^*, and then find a point $c \in \Gamma_l$ which is within $\sqrt{3}r^*$ distance away from c^*, i.e. with $d(c, c^*) \le \sqrt{3}r^*$. So, for each $s \in S$, there must exist $c \in \Gamma$ such that $d(s, c) \le d(s, c^*) + d(c^*, c) \le r^* + \sqrt{3}r^* = (1 + \sqrt{3})r^*$, and Cond. (1) holds. As the construction maps each optimal center $c^* \in C^* \cap S_l$ to one center $c \in \Gamma \cap S_l$, Cond. (2) and Cond. (3) hold according to the property of the optimal center set C^*. □

Based on this existence guarantee, our enumeration-based parameterized algorithm simply proceeds in two stages:

1) For each group $l \in [m]$, compute a λ-independent center set Γ_l with $\lambda = \sqrt{3}r^*$.
2) Enumerate all the possible center sets C with size bounded by k and satisfying the fairness constraint, and choose C with the smallest radius among all the computed center sets.

Lemma 2. *The algorithm above can correctly find Γ that satisfies the three conditions of Thm. 1.*

Proof. First, we show the algorithm can always output a Γ, which is, there always exists $c \in \Gamma_l$ with $d(c, c^*) \le \sqrt{3}r^*$ when $c^* \notin \bigcup_{l=1}^m \Gamma_l$ and $c^* \in S_l$. Suppose otherwise, then $d(c^*, c) > \sqrt{3}r^*$ holds for every $c \in \Gamma_l$, which indicates that c^* must be added to Γ_l and arises a contradiction.

Then, we show that all three conditions hold. Cond. (1) immediately holds following the construction of Γ.

For Cond. (2), we add at most one point (i.e., c or c^*) to Γ upon one point $c^* \in C^*$ (note that the point to be added might already exist in Γ). So $|\Gamma| \le |C^*| = k$ and hence condition (2) is true. For condition (3), the algorithm adds a point (either c or c^*) of S_l to Γ if and only if the processing point c^* belongs to S_l. That is, we have $|\Gamma \cap S_l| \le |C^* \cap S_l| \le k_l$ holds for each l. This completes the proof. □

Theorem 2. *The fair k-center problem admits a parameterized approximation algorithm with ratio $1 + \sqrt{3} \approx 2.732$ and runtime $O(2^{k(1+\log m)} \cdot mnk^2 \log n)$.*

Proof. The ratio can be directly derived from Thm. 1. That is because the enumeration can always find a Γ satisfying the three conditions as in Thm. 1 if such Γ exists; while on the other hand, Thm. 1 claims the existence of such Γ.

For the runtime, Stage 2 enumerates $O\left(\binom{2mk}{k}\right)$ sets (of k-centers), because there are at most $2mk$ points in $\bigcup_{l\in[m]}\Gamma_l$. Moreover, each enumerated set requires $O(nk)$ time to compute its maximum radius, so Stage 2 takes a total runtime $O\left(nk\cdot\binom{2mk}{k}\right)$. By employing Stirling's approximation formulation and via calculation, we get:

Lemma 3. $\binom{2mk}{k}$ *is bounded by* $O\left(2^{k(1+\log m)}\cdot mk\right)$.

Then, the total runtime of the parameterized algorithm sums up to $O(2^{k(1+\log m)}\cdot mnk^2\log n)$. Then by multiplying the factor $O(\log n)$ for employing the binary search for not knowing r^*, we complete the proof of Thm. 2. The detailed proof of Lem. 3 can be found in the full version.

3.2 The Parameterized Approximation in Streams

In this subsection, we introduce the general framework of our parameterized streaming algorithm, there are two stages:

1) **Streaming Stage**. Construct a λ-independent center set Γ_l for each group S_l *upon the stream* with $\lambda=\sqrt{3}r^*$.
2) **Post-streaming Stage**. Select centers from $\bigcup_{l=1}^{m}\Gamma_l$ to construct the desired center set Γ by enumeration similar to the offline parameterized approximation algorithm.

Because the enumeration here considers only the points in $\bigcup_l\Gamma_l$ rather than in the entire point set S, its compromised performance guarantee can be stated as below:

Lemma 4. *The parameterized streaming algorithm consumes a memory of* $O(k\log_\epsilon\alpha)$ *and achieves an approximation ratio* $1+2\sqrt{3}+\epsilon\approx 4.464$ *for any sufficiently small* $\epsilon>0$, *where* $\alpha=\Delta/\delta$ *is the* aspect ratio *for* $\Delta=\max_{p,q\in S}d(p,q)$ *denoting the diameter of* S *and* $\delta=\min_{p,q\in S,\,p\neq q}d(p,q)$ *being the minimum pairwise distance.*

Proof. We need only to bound the distance from any point $s\in S$ to the center set C constructed by the parameterized algorithm. For each input point $s\in S_l$, there must exist a point $i\in\Gamma_l$ such that $d(i,s)\le\sqrt{3}r^*$ according to the structure for λ-independent center set. If i is added to C, then $d(s,C)\le\sqrt{3}r^*$ immediately holds. Otherwise, we have $i\notin C$ and will show $d(s,C)\le(1+2\sqrt{3})r^*$ holds. Following Thm. 1, there exists $\Gamma\subseteq\bigcup_l\Gamma_l$, such that for any $i\in\bigcup_l\Gamma_l$, $d(i,\Gamma)\le(1+\sqrt{3})r^*$ holds. As C is constructed via enumeration, we have

$$\max_{i\in\bigcup_l\Gamma_l}d(i,C)\le\max_{i\in\bigcup_l\Gamma_l}d(i,\Gamma)\le(1+\sqrt{3})r^*.$$

Therefore, we have

$$d(s,C)\le d(s,i)+\max_{i\in\bigcup_l\Gamma_l}d(i,C)\le(1+2\sqrt{3})r^*,$$

This completes the proof. $\square$

Algorithm 1: The post-streaming algorithm

Input: A λ-independent center set Γ with $\lambda=2r^*$ and Γ_i ($i\in[m]$) with $\lambda=\sqrt{3}r^*$.
Output: A center set C.

Create a directed graph $G(V,E)$ with $V=\{s,t\}$ and $E=\emptyset$;
Create vertex set V_Γ with one-to-one mapping to points in Γ;
Create a vertex set V_f with a one-to-one mapping to points in $\Gamma_1,\ldots,\Gamma_m$;
Create a vertex set V_t of size m with a one-to-one mapping to group indices;
Update $V\leftarrow V\cup V_\Gamma\cup V_f\cup V_t$;
for *each group* $l\in[m]$ **do**
  Add an edge from the vertex corresponding to l in V_t to t with capacity k_l;
for *each* $a\in\Gamma$ *and each* $f\in V_f$ *corresponding to a point in* $\bigcup_{l\in[m]}\Gamma_l$ **do**
  if $d(a,f)\le(1+\sqrt{3})r^*$ **then**
    Add an edge from vertex $a\in V_\Gamma$ to vertex $f\in V_f$ with capacity 1;
    Add an edge from vertex $f\in V_f$ to the vertex $l\in V_t$ (where $f\in\Gamma_l$) with capacity 1;
for *each* $a\in\Gamma$ **do**
  Add an edge from s to the vertex for $a\in V_\Gamma$ with capacity 1;
Run Dinic's Max-Flow algorithm [Shimon *et al.*, 1975] on G from s to t;
if *the max flow value equals* $|\Gamma|$ **then**
  Construct C from the centers corresponding to edges used in the flow;
  return C;
else
  return *infeasible*.

Note that, by using the smaller value $\lambda=\sqrt{2}r^*$, the parameterized approximation ratio in Thm. 2 and the ratio in Lem. 4 can be improved to $1+\sqrt{2}\approx 2.414$ and $1+2\sqrt{2}+\epsilon\approx 3.828$, respectively.

4 Streaming Euclidean Fair k-Center in Polynomial Runtime

In this section, we present a one-pass streaming algorithm for Euclidean space, where the algorithm employs network flow as a building block, achieving a polynomial runtime and a ratio of $3+\sqrt{3}+\epsilon\approx 4.732$.

The key idea of our algorithm is first to construct a λ-independent center set Γ for S with $\lambda=2r^*$(ignoring groups) and for each group l, construct λ-independent center set Γ_l with a different $\lambda=\sqrt{3}r^*$ along the stream simultaneously; and then use the Γ as the solution except replacing some centers therein using $\bigcup_l\Gamma_l$ to satisfy the fairness constraint.

In general, our algorithm mainly proceeds in two phases:

(1) Upon the stream, construct $m+1$ independent center sets, including m independent center sets Γ_l for each group S_l ($l\in[m]$) regarding $\lambda=\sqrt{3}r^*$ and an independent center set Γ for $\lambda=2r^*$ ignoring the group division;

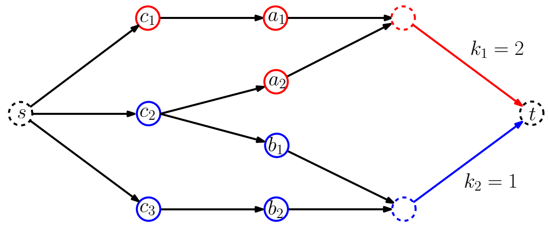

Figure 1: An example of executing Alg. 1 with $k = 3$, $k_1 = 2$, $k_2 = 1$. Let $\Gamma = \{c_1, c_2, c_3\}$ be a λ-independent center set for S ignoring the groups for $\lambda = 2r^*$. Assume that $\Gamma_1 = \{a_1, a_2\}, \Gamma_2 = \{b_1, b_2\}$ are for S_1, S_2 respectively with a different $\lambda = \sqrt{3}r^*$. Each edge has a default capacity of 1 except the edges entering t.

(2) Select at most k centers from the computed $\bigcup_l \Gamma_l \cup \Gamma$: Construct an auxiliary bipartite graph G where a maximal flow corresponds to a center set that can cover all the data points within the desired radius $(3 + \sqrt{3})r^*$.

The auxiliary graph $G = (V; E)$ for Phase (2) can be constructed as follows:

1) For the vertices of G, set $V = \{s, t\} \cup V_\Gamma \cup V_f \cup V_t$ where $\{s, t\}$ is the set of source and target nodes, V_Γ is a set with a one-to-one mapping to points in Γ, V_f is a set with a one-to-one mapping to points in $\bigcup \Gamma_l$ and V_t is a set with a one-to-one mapping to group indices.
2) We add four kinds of edges of G as follows:
   - For each $a \in V_\Gamma$, we add an edge from the source node s to the vertex a with capacity 1.
   - For each $a \in V_\Gamma$ and each $f \in V_f$, if $d(a, f) \leq (1 + \sqrt{3})r^*$, we add an edge from vertex a to vertex f with capacity 1.
   - For each $f \in V_f$, we add an edge from vertex f to the vertex $l \in V_t$ (where $f \in \Gamma_l$) with capacity 1.
   - For each group $l \in [m]$, we add an edge from the vertex corresponding to l in V_t to the target node t with capacity k_l.

The detailed algorithm is as depicted in Alg. 1 and an example is in Fig. 1.

Lemma 5. *In Alg. 1, two conditions hold: (1) The size of each $\Gamma_l, l \in [m]$, is less than $2k$. (2) The size of Γ is less than k.*

Proof. According to Lem. 1, $|\Gamma| \leq k$ is true and $|\Gamma_l| \leq 2k$ holds for each $l \in [m]$. □

Lemma 6. *In Alg. 1, for point $p \in \Gamma$, if $\forall i \in \Gamma_l, d(p, i) > (1 + \sqrt{3})r^*$ holds where $l \in [m]$, there exists no point $q \in S_l$ such that $d(p, q) \leq r^*$.*

Proof. Suppose that there exists a point $q \in S_l$ such that $d(p, q) \leq r^*$. Then, there must exist a point $i \in \Gamma_l$ such that $d(q, i) \leq \sqrt{3}r^*$. We have $d(p, i) \leq d(p, q) + d(q, i) \leq r^* + \sqrt{3}r^* = (1+\sqrt{3})r^*$, contradicting $d(p, i) > (1+\sqrt{3})r^*$. □

Lemma 7. *There exists a flow of value equal to $|\Gamma|$ in the auxiliary graph G if and only if there exists a center set $C \subseteq \bigcup_l \Gamma_l \cup \Gamma$ satisfying the fairness requirement and covering all the points of S within a radius $(3 + \sqrt{3})r^*$.*

Proof. If there exists a flow of value equal to $|\Gamma|$ in the auxiliary graph G, we need to prove C satisfies: (1) $|C| \leq k$; (2) $|C \cap S_l| \leq k_l$; (3) $\forall s \in S, d(s, C) \leq (3 + \sqrt{3})r^*$.

For $|C| \leq k$, the weight of edge from the vertex a in V_Γ to the vertex f in V_f is 1, that is the exchange is one-to-one, when the C created by the maximal flow algorithm, $|C| = |\Gamma| \leq k$ as Γ is an $2r^*$-independent center set for S.

Then, for $|C \cap S_l| \leq k_l$, according to Lem. 6, there always exists a feasible flow and the weight of edge from the vertex for l in V_l to t is k_l, so $|C \cap S_l| \leq k_l$.

For $\forall s \in S, d(s, C) \leq (3 + \sqrt{3})r^*$, there are two cases: (1) for each point $s \in S$, there exists a point $i \in \Gamma$ such that $d(s, i) \leq 2r^*$ and i is added to C, we have $d(s, C) \leq d(s, i) \leq 2r^*$; (2) for each point $s \in S$, there exists a point $i \in \Gamma$ such that $d(s, i) \leq 2r^*$, however, a point $j \in \Gamma_l, l \in [m]$ replace i to be added to C as $d(i, j) \leq (1 + \sqrt{3})r^*$, then $d(s, C) \leq d(s, j) \leq d(s, i) + d(i, j) \leq 2r^* + (1 + \sqrt{3})r^* = (3 + \sqrt{3})r^*$. That is, there exists a center set $C \subseteq \bigcup_l \Gamma_l \cup \Gamma$ satisfying the fairness requirement and covering all the points of S within a radius $(3 + \sqrt{3})r^*$.

If there exists a center set $C \subseteq \bigcup_l \Gamma_l \cup \Gamma$ satisfying the fairness requirement and covering all the points of S within a radius $(3 + \sqrt{3})r^*$. As $|C| = |\Gamma| \leq k$, there exists a flow of value equal to $|\Gamma|$ in the auxiliary graph G. □

Theorem 3. *For the center set C created by Alg. 1, we have: (1) $|C \cap S_l| \leq k_l$; (2) $|C| \leq k$; (3) $d(s, C) \leq (3 + \sqrt{3})r^*$ holds for each $s \in S$. In other word, Alg. 1 achieves the approximation ratio $3 + \sqrt{3} \approx 4.732$ for the streaming fair k-center problem for general m.*

Proof. For Cond. (1), as the construction for graph G, the weight of the edge from the vertex for l in V_l to t is k_l, the center set C created by Alg. 1 satisfies $|C \cap S_l| \leq k_l$.

For Cond. (2), first, $|\Gamma| \leq k$ holds according to lem. 5, then, the weight of the edge from the vertex $a \in \Gamma$ to the vertex f in V_f is 1 and the weight of the edge from the vertex f in V_f to the vertex l in V_l is 1. According to the conservation law of flow in the maximum flow algorithm, $|C| \leq k$ holds.

For Cond. (3), we bound the distance from any point $s \in S$ to C created by Alg. 1. For each point $s \in S$, there are two cases: (1) There exists a point $i \in \Gamma$ such that $d(s, i) \leq 2r^*$, and i is added into C as a center, thus $d(s, C) \leq 2r^*$; (2) There exists a point $i \in \Gamma$ such that $d(s, i) \leq 2r^*$, then there exists a point $j \in \Gamma_l, l \in [m], d(i, j) \leq (1 + \sqrt{3})r^*$ and i is replaced by j to add to center set C, we have $d(s, C) \leq d(s, j) \leq d(s, i) + d(i, j) \leq 2r^* + (1 + \sqrt{3})r^* = (3 + \sqrt{3})r^*$. □

Following the above theorem, and accounting for the cost of guessing r^* over the stream, we eventually obtain a streaming algorithm with an approximation ratio $(3 + \sqrt{3} + \epsilon)$ and a memory complexity $k \log_\epsilon \alpha$ by running Alg. 1 for each guessed value of r^* with binary search. Moreover, by using the smaller threshold $\lambda = \sqrt{2}r^*$ to generate Γ_l during streaming and modifying Alg. 1 accordingly via replacing $\sqrt{3}r^*$ with

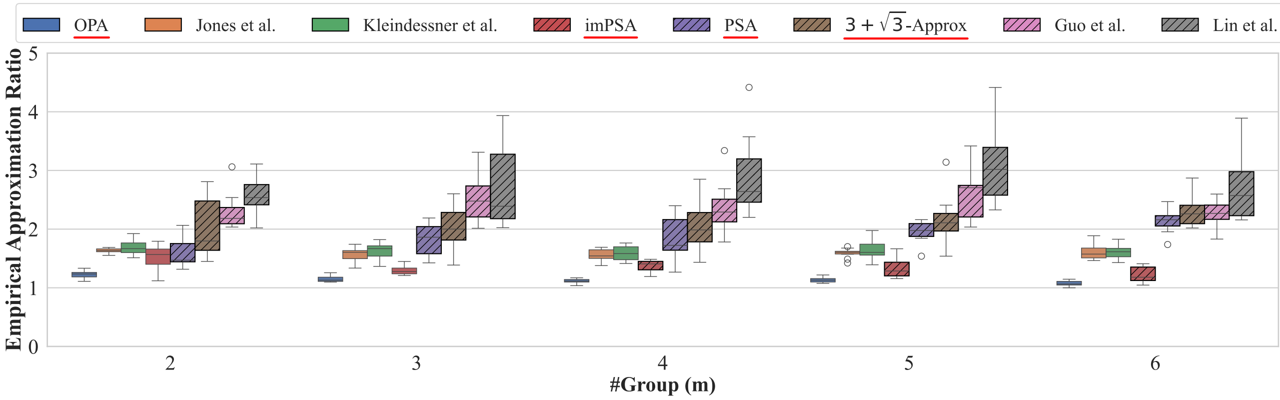

Figure 2: Empirical approximation ratio (cost/r^*) of our algorithms (indicated by a red underline) compared with other baselines. (Solid colors indicate the offline algorithms' result, whereas hatched boxes represent the results of streaming algorithms.)

$\sqrt{2}r^*$, the approximation ratio can be improved to 4.42, which slightly improves upon the previous state-of-the-art ratio of 4.464 [Guo *et al.*, 2026a].

5 Experimental Results

In this section, we evaluate our algorithms on both simulated and real-world datasets, comparing them to three approximation algorithms as baselines. All experiments were averaged over multiple runs, implemented in Python 3.8, and executed on a 12th Gen Intel(R) Core(TM) i9 with 64 GB of RAM.[1]

5.1 Experimental Setting

Datasets. Following the approach in previous work [Kleindessner *et al.*, 2019], we used their method to construct a simulated dataset with a known optimal solution for the k-center problem. In addition, we apply our algorithms to three real-world datasets from UCI [Asuncion and Newman, 2007]: Wholesale, Student and Adult. Following the previous work [Jones *et al.*, 2020; Chen *et al.*, 2019; Guo *et al.*, 2026b], we utilized numeric features for clustering and selected multiple categorical attributes to construct datasets with the fair requirement.

Algorithms. To evaluate our offline parameterized algorithm, which achieves a $(1+\sqrt{3})$-approximation ratio (OPA; see Section 3.1), we compare it against two baseline methods: the 5-approximation algorithm [Kleindessner *et al.*, 2019] and the 3-approximation algorithm [Jones *et al.*, 2020].

To assess the clustering quality of our online algorithms for the fair k-center problem, we implement a $(1+2\sqrt{3}+\epsilon)$-approximation parameterized streaming algorithm (PSA; see Section 3.2) and an improved algorithm with $1+2\sqrt{2}+\epsilon$ ratio (imPSA), and a $(3+\sqrt{3}+\epsilon)$-approximation algorithm with polynomial time (denoted as $(3+\sqrt{3})$-approx; see Section 4). We compare these approaches with two online baselines: the $(7+\epsilon)$-approximation algorithm [Lin *et al.*, 2024] and the $(5+\epsilon)$-approximation algorithm [Guo *et al.*, 2026b].

[1]Our code can be found in GitHub https://github.com/ChaoqiJia/FPT_EuclideanFairk-Center.

Constraints Settings. Following the fairness principle of disparate impact as outlined by Feldman *et al.*, we restricted the selection to k_l data points from the lth group to serve as centers. We then evaluated the clustering quality by varying the parameter m and the fair ratio across these datasets.

Metrics. We use the *cost* metric, as defined in Section 2, to compare the quality of clustering across the datasets based on their average values. Additionally, we measure runtime in seconds.

5.2 Experimental Analysis

In this subsection, we analyze the approximation ratios and running times of all algorithms on a small simulated dataset by varying the number of groups m, with focus on our proposed offline and streaming methods: $(3+\sqrt{3})$-Approx, OPA, PSA, and imPSA. We then compare the algorithms on three real-world datasets in terms of clustering cost. Finally, we evaluate how the clustering cost varies under different fairness requirement ratios.

Approximation Factor. We compare algorithm performance by evaluating the relative solution ratio against the provided optimal radius r^* on the simulated dataset. The ratio of the evaluation result can be referred to as *empirical approximation ratio*, and the maximum value represents the worst-case. Our target in this experiment is to validate the approximation factors achieved by our algorithms ($(3+\sqrt{3}+\epsilon)$-Approx, OPA, PSA and imPSA). In Figure 2, we compared our algorithms with the baselines in the settings of $|S| = 200$, $k = 9$, and increased the number of groups m from 2 to 6.

Across all values of the number of groups m, the offline methods (solid-color boxes) consistently achieve lower empirical approximation ratios than the streaming methods (hatched boxes). This holds even though some offline baselines (e.g., Kleindessner et al.) have weaker theoretical guarantees than PSA ($(1+2\sqrt{3}+\epsilon)$-approx) and $(3+\sqrt{3}+\epsilon)$-approx, while only imPSA breaks the observation. We attribute the stronger empirical performance of the offline methods to their access to the full dataset, whereas streaming algorithms must operate under strict memory constraints and can only exploit a limited subset of data points. This interpretation is further

Algorithms	Approx. Ratio	A-Gender	A-Race	S-Address	S-School	S-Sex	W-Location
OPA	$1+\sqrt{3}$	–	–	**1.356**	**1.376**	**1.349**	**0.633**
[Jones *et al.*, 2020]	3	0.212	0.232	1.726	1.668	1.663	0.749
[Kleindessner *et al.*, 2019]	5	0.226	0.226	1.702	1.578	1.834	0.749
imPSA	$1+2\sqrt{2}+\epsilon$	-	-	**1.502**	**1.623**	**1.503**	**0.595**
PSA	$1+2\sqrt{3}+\epsilon$	0.243	0.243	1.568	1.779	1.633	0.632
$3+\sqrt{3}$**-Approx**	$3+\sqrt{3}+\epsilon$	**0.234**	**0.229**	1.684	1.832	1.540	0.812
[Guo *et al.*, 2026b]	$5+\epsilon$	0.307	0.296	1.834	1.788	1.834	0.844
[Lin *et al.*, 2024]	$7+\epsilon$	0.310	0.701	1.823	1.788	1.834	0.911

Table 1: Evaluation of clustering cost for fair k-center on real-world datasets.

supported by imPSA, which uses more memory than the other streaming algorithms and correspondingly achieves better empirical performance. In addition, our results report empirical performance averaged over 10 runs on 10 simulated datasets for each group size m; the worst-case instances that drive the theoretical approximation guarantees may not arise in our experiments.

When we analyze the offline and streaming results separately, the empirical approximation ratios are consistent with the theoretical guarantees: algorithms with smaller approximation factors tend to yield smaller observed ratios, and the results remain well within the corresponding worst-case bounds. This agreement indicates strong alignment between our empirical findings and theoretical analysis. Moreover, the experimental results exhibit a similar trend as we vary the number of groups. This suggests that the empirical performance observed for $m = 2$ generalizes well to larger values of m.

The parameterized methods (i.e., OPA, PSA and imPSA) generally achieve better empirical approximation ratios compared with the baselines. In particular, OPA ($1+\sqrt{3}$-Approx) performs best among the offline algorithms, while imPSA ($1+2\sqrt{2}$-Approx) typically outperforms the generic streaming baselines (Guo *et al.*; Lin *et al.*). We reason that the parameterized algorithm spent more time and more memory space optimizing the selection of centers.

Clustering on Real World Datasets. In Table 1, we evaluate clustering cost on three real-world datasets under different fairness group settings, setting the number of centers to $k = 1\%$ of the dataset size. Under this setting, the *Adult* yields a relatively large k (about 300), whereas the *Student* and *Wholesale* datasets have k less than 10. Consistent with the simulated data, these algorithms exhibit similar empirical trends across datasets. We attribute the strong performance of our methods in part to the larger pool of available candidate centers. However, we also show that the optimal solution restricted to the candidate center set need not coincide with the best performance on the full dataset, especially when k is large. For instance, $(3+\sqrt{3})$-Approx occasionally achieves slightly lower cost than PSA on *Adult*.

Robustness. In Figure 3, we vary the fairness constraint ratio on the *Student* dataset using the *school* attribute. We start from the ratio induced by the data distribution when selecting 10% of the points as centers, which corresponds to a constraint range of $5/35$ in this dataset, and then gradually adjust the ratio until it reaches $35/5$.

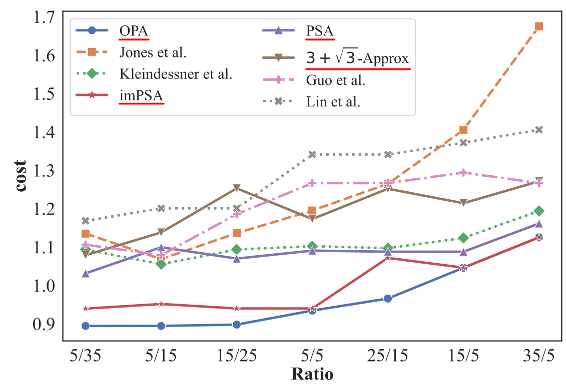

Figure 3: Cost of our algorithms (indicated by a red underline) compared with other baselines with the different balanced fair center size.

In Figure 3, we observe that all algorithms are affected by the fairness ratio: the clustering cost increases as the ratio becomes more unbalanced. We attribute this trend to the fact that our methods first construct a candidate center set without using group labels. Thus, when the required center ratio aligns with the dataset's natural group distribution, the candidate set is more likely to cover points from each group in a way that satisfies the fairness constraints, which is also representative of many real-world settings. In contrast, this experiment considers deliberately mismatched ratios that deviate from the data distribution. In this case, the cost increases, but the relative ranking of the algorithms remains similar.

6 Conclusion

In this paper, we first propose a parameterized approximation algorithm with a ratio of $1+\sqrt{3} \approx 2.732$ for the offline Euclidean fair k-center clustering problem. By applying this algorithm in the post-streaming processing stage, we obtain a streaming algorithm with a ratio of $1+2\sqrt{3}+\epsilon \approx 4.464$. These ratios can be further improved to $1+\sqrt{2} \approx 2.414$ and $1+2\sqrt{2}+\epsilon \approx 3.828$, respectively. To ensure polynomial running time, we devise another one-pass streaming algorithm that incorporates network flow techniques into the milestone algorithm of [Jones *et al.*, 2020], achieving an approximation ratio of $3+\sqrt{3}+\epsilon \approx 4.732$. This ratio can be further improved to 4.42, improving upon the previous state-of-the-art ratio 4.464 in Euclidean space due to Guo *et al.*.

Acknowledgments

This work is supported by National Natural Science Foundation of China (No. 12271098) and Key Project of the Natural Science Foundation of Fujian Province (No. 2025J02011).

References

[Agarwal and Procopiuc, 2002] Pankaj K Agarwal and Cecilia Magdalena Procopiuc. Exact and Approximation Algorithms for Clustering. *Algorithmica*, 33:201–226, 2002.

[Asuncion and Newman, 2007] Arthur Asuncion and David Newman. UCI Machine Learning Repository, 2007.

[Badanidiyuru *et al.*, 2014] Ashwinkumar Badanidiyuru, Baharan Mirzasoleiman, Amin Karbasi, and Andreas Krause. Streaming Submodular Maximization: Massive Data Summarization on the Fly. In *Proceedings of the 20th ACM Special Interest Group on Knowledge Discovery and Data Mining (KDD-14)*, pages 671–680, 2014.

[Bandyapadhyay *et al.*, 2024] Sayan Bandyapadhyay, Zachary Friggstad, and Ramin Mousavi. Parameterized Approximation Algorithms and Lower Bounds for k-Center Clustering and Variants. *Algorithmica*, pages 1–18, 2024.

[Bera *et al.*, 2022] Suman K Bera, Syamantak Das, Sainyam Galhotra, and Sagar Sudhir Kale. Fair k-Center Clustering in MapReduce and Streaming Settings. In *Proceedings of the 31st ACM Web Conference (WWW-22)*, pages 1414–1422, Virtual Event, Lyon, France, 2022.

[Ceccarello *et al.*, 2024] Matteo Ceccarello, Andrea Pietracaprina, and Geppino Pucci. Fast and Accurate Fair k-Center Clustering in Doubling Metrics. In *Proceedings of the 33rd ACM Web Conference (WWW-24)*, pages 756–767, 2024.

[Charikar *et al.*, 1997] Moses Charikar, Chandra Chekuri, Tomás Feder, and Rajeev Motwani. Incremental Clustering and Dynamic Information Retrieval. In *Proceedings of the 29th Annual ACM Symposium on Theory of Computing (STOC-97)*, pages 626–635, 1997.

[Charikar *et al.*, 2003] Moses Charikar, Liadan O'Callaghan, and Rina Panigrahy. Better Streaming Algorithms for Clustering Problems. In *Proceedings of the 35th Annual ACM Symposium on Theory of Computing (STOC-03)*, pages 30–39, 2003.

[Chen *et al.*, 2019] Xingyu Chen, Brandon Fain, Liang Lyu, and Kamesh Munagala. Proportionally Fair Clustering. In *Proceedings of the 36th International Conference on Machine Learning (ICML-19)*, pages 1032–1041. PMLR, 2019.

[Chen *et al.*, 2024] Xianrun Chen, Sai Ji, Chenchen Wu, Yicheng Xu, and Yang Yang. An Approximation Algorithm for Diversity-aware Fair k-Supplier Problem. *Theoretical Computer Science*, 983:114305, 2024.

[Chierichetti *et al.*, 2017] Flavio Chierichetti, Ravi Kumar, Silvio Lattanzi, and Sergei Vassilvitskii. Fair Clustering through Fairlets. In *Proceedings of the 31st International Conference on Neural Information Processing Systems (NeurIPS-17)*, pages 5036–5044, 2017.

[Chiplunkar *et al.*, 2020] Ashish Chiplunkar, Sagar Kale, and Sivaramakrishnan Natarajan Ramamoorthy. How to Solve Fair k-Center in Massive Data Models. In *Proceedings of the 37th International Conference on Machine Learning (ICML-20)*, pages 1877–1886, 2020.

[Feder and Greene, 1988] Tomás Feder and Daniel Greene. Optimal algorithms for approximate clustering. In *Proceedings of the twentieth annual ACM symposium on Theory of computing*, pages 434–444, 1988.

[Feldman *et al.*, 2015] Michael Feldman, Sorelle A Friedler, John Moeller, Carlos Scheidegger, and Suresh Venkatasubramanian. Certifying and Removing Disparate Impact. In *proceedings of the 21th ACM SIGKDD international conference on knowledge discovery and data mining (KDD-15)*, pages 259–268, 2015.

[Gadekar *et al.*, 2025] Ameet Gadekar, Aristides Gionis, and Suhas Thejaswi. Fair Clustering for Data Summarization: Improved Approximation Algorithms and Complexity Insights. In *Proceedings of the 34th ACM on Web Conference (WWW-25)*, pages 4458–4469, 2025.

[Gonzalez, 1985] Teofilo F Gonzalez. Clustering to Minimize the Maximum Intercluster Distance. *Theoretical computer science*, 38:293–306, 1985.

[Goyal and Jaiswal, 2023] Dishant Goyal and Ragesh Jaiswal. Tight FPT Approximation for Constrained k-Center and k-Supplier. *Theoretical Computer Science*, 940:190–208, 2023.

[Grünbaum *et al.*, 1967] Branko Grünbaum, Victor Klee, Micha A Perles, and Geoffrey Colin Shephard. *Convex Polytopes*, volume 16. Springer, 1967.

[Guha, 2009] Sudipto Guha. Tight Results for Clustering and Summarizing Data Streams. In *Proceedings of the 12th International Conference on Database Theory (ICDT-09)*, pages 268–275, 2009.

[Guo *et al.*, 2024] Longkun Guo, Chaoqi Jia, Kewen Liao, Zhigang Lu, and Minhui Xue. Efficient Constrained k-Center Clustering with Background Knowledge. *Proceedings of the 38th AAAI Conference on Artificial Intelligence (AAAI-24)*, 38(18):20709–20717, Mar. 2024.

[Guo *et al.*, 2025] Longkun Guo, Chaoqi Jia, Kewen Liao, Zhigang Lu, and Minhui Xue. Near-optimal algorithms for instance-level constrained k-center clustering. *IEEE Transactions on Neural Networks and Learning Systems*, 2025.

[Guo *et al.*, 2026a] Longkun Guo, Chaoqi Jia, and Chao Chen. Fair k-Center Clustering on Massive Social Network Data Streams. In *Proceedings of the 35th ACM Web Conference (WWW-26)*, pages 4689–4700, 2026.

[Guo *et al.*, 2026b] Longkun Guo, Zeyu Lin, Chaoqi Jia, and Chao Chen. Improved Streaming Algorithm for Fair k-Center Clustering. In *Proceedings of the 40th AAAI Conference on Artificial Intelligence (AAAI-26)*, volume 40, 2026.

[Hochbaum and Shmoys, 1985] Dorit S Hochbaum and David B Shmoys. A Best Possible Heuristic for the k-Center Problem. *Mathematics of operations research*, 10(2):180–184, 1985.

[Huang *et al.*, 2019] Lingxiao Huang, Shaofeng Jiang, and Nisheeth Vishnoi. Coresets for Clustering with Fairness Constraints. *Advances in Neural Information Processing Systems*, 32, 2019.

[Jia *et al.*, 2026a] Chaoqi Jia, Longkun Guo, Kewen Liao, Zhigang Lu, Chao Chen, and Jason Xue. Approximation Algorithm for Constrained k-Center Clustering: A Local Search Approach. *Proceedings of the 40th AAAI Conference on Artificial Intelligence (AAAI-26)*, 40(43):36982–36990, Mar. 2026.

[Jia *et al.*, 2026b] Chaoqi Jia, Weihong Wu, Longkun Guo, Zhigang Lu, Chao Chen, and Kok-Leong Ong. Optimized Algorithms for Text Clustering with LLM-Generated Constraints. *Proceedings of the 40th AAAI Conference on Artificial Intelligence (AAAI-26)*, 40(27):22229–22237, Mar. 2026.

[Jones *et al.*, 2020] Matthew Jones, Huy Nguyen, and Thy Nguyen. Fair k-Centers via Maximum Matching. In *Proceedings of the 37th International Conference on Machine Learning (ICML-20)*, pages 4940–4949. PMLR, 2020.

[Kleindessner *et al.*, 2019] Matthäus Kleindessner, Pranjal Awasthi, and Jamie Morgenstern. Fair k-Center Clustering for Data Summarization. In *Proceedings of the 36th International Conference on Machine Learning (ICML-19)*, pages 3448–3457. PMLR, 2019.

[Lin *et al.*, 2024] Zeyu Lin, Longkun Guo, and Chaoqi Jia. Streaming Fair k-Center Clustering over Massive Dataset with Performance Guarantee. In *Proceedings of the 28th Pacific-Asia Conference on Knowledge Discovery and Data Mining (PAKDD-24)*, pages 105–117. Springer, 2024.

[Mahabadi and Trajanovski, 2023] Sepideh Mahabadi and Stojan Trajanovski. Core-sets for Fair and Diverse Data Summarization. *Advances in Neural Information Processing Systems*, 36:78987–79011, 2023.

[Matthew McCutchen and Khuller, 2008] Richard Matthew McCutchen and Samir Khuller. Streaming Algorithms for k-Center Clustering with Outliers and with Anonymity. *Approximation, Randomization and Combinatorial Optimization. Algorithms and Techniques*, pages 165–178, 2008.

[Nagarajan *et al.*, 2020] Viswanath Nagarajan, Baruch Schieber, and Hadas Shachnai. The Euclidean k-Supplier Problem. *Mathematics of Operations Research*, 45(1):1–14, 2020.

[Shimon *et al.*, 1975] Shimon, Even, R., Endre, and Tarjan. Network Flow and Testing Graph Connectivity. *SIAM Journal on Computing*, 4(4):507–518, 1975.

[Wu *et al.*, 2023] Xiaoliang Wu, Qilong Feng, Jinhui Xu, and Jianxin Wang. The Fair k-Center with Outliers Problem: FPT and Polynomial Approximations. In *Proceedings of the 17th International Workshop on Frontiers in Algorithmics (FAW-23)*, pages 225–238. Springer, 2023.

[Zhou *et al.*, 2025] Jianqi Zhou, Zhongyi Zhang, Yinghui Wen, and Jiong Guo. From Metric to General Graphs: FPT Constant-Factor Approximation Algorithms for Three Location Problems. In *Proceedings of the 31st International Computing and Combinatorics Conference (COCOON-25)*, pages 297–310. Springer, 2025.